Cooperation among Access Points for Enhanced Quality of Service in Dense Wireless Environments

Josephina Antoniou*, Vicky Papadopoulou-Lesta†, Lavy Libman‡, Andreas Pitsillides§, Hooman Reisi Dehkordi‡
* School of Sciences, University of Central Lancshire Cyprus, Pyla, Cyprus
† School of Sciences, European University Cyprus, Nicosia, Cyprus
‡ School of Computer Science and Engineering, UNSW, Sydney, Australia
§ Department of Computer Science, University of Cyprus, Nicosia, Cyprus

***Abstract*—The high popularity of Wi-Fi technology for wireless access has led to a common problem of densely deployed access points (APs) in residential or commercial buildings, competing to use the same or overlapping frequency channels and causing a degradation to the user experience due to excessive interference. This degradation is partly caused by the restriction where each client device is allowed to be served only by one of a very limited set of APs (e.g. belonging to the same residential unit), even if it is within range of (or even has a better signal quality to) many other APs. In this paper, we propose a cooperative strategy to mitigate the interference and enhance the quality of service in dense wireless deployments, by having neighboring APs agree to take turns (e.g. in round-robin fashion) to serve each other's clients. We present and analyze a cooperative game-theoretic model of the incentives involved in such cooperation and identify the conditions under which cooperation would be beneficial for the participating APs.**

I. Introduction

The impressive increase in the density of wireless networks in urban residential areas, due to the low cost and easy deployment of wireless technologies, often results in unmanaged deployments of IEEE 802.11 (Wi-Fi) home networks with degraded experience for the home users. Very frequently in such deployments, an Access Point (AP) can be located within range of dozens of other APs, competing for the limited number of channels offered by the IEEE 802.11 standard [1]. Wireless APs that operate in the same geographical region without any coordination typically cause degradation to their users' experience. This is due to the fact that, in the current standards, at any given time every terminal must be rigidly associated with one particular AP. This leads to a competition between colocated APs for the same communication resource (radio channel) and a reduced quality of service due to the resulting interference.

To mitigate the problem, it would be beneficial for individual APs that are in physical proximity to each other to form *cooperative groups*, where one member of the group would serve the terminals of all group members in addition to its own terminals, provided that the signal strength of the AP is sufficient to support the user activity. The rest of the APs in the group can be silent or even turned off, thereby reducing interference. The group members can take turns at regular intervals to serve all the terminals, as long as the coverage of the group does not change. Clearly, it is important that such groups include only members whose signal strength and/or available bandwidth are sufficient to serve all group members without loss of the quality of service.

Since there is no centralized entity that can control the APs and force them to form cooperative groups, the creation of such groups must arise from a distributed process where each AP makes its own decisions independently and rationally for the benefit of itself and its terminals. Accordingly, we use game theory [7] in order to model and investigate the feasibility of this cooperative operation among APs that selfishly maximize their own individual benefits, under no central authority. We build upon and expand on earlier works where a game-theoretic approach towards reducing interference in dense wireless deployments was presented [2], [3].

In this work, we explore the *graphical strategic game* introduced by [2] in order to find those conditions that enable cooperation of the selfish APs, captured through Nash equilibria [5], [6] of the corresponding game in more general network topologies. In particular, we consider the case where the APs are located very densely. In such dense placement of APs, interference appears between almost any pair of APs. Thus, we model these placements through a corresponding directed, weighted *clique* graph; i.e. for every pair of nodes of the graph, there exists a bidirectional edge connecting them.

The weight of a (directed) edge connecting an AP to its neighbor represents the measurement of the signal power received by the neighbor when the AP is ON. This signal is treated as interference when both the AP and its neighbor transmit at the same time over the same channel, but at the same time it indicates the strength of the (useful) signal that can be used to serve the neighbor's clients when the two APs are in a cooperation agreement. The dense placements of APs are usually of relatively small size, compared to the number of clients in the area. Given the setting outlined above, it is reasonable to assume that APs can serve, during their ON transmission mode, not only their clients but also the clients of their neighbors that are in cooperation agreement with them.

It must be emphasized that cooperation between APs belonging to different service sets is not impossible even if they are required to use secure encryption/authentication (such as WPA in the 802.11 standard). At first sight, it may seem that such a cooperation would require the APs to share their secret passwords, which is of course undesirable in an uncoordinated deployment. However, there are known techniques that allow a group of cooperating APs to establish a common "trust group", and share a secret key and/or authenticate their clients without a need to expose their passwords [4].

Our contribution in this paper focuses on proving the necessary and sufficient conditions in order for all APs of the (clique) graph to join a cooperation group, where all APs maximize their users' quality of service.

II. A Graph-Theoretic Model

A. Background

1) Graph Theory: A weighted ***digraph*** $G(V, \overrightarrow{E}, \overrightarrow{W})$ consists of three types of elements, namely *nodes* constituting the set V, *edges* constituting the set $\overrightarrow{E}$, and finally the *weights* of the edges constituting the set $\overrightarrow{W}$. An edge e is specified by the *ordered* pair $e = (u, v) \in E$ iff there exists a (directional) connection (or link) *from* node u *to* node v, where $u, v \in V$ with positive weight $w(u, v) > 0$ in the graph G. $w(u, v)$ is positive if and only if $(u, v) \in \overrightarrow{E}$.

