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Reducing the Number of Decision Variables in Ready Mixed Concrete for Optimally Solving Small Instances in a Practical Time

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Abstract: In this paper a new formulation is presented for optimally solving single depot Ready Mixed Concrete (RMC) with a homogeneous fleet. Most introduced models in the literature deal with individual deliveries; however, the proposed model is based on customers' specifications. This results in a reduction in the number of constraints as well as in the number of variables. Therefore, the solution space of the proposed formulation is smaller than similar ones introduced in the literature. Consequently, it can be solved faster than similar formulations, which provides an opportunity for optimally solving small to medium sized RMC problems. In practice, this formulation can be used for solving resource allocation tasks in RMC.

1. Introduction

Resource allocation problems arise in many civil engineering projects. For Ready Mixed Concrete (RMC) dispatching, a few models have been introduced to optimally allocate the available resources in order to minimize the cost function. The RMC dispatching problem involves routing and assignment decisions that are made jointly (Asbach et al., 2009, Chen et al., 2009, Maghrebi et al., 2013b, Schmid et al., 2010), and is remotely similar to the Vehicle Routing Problem (VRP) which is known to be a NP-hard problem (Chen et al., 2009, Lenstra and Kan, 1981, Golden and Wong, 1981, Dethloff, 2001, Solomon, 1987, Bianchi et al., 2006, Min, 1989, Cordeau et al., 2002, Dullaert et al., 2002, Beullens et al., 2003). This means that with the available computing facilities it cannot solve medium and large scale RMC dispatching problems in polynomial time (Maghrebi et al., 2014, Maghrebi et al., 2013a). In this paper, we present a novel Integer Programming (IP) model for a particular variant of the RMC dispatching problem. In the studied variant, we focus on a case where no time restrictions are imposed on the dispatching schedule and where the vehicle fleet is homogeneous. In the proposed formulation, the number of integer decision variables is significantly lower than the one required by other existing models. Our approach attempts to reduce the number of decision variables by deriving valid properties of the model as well as the number of constraints to have a manageable solution space. Finally, we propose an exact Mixed Integer Linear Programming (MILP) reformulation that can be solved by commercial optimization software and which might provide an opportunity for optimally solving small instances in a reasonable time. This paper comprises three main sections, excluding the introduction. First, the related introduced models are discussed in detail. Second, the new formulation is presented. Finally, the reduction in the size of problem is analytically proved.

2. Related Works

In this section the introduced mathematical models are discussed although we will not consider the implemented heuristic solution for each model. Only a few mathematical models have been introduced for the RMC dispatching problem and most of these were developed in the past five years. From a general point of view, we can divide the introduced models into two categories: (i) models for a single depot, and (ii) models for multi-depots. The first category is for small to medium sized RMCs which only have an active batch plant and an assumed homogeneous fleet. In the second category there are multiple batch plants (depots) for mixing fresh concrete and loading it into trucks, and typically a wide range of trucks is available in their fleets. The number of decision variables in the models of the second category is much higher than for the ones in the first category. Furthermore, the introduction of time restrictions on the schedule of trucks requires additional variables. In this paper, a new model for the single depot RMC dispatching problem is presented. The main contribution of this approach is to reduce the number of decision variables and consequently provide a lighter formulation that can be used to solve some variants of the RMC dispatching problem optimality in a reasonable time.

Form the first category, Feng and Wu (2000) introduced a single depot model by focusing on minimizing idle times; this model was developed to be solved heuristically because there are few *IF* in the constraints. Feng and Wu introduced a more advanced model in 2006 (Feng and Wu, 2006). Naso et al. (2007) introduced one the most advanced RMC models so far. It can cover a multi-depots RMC but with a homogeneous fleet by considering multi objectives. Their model can take into account the hired trucks as well as the out-sourced deliveries. This model deals with deliveries to a customer and the assigned truck/s and depot/s for each delivery. The only drawback of this model is the large number of decision variables as well as the number of side constraints. All the decision variables are binary; therefore, computing time is a challenge in this model when the optimum solution is desirable. Yan et al. (2008) presented a new formulation for a single depot with a homogeneous fleet; similar to (Naso et al., 2007), it splits a customer depending on the number of required deliveries. Naso et al. A wide variant of RMC formulation were introduced by Yan and his colleagues; such as when the overtime is considered (Yan and Lai, 2007), or covering the incidents (Yan et al., 2012) and also associating stochastic travel times (Yan et al., 2012). Lin et al. (2010) presented a new model by focusing on minimizing the waiting time when there is uncertainty in demand. They assumed RMC dispatching as a job shop problem when the construction site represents a job and trucks correspond with a workstation. This model can be used for a single depot with a heterogeneous fleet. Another model in this context was presented by Schmid et al. (2009) for a single depot with a heterogeneous fleet. Their model forces MIP to avoid unsupplied customers by penalizing the unsatisfied customers in the objective function; later on a new version of this model was introduced in 2010 (Schmid et al., 2010). Asbach et al. (2009) introduced a novel model whose structure is much simpler than that of other introduced models and which can be used for modelling multi-depots and a heterogeneous fleet. In this formulation, a depot is divided into a set of sub-depots based on the number of possible loadings at that depot. Similarly, a customer is divided into a set of sub-customers according to the number of required deliveries. According to these assumptions, their formulation is no longer needed to deal with customers who require more than one delivery, which results in a reduction in the number of side constraints. However, as a result of replacing the customers with deliveries, the number of decision variables is increased.

In summary, there are two challenges to be met in order for the aforementioned models to be solved optimally. First is the number of decision variables; second is number of side constraints. These models may fail to solve even small instances with available computing facilities in practical time (Cordeau et al., 2006). For this reason, so far all the introduced models have been solved heuristically for large instances (Asbach et al., 2009, Maghrebi et al., 2014) . In this paper, we introduce a new mathematical model that can solve small to medium sized RMC problems with a single depot and homogeneous vehicle fleet faster than similar methods.

3. Problem Formulation

In this section, we present a Mixed-Integer Linear Programming (MILP) model to solve the RMC Dispatching Problem (RDP). The RDP seeks to find the optimal routing of a set of vehicles to service a set of customers, such that transportation costs are minimized. Hence the RDP combines routing and assignment decisions which need to be addressed simultaneously in order to achieve optimal solutions.

3.1. Model Assumptions

The following notation is used in this paper:

N	set of nodes
A	set of links
C	set of customers' nodes
D	set of depot nodes
N_s	set of start nodes
N_e	set of end nodes
K	set of vehicles
q_j	demand of customer $j \in C$
C_k	capacity of vehicle $k \in K$
w_i	time of arrival at node $i \in N$
s_i	service time at depot $i \in D$
β_j	penalty for not servicing customer j $\forall$ j $\in C$
z_{ijk}	travel cost on link $(i,j) \in A$ for vehicle k $\in K$
γ	maximum concrete haul time
T	total service time

In recent works such as: (Maghrebi et al., 2014, Asbach et al., 2009) used a model for the RDP where each depot node is broken down into a set of sub-depot nodes according to the number of possible loading slots at each depot, and where each customer is broken down into a set of sub-customer nodes according to the number of required deliveries at each customer. The sub-depot break down can be achieved optimally by determining the number of available time slots at each depot node $i \in D$, that is $\left\lfloor \frac{T}{s_i} \right\rfloor$ and generates a total number of sub-depot nodes of $\mathcal{O}(|D|S_{max}^{D})$ where

$$S_{max}^{D} \equiv \left\lfloor \frac{T}{min_{\ i \in D\{s_i\}}} \right\rfloor \tag{3}$$