For any node v, the set Neigh_G denotes the set of (other) nodes of the graph connected to that node via a communication link, i.e., $\mathsf{Neigh}_G = \{u \in V \mid (u, v) \in \overrightarrow{E},\ (v, u) \in \overrightarrow{E}\}$. When clear from the context, we omit the graph subscript. A weighted digraph $G(V, \overrightarrow{E}, \overrightarrow{W})$ is a ***clique*** iff for every pair $u, v \in V$, there exists a pair $(u, v) \in \overrightarrow{E}$, $(v, u) \in \overrightarrow{E}$ and $w(v, u) > 0$. A graph G is said to be of size k if $|V| = k$. Throughout the paper, for an integer $n \geq 1$, denote $[n] = \{1, \ldots, n\}$. This paper is concentrated on weighted, directed clique graphs. In the following, when we refer to a clique graph, we imply a weighted, directed clique graph.

2) Game Theory: A simple (one-shot) ***strategic game*** Γ is defined by a set of players N and a set of available strategies (behaviors) for each player $i \in N$, denoted as S_i. A (pure) profile $\boldsymbol{\sigma}$ of the game Γ specifies a particular strategy $\boldsymbol{\sigma}_i$, one for each player $i \in N$. For a profile $\boldsymbol{\sigma}$, each player i has a *utility* that depends on the specific player's current strategy as well as the other players' current strategies, and is denoted as $\mathsf{U}_{\boldsymbol{\sigma}}(i)$.

A profile $\boldsymbol{\sigma}$ of the game is said to be a Nash equilibrium [5], [6] if, for every player, the utility in that profile is not smaller that the utility gained if that player unilaterally changes its strategy to any other $x_i \neq \boldsymbol{\sigma}_i$, $x_i \in S_i$. Denote as $\boldsymbol{\sigma}^{x_i}$ the new profile obtained. Then, if $\boldsymbol{\sigma}$ is a Nash equilibrium, it must be that $\mathsf{U}_{\boldsymbol{\sigma}}(i) \geq \mathsf{U}_{\boldsymbol{\sigma}^{x_i}}(i)$, for every $i \in N$.

Graphical games are games in which the player strategies (and, consequently, their utilities) are related to, and defined in terms of, a graph G. The components of the graph (i.e. its nodes and edges) are said to capture the *locality* properties among the players, or, in other words, how the game is affected by the player's positioning.

B. A Mathematical Framework

1) The Graph: We consider a set of nodes $V = \{v_1, v_2, v_3, \ldots, v_n\}$ corresponding to the set of APs located in a geographical area. For simplicity, we assume that each server serves one client. At a time t, we say that server node v (an AP) is active or ON, if it is transmitting (to its own client). Otherwise, we say that the node is inactive or OFF.

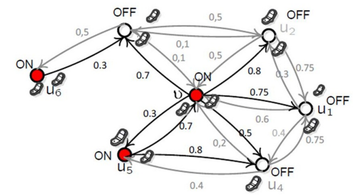

Fig. 1. An example weighted graph in operation, at some given time t. The ON nodes and their outgoing edges are highlighted.

We are concerned primarily with the decisions made by the server nodes (the APs); therefore when we refer to a node henceforth we refer to an AP.

Consider two nodes, u, v which are within range of each other's signals. In particular, node v (and its clients) may receive information from node u when it is OFF while node u is ON, i.e. when node u broadcasts. If the quality of the signal received by node v (and its clients) from node u is above some lower bound value assumed by the node v, we may consider that there exists a *directed* edge from node u to node v, denoted as (u, v). Moreover, we can quantify the quality of the received signal by having a weight $w(u, v)$ associated with the directed edge (u, v). For purposes of normalization, we define that $w(u, v) \in [0, 1]$. The weight value is analogous to the quality of the received signal, i.e. good-quality reception corresponds to a value of $w(u, v)$ close to 1. In this work these values are assumed to be known; if necessary, they can be discovered through a distributed process initiated and executed locally at any AP.

Summing up, from this setting of APs and the communication of their clients, we derive the following weighted graph:

Definition 1: (**The weighted digraph**) [2] Consider a set of nodes (APs) $V = \{v_1, \cdots, v_n\}$ located in a geographical area. Consider two nodes u and v such that (the clients of) node v can receive transmissions from node u when (the server of) v is OFF while node u is ON, with a quality above some lower bound value. Then, we define a directed edge to exist from node u to node v, denoted as (u, v), of a positive weight $w(u, v) \in (0, 1)$ representing the quality of the received signal at node v.

We further assume that if $w(u, v) > 0$, then $w(v, u) > 0$ for all nodes $v, u \in V$.

An example weighted graph is illustrated in Figure 1.

2) Time: As in [2], a basic unit of time period T is considered, e.g. 1 hour, and we split the time period T into x smaller time slots $T_1, T_2, \ldots T_x$ such that for each $T_k \in T$ there exists at least one node that in a group of nodes that may alternate between the ON and OFF node and remains ON/OFF for the whole time slot T_k. So, $\bigcup_{T_k} = T$ and $T_k \bigcap T_l = \emptyset, k \neq l$, i.e. the sum of the time slots is the time period T and no two time slots overlap. By $|T_k|$ we denote the time elapsed from the beginning of time slot T_k until the end of the time slot T_k.

Fix a time slot T_k. Then, we denote: $\mathsf{ON}(T_k) = \{v \in V \mid \text{ node } v \text{ is } \mathsf{ON} \text{ in time slot } T_k\}$ and $\mathsf{OFF}(T_k) = \{v \in V \mid \text{node } v \text{ is } \mathsf{OFF} \text{ in time slot } T_k\}$. So, for node v the time period T can be partitioned into two sets

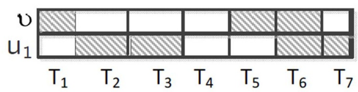

Fig. 2. An example of a time split for nodes v, u_1. In the shaded areas the corresponding APs are ON.

$\mathsf{ON\_Time}_T(v) = \{T_k \in T \mid v \in \mathsf{ON}(T_k)\}$ and $\mathsf{OFF\_Time}_T(v) = \{T_k \in T \mid v \in \mathsf{OFF}(T_k)\}$. Denote $\mathsf{Mode}_T(v) = \{\mathsf{ON}_T(v), \mathsf{OFF}_T(v)\}$.

For the example graph of Figure 1, $\mathsf{ON}(t) = \{v, u_5, u_6\}$. For a sample time split of nodes v, u_1 see Figure 2.

3) (Non-)Cooperative Neighbors: Given a weighted digraph G, for any node $v \in V$, $\mathsf{Neigh}(v)$ denotes the set of neighbors of node v in G, i.e., $\mathsf{Neigh}(v) = \{u \in V \mid (u, v) \text{ and } (v, u) \in \overrightarrow{E}\}$. For the example graph of Figure 1, $\mathsf{Neigh}(v) = \{u_1, u_2, u_3, u_4, u_5\}$. Within a given time slot T, node v may be in *agreement* or in *cooperation* with some of its neighboring nodes. For any node v, being in agreement with node u, $u \neq v$ means that: (*i*) node v (or u) transmits *only* when u (respectively, v) does not transmit; and (*ii*) node v (u) serves the client(s) of the other node, in addition to its own client(s), during the time that it (v or u, respectively) is ON. The set of the neighbors of node v that are in agreement during time T, is denoted by $\mathsf{Coop}_T(v) \subseteq \mathsf{Neigh}(v)$. Thus, $\mathsf{ON\_Time}_T(v) \cap \mathsf{ON\_Time}_T(u) = \emptyset$. On the other hand, the set of neighbors with which v is not in agreement with is denoted as $\mathsf{NCoop}_T(v)$, where $\mathsf{NCoop}_T(v) \subseteq \mathsf{Neigh}(v)$ and it may be that $\mathsf{OFF\_Time}_T(v) \cap \mathsf{OFF\_Time}_T(u) \neq \emptyset$.

4) Experienced Quality: As in [2], for any node $v \in V$, the quality experienced by a node's clients can be quantified through measurements of the signal strength received at the clients of node v, at any time slot T_k during the time period T and it is denoted as $\mathsf{QoE}_{T_k}(v)$. There are two cases: when the node is ON and when it is OFF.

ON Operation: During any time slot T_k, if a node is ON, there are two possibilities: (*i*) none of its neighbors transmit at that time slot or (*ii*) some of them do. In the first case,

$$\mathsf{QoE}_{T_k}(v) = 1. \tag{1}$$

In the second case,

$$\mathsf{QoE}_{T_k}(v) = \max\{0, 1 - \sum_{\substack{u \in \mathsf{Neigh}_T(v) \\ u \in \mathsf{ON}(T_k)}} w(u, v)\} \tag{2}$$

OFF Operation: When node v is OFF, the quality of the signal received at node v's clients, and hence the quality experienced, depends on the number of neighbors in cooperation with node v (i.e. in the set $\mathsf{Coop}_{T_k}(v)$) that are ON at time T_k, and are serving the clients of node v as well as their own clients. If there exists only one such neighboring node u, the quality of the signal received at node v is captured by the weight $w(u, v)$ of edge (u, v). However, if there exist more than one neighboring nodes not in cooperation with node v that are ON at the same time, this results in interference received at node v, degrading the experienced quality at the node's clients. Thus, for the case of the OFF operation of node v its experienced quality can be denoted as:

$$\mathsf{QoE}_{T_k}(v) = \max\{0, (w(m, v) - \sum_{\substack{u \in \mathsf{Neigh}_T(v), u \neq m \\ u \in \mathsf{ON}(T_k)}} w(u, v))\}, \tag{3}$$

where $m = \arg\max_{\substack{u \in \mathsf{Coop}_T(v) \\ u \in \mathsf{ON}(T_k)}} \{w(u, v)\}$.