This is the maximum number of sub-depot nodes obtained from a depot after break down. The breakdown of a customer $j \in C$ into sub-customer nodes depends on its demand q_j as well as on the capacity of the fleet of vehicles. If the fleet is homogeneous, that is $\forall\, k \in K, c_k = c$ then the number of required deliveries at customer i is $\left\lfloor \frac{q_j}{c} \right\rfloor$ and the total number of sub-customer nodes generated is $\mathcal{O}(|C|S_{max}^{C})$ where

$$S_{max}^{C} \equiv \left\lfloor \frac{max_{\ j \in C\ \{s_i\}}}{c} \right\rfloor \tag{4}$$

is the maximum number of sub-customer nodes obtained from a customer after break down. However, in the context of a heterogeneous fleet of vehicles, the number of required deliveries for a customer node cannot be determined *a priori* since this number depends on the type of vehicle assigned to the customer. Hence, if a lower limit on the vehicle capacity is used, the total number of sub-customer nodes generated is $\mathcal{O}(|C|\tilde{S}_{max}^{C})$ where

$$\tilde{S}^{C}_{max} \equiv \left\lceil \frac{max_{\ j \in C\ \{q_j\}}}{min_{\ k \in K \{c_k\}}} \right\rceil \qquad (5)$$

Therefore, this modelling approach may dramatically increase the number of decision variables and constraints in the model for the RDP. To improve the computational complexity of the model for the RDP, we propose an innovative formulation that does not require the creation of virtual sub-nodes. Our approach is based on the rationale that the route of each vehicle can be broken down according to the different stages of its service. We distinguish four types of trips for each vehicle:

1. **Start–Depot:** trips from the start nodes to the first assigned depot nodes in the route.
2. **Depot–Customer:** trips from the depot nodes to the customer nodes (the vehicle carry RMC).
3. **Customer–Depot:** trips from the customer nodes to the depot nodes (the vehicle is empty and will be loaded at the next depot).
4. **Customer–End:** trips from the customer nodes to the end nodes.

By deduction, the first and fourth types of trips occur only once in the route of each vehicle. In contrast, the second and third types of trips may occur any number of times, even if a single pair (of depot and customer) is considered. We consider several variants of the RDP.

- **Static-homogeneous-RDP**--- in this variant we assume that there is no service time limit and that the fleet of vehicles is homogeneous; hence, there are no restrictions on the accessibility to depot nodes or transit times between depot and customer nodes and each vehicle in the fleet has the same capacity.
- **Static-heterogeneous-RDP**--- in this variant we assume that there is no service time limit but we assume a heterogeneous fleet of vehicles; delivery vehicles may have different capacities c_k and/or routing costsz_{ijk}.
- **Dynamic-RDP**--- in this variant we assume that a total service time T is used to limit the makespan of the deliveries and that each loading operation at a depot $i \in D$ takes s_i units of time. Furthermore, a maximum RMC haul time of γ units of time is enforced. Hence, the routing of a vehicle from a depot to a customer is constrained by slot availability at depot nodes and transit time in the network.

In this paper, we focus on the Static-Homogeneous-RDP and propose a light formulation for this variant of the RMC dispatching problem.

3.2. Properties of the Static-homogeneous-RDP

In the static-homogeneous case, we assume that the capacity of each vehicle is constant across the fleet and that transportation costs are not vehicle-dependent. Let $c = c_k$ be the vehicle capacity; in this section the transportation costs z_{ijk} are denoted $z_{ijk}, \forall \in A$. With the Static-Homogeneous-RDP problem it can be observed that, at the optimum, the number of trips required to service a customer is limited by the capacity of the vehicles, and that only the cheapest depot with regards to transportation costs is used.

Proposition 1. *At the optimum, the number of trips required to complete an assignment, i.e. service a customer $j \in C$ is*

$$2\left\lfloor\frac{q_j}{c}\right\rfloor - 1 \tag{6}$$

and the transportation costs involved with the completion of this assignment is

$$\left(2\left\lfloor\frac{q_j}{c}\right\rfloor - 1\right) z_j^* \quad \textit{such that } z_j^* = \underset{i \in D}{\operatorname{argmin}}\{z_{ij}\} \tag{7}$$