C. The Graphical Game

Using the above mathematical framework, we now formally define the resulting strategic game. This is a one-shot game where the players are the nodes (APs). In any profile, given the decisions of the players (namely, whether to operate in ON or OFF mode) during each time slot $T_k \in T$, the utility of player $v \in V$ is equal to the quality of experience of its clients, i.e., $\mathsf{QoE}_{T_k}(v)$. A formal definition of the game follows:

Definition 2: Consider an one-shot strategic game Γ played on a weighted digraph $G(V, \overrightarrow{E}, \overrightarrow{W})$. The set of players of the game is V. A profile $\boldsymbol{\sigma}$ of the game is associated with the basic time period T of the scenario described. T is split into time slots $T_1, T_2, \ldots T_x$, such that $\bigcup_{T_k} = T$ and $T_k \bigcap T_l = \emptyset, k \neq l$, T_k corresponding to the time slot allowing alterations between ON and OFF operations of the nodes.

The strategy of any player (node) $v \in V$ in a profile $\boldsymbol{\sigma}$ is defined as follows:

$$\boldsymbol{\sigma}_v = (\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(v), \mathsf{OFF\_Time}_{\boldsymbol{\sigma}}(v), \mathsf{Coop}_{\boldsymbol{\sigma}}(v)),$$

where $\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(v) = \{T_k \in T | v \in \mathsf{ON}_{\boldsymbol{\sigma}}(T_k)\}$, $\mathsf{OFF\_Time}_{\boldsymbol{\sigma}}(v) = \{T_k \in T | v \in \mathsf{OFF}_{\boldsymbol{\sigma}}(T_k)\}$, and $\mathsf{ON}_{\boldsymbol{\sigma}}(T_k)$ ($\mathsf{OFF}_{\boldsymbol{\sigma}}(T_k)$) denotes the set of nodes in V that are in ON (OFF) operation during T_k according to $\boldsymbol{\sigma}$. $\mathsf{Coop}_{\boldsymbol{\sigma}}(v) \subseteq \mathsf{Neigh}(v)$ is the set of neighboring nodes of node v, with which node v has decided to cooperate with in $\boldsymbol{\sigma}$. Cooperation means that for each such cooperative neighbor u of node v, (*i*) $\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(v) \cap \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(u) = \emptyset$ and (*ii*) the two nodes are in agreement to serve each other's clients.

For player (node) v denote $\mathsf{MaxCoop}_{\boldsymbol{\sigma}}(T_k, v) = \{m \in \mathsf{Neigh}(v) | w(m, v) = \max_{\substack{u \in \mathsf{Coop}_{\boldsymbol{\sigma}}(v) \\ u \in \mathsf{ON}_{\boldsymbol{\sigma}}(T_k)}} w(u, v)\}$. Then, the utility of player (node) v corresponding to the QoE for the ON/OFF operations of equations (1), (2), (3), is given by:

$$\begin{aligned} \mathsf{U}_{\boldsymbol{\sigma}}(v) = & \sum_{T_k \in \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(v)} \mathsf{ONU}_{\boldsymbol{\sigma}}(v, T_k) \cdot |T_k| \\ & + \sum_{T_k \in \mathsf{OFF\_Time}_{\boldsymbol{\sigma}}(v)} \mathsf{OFFU}_{\boldsymbol{\sigma}}(v, T_k) \cdot |T_k| \end{aligned} \tag{4}$$

where the quality experienced for the ON operation of the node is determined by equations (1) and (2):

$$\mathsf{ONU}_{\boldsymbol{\sigma}}(v, T_k) = \max\left\{0, 1 - \sum_{\substack{u \in \mathsf{Neigh}(v) \\ u \in \mathsf{ON}_{\boldsymbol{\sigma}}(T_k)}} w(u, v)\right\} \tag{5}$$

and similarly

$$\begin{aligned} \mathsf{OFFU}_{\boldsymbol{\sigma}}(v, T_k) = \max\{0, (w(\mathsf{MaxCoop}_{\boldsymbol{\sigma}}(T_k, v), v) \\ - \sum_{\substack{u \in \mathsf{Neigh}(v) \\ u \in \mathsf{ON}_{\boldsymbol{\sigma}}(T_k) \\ u \neq \mathsf{MaxCoop}_{\boldsymbol{\sigma}}(T_k, v)}} w(u, v))\} \end{aligned} \tag{6}$$

III. Agreement Equilibria for Clique Networks

In this section, we consider only game instances Γ where the graph G is a clique of size q. For simplicity, in this section, any time we refer to a graph we imply a clique graph.

A. Agreement Profiles

We now introduce and analyze profiles where all players of the graph are in cooperation with each other. Specifically, we first formally define:

Definition 3: A profile $\boldsymbol{\sigma}$ is called ***agreement*** if all nodes of the graph G agree to cooperate with each other in $\boldsymbol{\sigma}$.

Applying agreement profiles on clique graphs, we observe:

Observation 1: **Agreement profiles on clique graphs.** Consider an agreement profile $\boldsymbol{\sigma}$ in the game Γ, over a clique graph G of size q. Then, consider any time slot $T_k \in \mathsf{ON}_{\boldsymbol{\sigma}}(T)$ and the corresponding (single) node k which is ON during T_k. Thus, by equation (5), $\mathsf{ONU}_{\boldsymbol{\sigma}}(k, T_k) = 1$. Also, the node k serves all other nodes of the clique during the time T_k. Thus, for any player $j \neq k$ (who is therefore OFF during time slot T_k), $\mathsf{OFFU}_{\boldsymbol{\sigma}}(j, T_k) = w(k, j)$.

Next we prove that when discussing agreement profiles, we may, equivalently and without loss of generality, refer to the simpler subclass of the fully allocated agreement profiles.

We first define the following equivalence relation:

Definition 4: A profile $\boldsymbol{\sigma}$ is said to be ***equivalent*** to another profile $\boldsymbol{\sigma}'$ if each player $i \in [q]$ has $\mathsf{U}_{\boldsymbol{\sigma}}(i) = \mathsf{U}_{\boldsymbol{\sigma}'}(i)$.

We prove:

Claim 1: Consider any agreement profile $\boldsymbol{\sigma}$ of a game Γ over a clique graph G of size q. Then there exists an ordered agreement profile $\boldsymbol{\sigma}'$ such that $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$ are equivalent.

Proof: Consider any player i in the profile $\boldsymbol{\sigma}$. Note that the time slots in which the player is ON may not be continuous. Let $T_{i_1}, \ldots, T_{i_x}$ be the time slots where the player i is ON in $\boldsymbol{\sigma}$. Since $\boldsymbol{\sigma}$ is an agreement profile, $\mathsf{ONU}_{\boldsymbol{\sigma}}(i, \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)) = 1$, and for each $t_k \in \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(k)$, $k \neq i$, $\mathsf{OFFU}_{\boldsymbol{\sigma}}(i, t_k) = w(k, i)$. Thus, from equation (4) and Observation 1, we obtain:

$$\begin{aligned}\mathsf{U}_{\sigma}(i) &= \mathsf{ONU}_{\boldsymbol{\sigma}}(i, \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)) \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)| \\ &\quad + \sum_{\substack{t_k \in \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(k) \\ k \in [q]\setminus i}} \mathsf{OFFU}_{\boldsymbol{\sigma}}(i, t_k) \cdot |t_k| \\ &= 1 \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)| + \sum_{k \in [q]\setminus i} w(k, i) \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(k)| \qquad (7)\end{aligned}$$

Now, construct a modified profile $\boldsymbol{\sigma}'$ such that for each player $i \in [q]$, we set its ON mode to a *single* time slot T_i which is equal to the total duration of its ON time slots in $\boldsymbol{\sigma}$; i.e., $|T_i| = |T_{i_1}| + |T_{i_2}| + \cdots + |T_{i_x}|$. Moreover, we start the ON time slot of the first player at time 0 and continue until time $|T_1|$, for the second player we start its (single) time slot at time $|T_1| + 1$ and continue until time $|T_1| + |T_2|$, and so on, so that player k is ON only during the time slot T_k and $|T_k| = |T_{k_1}| + |T_{k_2}| + \cdots + |T_{k_x}|$.

Note that, by construction, $\boldsymbol{\sigma}'$ is ordered, and it is an agreement profile (since the time slots allocated to different players are non-overlapping). Moreover, $\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(i) = \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)$ for each $i \in [q]$. It follows that for any player $i \in [q]$, $\mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i) = 1$.

Also, note that for each time slot $T_k \in T\setminus T_i$, only player k is ON and serves all other players. Thus, during the time slot T_k, player i (which is in OFF mode) is served by player k. So, $\mathsf{OFFU}_{\boldsymbol{\sigma}}(i, T_k) = w(k, i)$. It follows that,

$$\begin{aligned}\mathsf{U}_{\boldsymbol{\sigma}'}(i) &= \mathsf{ONU}_{\boldsymbol{\sigma}'}(i, \mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(i)) \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(i)| \\ &\quad + \sum_{\substack{T_k \in \mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(k) \\ k \in [q]\setminus i}} \mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k) \cdot |T_k| \\ &= 1 \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(i)| + \sum_{k \in [q]\setminus i} w(k, i) \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(k)|.\end{aligned}$$

Since $|\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(i)| = |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)|$ and for each player $k \in [q]\setminus i$, $|\mathsf{ON\_Time}_{\boldsymbol{\sigma}'}(k)| = |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(k)|$, the above equation becomes,

$$\mathsf{U}_{\boldsymbol{\sigma}'}(i) = 1 \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)| + \sum_{k \in [q]\setminus i} w(k, i) \cdot |\mathsf{ON\_Time}_{\boldsymbol{\sigma}}(k)| \qquad (8)$$

By equations (7) and (8), it follows that $\mathsf{U}_{\boldsymbol{\sigma}'}(i) = \mathsf{U}_{\boldsymbol{\sigma}}(i)$. ■

We next prove a useful property of agreement profiles:

Claim 2: Assume an agreement profile $\boldsymbol{\sigma}$ for the game Γ over a clique graph G of size q. Then, for any player $i \in [q]$,

$$\mathsf{U}_{\boldsymbol{\sigma}}(i) = |T_i| + \sum_{T_k \in T\setminus T_i} w(k, i) \cdot |T_k|.$$

Proof: By Claim 1, assume without loss of generality that $\boldsymbol{\sigma}$ is an ordered agreement profile (equivalently, if $\boldsymbol{\sigma}'$ is the equivalent ordered agreement profile, then set $\boldsymbol{\sigma} = \boldsymbol{\sigma}'$ without changing the utility values). Consequently, node i is the only node in ON operation during time slot T_i. Thus, by equation (5), $\mathsf{ONU}_{\boldsymbol{\sigma}}(i, T_i) = 1$. Also, the node is OFF for the rest of the time and during any other time slot $T_k \in T\setminus T_i$ it is served by node k which is the only one node on ON operation during time slot T_k, for each $T_k \in T\setminus T_i$. Thus, by equation (6), $\mathsf{ONU}_{\boldsymbol{\sigma}}(i, T_k) = w(k, i)$ for each $T_k \in T\setminus T_i$. We therefore obtain, by equation (4),

$$\begin{aligned}\mathsf{U}_{\boldsymbol{\sigma}}(i) &= \sum_{T_k \in \mathsf{ON\_Time}_{\boldsymbol{\sigma}}(i)} \mathsf{ONU}_{\boldsymbol{\sigma}}(i, T_k) \cdot |T_k| \\ &\quad + \sum_{T_k \in \mathsf{OFF\_Time}_{\boldsymbol{\sigma}}(i)} \mathsf{OFFU}_{\boldsymbol{\sigma}}(i, T_k) \cdot |T_k| \\ &= 1 \cdot |T_i| + \sum_{T_k \in T\setminus T_i} w(k, i) \cdot |T_k|,\end{aligned}$$

as claimed. ■

Claim 3: Assume an agreement profile $\boldsymbol{\sigma}$ of a game Γ over a clique graph G of size q. Then no player can increase its utility by unilaterally decreasing its ON time and increasing its OFF time.

Proof: By Claim 1, assume without loss of generality that $\boldsymbol{\sigma}$ is an ordered agreement profile. Consider any player $i \in [q]$. Then by Claim 2, the node gets a utility of

$$\mathsf{U}_{\boldsymbol{\sigma}}(i) \quad = \quad |T_i| + \sum_{T_k \in T\setminus T_i} w(k, i) \cdot |T_k|. \qquad (9)$$

If the player i increases its OFF period by some $|t| > 0$, it follows that its new ON time slot will be $|T_i'| = |T_i| - |t|$, thus $t \leq |T_i|$. Let $\boldsymbol{\sigma}'$ be the resulting profile. Since $\boldsymbol{\sigma}$ is an agreement profile, there is no other player that is ON during time period t. So, its $\mathsf{OFFU}_{\boldsymbol{\sigma}'}(i)$ remains the same as in $\boldsymbol{\sigma}$.

Moreover, its $\mathsf{ONU}\boldsymbol{\sigma}'(i)$ must be decreased by t. Summing up, in the resulting profile its new utility becomes:

$$\mathsf{U}_{\boldsymbol{\sigma}'}(i) = \mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i') \cdot |T_i'| + \sum_{T_k \in T \setminus T_i'} \mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k) \cdot |T_k|$$
$$= 1 \cdot (|T_i| - |t|) + \sum_{T_k \in T \setminus T_i'} w(k, i) \cdot |T_k|$$
$$= -|t| + \left(|T_i| + \sum_{T_k \in T \setminus T_i} w(k, i) \cdot |T_k| \right) = -|t| + \mathsf{U}_{\boldsymbol{\sigma}}(i).$$

Since $t > 0$, it follows that $\mathsf{U}_{\boldsymbol{\sigma}}(i) > \mathsf{U}_{\boldsymbol{\sigma}'}(i)$ so that the player does not increase its utility by unilaterally increasing its OFF mode operation. ■

B. Necessary Conditions for Nash equilibria

Next we prove a necessary condition for an agreement profile to be a Nash equilibrium of the game.

Proposition 1: Consider an agreement profile $\boldsymbol{\sigma}$ for a game Γ over a clique graph G of size q. Then, if $\boldsymbol{\sigma}$ is a Nash equilibrium then for any of the players $i, j \in [q]$, it holds that $w(i, j) \geq \frac{1}{2}$.

Proof: Recall that by Claim 1, we may assume that $\boldsymbol{\sigma}$ is an agreement, continuous $\mathsf{ON/OFF}$ time slots profile. Also, by Claim 2, the utility of player i is:

$$\mathsf{U}_{\boldsymbol{\sigma}}(i) \quad = \quad |T_i| + \sum_{T_k \in T \setminus T_i} w(k, i) \cdot |T_k|. \tag{10}$$

Since $\boldsymbol{\sigma}$ is a Nash equilibrium, no player that can increase its utility by unilaterally increase its ON time period. Assume now, by way of contradiction, that there exists a player $i \in [q]$, such that $w(j, i) < \frac{1}{2}$ for some player $j \in [q]$. Assume now that the player i increases its ON time period to:

$$|\mathsf{ON\_Time}_{\sigma'}(i)| = |T_i'| = |T_i| + |t_j|,$$

where $t_j \in T_j$. Denote the resulting profile by $\boldsymbol{\sigma}'$. Recall that node j was the only node in ON mode at the time slot T_j. Thus, during time t_j where the node i switches to be ON, the utility of node i is decreased due to the interference caused by node j. In particular, by equation (5), $\mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i) = 1$ while $\mathsf{ONU}_{\sigma'}(i, t_j) = (1 - w(j, i))$.

Moreover, observe that in $\boldsymbol{\sigma}'$ the player is OFF for a duration of time equal to $|\mathsf{OFF\_Time}_{\sigma'}(i)| = \sum_{T_k \in T \setminus T_i} |T_k| - |t_j|$. For each $T_k \in T \setminus \{T_i \cup T_j\}$ where the node i is OFF and served by node k, by equation (6), its utility is $\mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k) = w(k, i)$. During the decreased time period $T_j \setminus t_j$, it also gets $\mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_j \setminus t_j) = w(k, i)$.

In summary,

$$\mathsf{U}_{\boldsymbol{\sigma}'}(i) = \mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i') \cdot |T_i'| + \sum_{T_k \in T \setminus T_i'} \mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k) \cdot |T_k|$$
$$= 1 \cdot |T_i| + (1 - w(j, i)) \cdot |t_j| + \sum_{T_k \in T \setminus T_i \cup T_j} w(k, i) \cdot |T_k|$$
$$+ w(j, i) \cdot (|T_j| - |t_j|) = |t_j| + \mathsf{U}_{\sigma}(i) - 2w(j, i) \cdot |t_j|, \tag{11}$$

by equation (10).

Since $\boldsymbol{\sigma}$ is a Nash equilibrium, it must be that

$$\mathsf{U}_{\sigma}(i) \geq \mathsf{U}_{\sigma'}(i) = \mathsf{U}_{\sigma}(i) - 2w(j, i) \cdot |t_j| + |t_j|,$$

by equation (11). Consequently, $2w(j, i) \cdot |t_j| \geq |t_j|$, therefore $w(j, i) \geq \frac{1}{2}$, which is a contradiction since $w(j, i) < \frac{1}{2}$ by assumption. The proposition is therefore proved. ■

C. Sufficient Conditions for Nash equilibria

In this section, we prove a sufficient condition for an agreement profile to be a Nash equilibrium.

Theorem 1: Assume an agreement profile $\boldsymbol{\sigma}$ for the game Γ over a clique graph G of size q. If $w(i, j) \geq \frac{1}{2}$, for every pair $i, j \in [q]$, then $\boldsymbol{\sigma}$ is a Nash equilibrium.

Proof: Again, by Claim 1, we may assume without loss of generality that $\boldsymbol{\sigma}$ is an ordered agreement profile. By Claim 2, the utility of player i is:

$$\mathsf{U}_{\boldsymbol{\sigma}}(i) \quad = \quad |T_i| + \sum_{T_k \in T \setminus T_i} w(k, i) \cdot |T_k|. \tag{12}$$

Assume now that $\boldsymbol{\sigma}$ is *not* a Nash equilibrium. Then, for at least one player $i \in [q]$, there exists an alternation of its $\mathsf{ON/OFF}$ time slots so that its utility is increased compared to $\boldsymbol{\sigma}$. Note first that by Claim 3, the node can not gain more by increasing its OFF time period. Accordingly, assume that the player unilaterally increases its ON time period as follows: $T_i' = T_i \bigcup_{\substack{t_k \in T_k, \\ T_k \in T \setminus T_i}}$ where $t_k \geq 0$ for all k, and $t_k > 0$ for at least one k. Thus,

$$|\mathsf{ON\_Time}_{\sigma'}(i)| = |T_i'| = |T_i| + \sum_{\substack{t_k \in T_k, \\ T_k \in T \setminus T_i}} |t_k|.$$

Recall that, in $\boldsymbol{\sigma}$, each node $k \in [q]$ is the only node in ON mode at the time slot T_k. Thus, during time $t_k \in T_k$ where the node i switches to ON operation in $\boldsymbol{\sigma}'$, the node gets decreased utility due to the interference received by node k. In particular, by equation (5), $\mathsf{ONU}_{\boldsymbol{\sigma}'}(i, t_k) = 1 - w(k, i)$ for each such $t_k \in T_k$, while $\mathsf{ONU}_{\sigma'}(i, T_i) = 1$.

Moreover, observe that in $\boldsymbol{\sigma}'$ the player is OFF for a duration of time equal to $|\mathsf{OFF\_Time}_{\sigma'}(i)| = \sum_{T_k \in T \setminus T_i} (|T_k| - |t_k|)$. Thus, equation (4) becomes:

$$\mathsf{U}_{\boldsymbol{\sigma}'}(i) = \mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i') \cdot |T_i'| + \sum_{T_k \in T \setminus T_i'} \mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k) \cdot |T_k|$$
$$= \mathsf{ONU}_{\boldsymbol{\sigma}'}(i, T_i) \cdot |T_i| + \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} \mathsf{ONU}_{\sigma'}(i, t_k) \cdot |t_k|$$
$$+ \sum_{T_k \in T \setminus T_i} \mathsf{OFFU}_{\boldsymbol{\sigma}'}(i, T_k \setminus t_k) \cdot (|T_k| - |t_k|)$$
$$= 1 \cdot |T_i| + \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} (1 - w(k, i)) \cdot |t_k|$$
$$+ \sum_{T_k \in T \setminus T_i} w(k, i) \cdot (|T_k| - |t_k|)$$
$$= |T_i| + \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} |t_k| - 2 \cdot \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} w(k, i) \cdot |t_k|$$
$$+ \sum_{T_k \in T \setminus T_i} w(k, i) \cdot |T_k| \tag{13}$$

Since, by assumption, $\boldsymbol{\sigma}$ is not a Nash equilibrium, $\mathsf{U}_{\sigma'}(i) > \mathsf{U}_{\sigma}(i)$. Thus, by equations (12) and (13), combined with $\mathsf{U}_{\sigma}(i) < \mathsf{U}_{\sigma'}(i)$, it must be that

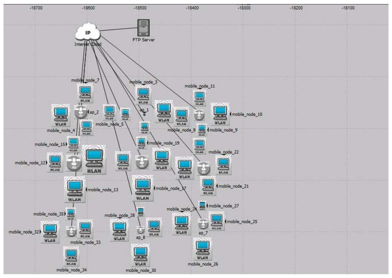

Fig. 3. The 9-AP configuration used by the simulation scenarios.

$$|T_i| + \sum_{T_k \in T \setminus T_i} w(k,i) \cdot |T_k| < |T_i| + \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} |t_k|$$

$$- 2 \cdot \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} w(k,i) \cdot |t_k| + \sum_{T_k \in T \setminus T_i} w(k,i) \cdot |T_k|$$

It follows that $2 \cdot \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} w(k,i) \cdot |t_k| < \sum_{\substack{t_k \in T_k \\ T_k \in T \setminus T_i}} |t_k|$, which is a contradiction, since for all $i, k \in [q]$, $w(k,i) \geq \frac{1}{2}$. Therefore, $\boldsymbol{\sigma}$ is indeed a Nash equilibrium. ■

D. A Characterization for Nash equilibria

Proposition 1 implies that the condition $w(i,j) \geq \frac{1}{2}$, for any pair $i, j \in [q]$ is a necessary condition in order an agreement profile to be a Nash equilibrium. Moreover, Theorem 1 implies that the condition is also sufficient. Thus,

Corollary 1: An agreement profile of the game Γ over a clique graph G of size q is a Nash equilibrium if and only if $w(i,j) \geq \frac{1}{2}$, for all pairs $i, j \in [q]$.

IV. Simulation Evaluation

In this section, we demonstrate the potential benefits of cooperation by simulation. Specifically, we implement the scenario of densely-deployed APs in a widely used network simulator (OPNET) and compare the performance, in terms of signal-to-noise ratio (SNR), between the default (non-cooperative) case where all APs serves their own clients independently at the same time, and the cooperative case where all clients are served by one AP at a time.

The following scenarios are statically configured, i.e. the configuration of nodes does not change throughout the simulation (Figure 3). A single content-generating server in the background is assumed to be connected via a fast wired backhaul router to all the APs. At a time of 100 sec after the start of the simulation, all the nodes start retrieving content from the server via FTP. All the wireless connections between the APs and the clients use standard WiFi at a maximum 802.11g rate (54Mbps). All the active APs are configured to use the same resource (Channel 1).

Using this configuration, we study two separate scenarios: where the wireless clients are served by their own APs (default); and where they are served by the central AP (cooperative). Figures 4 and 5, respectively, present the SNR values recorded for each of the 9 players in these scenarios. We observe that the SNR shows a sizeable increase (more for some users than others), clearly indicating the reduced interference

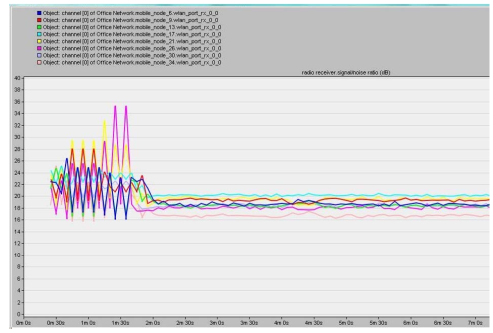

Fig. 4. The SNR values of the 9 users in a non-cooperative scenario.

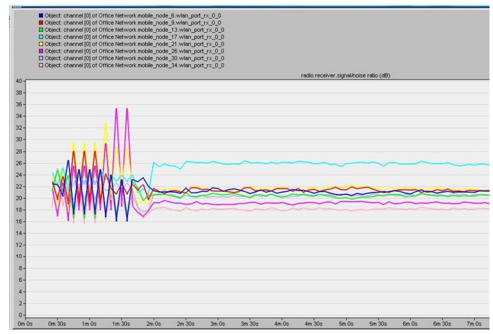

Fig. 5. The SNR values of the 9 users in a cooperative scenario.

in the environment, since the signal of the serving AP was not modified in any way.

V. Conclusion and Future Work

We considered a method to mitigate the interference caused by individual wireless Access Points (AP) located in a dense area, using cooperation such that the AP serve each other's clients at different times. We modeled the situation using a graphical game, particularly focusing on the case where the underlying graph is a clique with heterogeneous edge weights. We characterized the conditions under which agreements between all APs to jointly serve each other's clients are possible and achieve the maximum benefit for their clients. The implementation and detailed evaluation of a practical cooperation protocol is the subject of ongoing work.

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