Proof:
Since there is no restriction on the service time at a depot, at the optimum, all the trips from a depot to a customer are executed between the cheapest depot from that customer; otherwise the routing decision is not optimal. If $c \geq q_j$ then a single delivery is sufficient to service a customer j and the number of trips from the cheapest depot to a customer j is 1. If $c < q_j$,then $\left\lfloor\frac{q_j}{c}\right\rfloor$ is the number of trips of the second type (Depot–Customer) required to service a customer j. In this case, we can observe that the number of trips of the third type between the cheapest depot and j is equal to $\left\lfloor\frac{q_j}{c}\right\rfloor$ if the vehicle is routed to the same depot for its next assignment or $\left\lfloor\frac{q_j}{c}\right\rfloor - 1$ otherwise. Hence, there is always an even number of trips between the cheapest depot and a customer j minus a final trip of the third or fourth type for the vehicle's next assignment. The transportation cost for this assignment is obtained by pricing each trip with z_j^* which by construction minimizes the routing costs for the trip of the second type.

3.3. Decision Variables and Objectives

We introduce two types of decision variables to formulate a model for the RDP. Namely, the routing variables x_{ijk} defined as

$$\forall\, (i,j) \in A, \forall\, k \in K, \quad x_{ijk} \equiv \begin{cases} 1 & \textit{if vehicle } k \textit{ uses link } (i,j); \\ 0 & \textit{otherwise}. \end{cases} \tag{1}$$

And the assignment variables y_i defined as

$$\forall\, j \in C, \quad y_j \equiv \begin{cases} 1 & \textit{if customer } j \textit{ is serviced}; \\ 0 & \textit{otherwise}. \end{cases} \tag{2}$$

Proposition.1 provides a convenient way to evaluate the transportation costs involved in the static-homogeneous RDP; to address this problem we propose to break down the objective function into four terms according to the four types of trips involved in the route of a delivery vehicle. The Static-Homogeneous-RDP can be represented using the following model.

4. Integer Program for the Static-RDP

$$min \sum_{j\in C} \left[y_j\left(2\left\lfloor\frac{q_j}{c}\right\rfloor - 1\right)\right] \sum_{i\in D}\sum_{k\in K} z_{ijk}x_{ijk} + (1-y_i)\beta_j + \sum_{i\in N_s}\sum_{j\in D}\sum_{k\in K} z_{ijk}x_{ijk} + \sum_{i\in C}\sum_{j\in N_e}\sum_{k\in K} z_{ijk}x_{ijk} + \sum_{i\in C}\sum_{j\in D}\sum_{k\in K} z_{ijk}x_{ijk} \qquad \textbf{(8)}$$

Subject to:

$$\sum_{i\in N_s}\sum_{j\in D} x_{ijk} = 1 \qquad \forall k \in K \qquad (9)$$

$$\sum_{i\in C}\sum_{j\in N_e} x_{ijk} = 1 \qquad \forall k \in K \qquad (10)$$

$$\sum_{i\in N} x_{ijk} - \sum_{j\in N} x_{ijk} = 0 \qquad \forall i \in C \cup D, \forall k \in K \qquad (11)$$

$$\sum_{i\in D}\sum_{K\in D} x_{ijk} \geq y_j \qquad \forall i \in C \qquad (12)$$

$$x_{ijk} \in \{0,1\} \qquad \forall (i,j) \in A, \forall k \in K \qquad (13)$$

$$y_j \in \{0,1\} \qquad \forall j \in C \qquad (14)$$

The above model contains a quadratic objective function which involves the product of binary decision variables x_{ijk} andy_j. This quadratic form can be linearized by introducing an auxiliary decision variable$\beta_{ijk} \in [0,1]$, defined as

$$\forall\ (i,j) \in A, \forall\ k \in K, \quad \beta_{ijk} \equiv x_{ijk} * y_j$$

and the following side constraints

$$\beta_{ijk} \leq x_{ijk} \qquad \forall (i,j) \in A, \forall k \in K \qquad (15)$$

$$\beta_{ijk} \leq y_j \qquad \forall (i,j) \in A, \forall k \in K \qquad (16)$$

$$\beta_{ijk} \geq x_{ijk} + y_j - 1 \qquad \forall (i,j) \in A, \forall k \in K \qquad (17)$$

4.1. Model Evaluation

The obtained MILP for the Static-Homogeneous-RDP requires

- $|A| * |K|$ (binary) routing variables x_{ijk}
- $|C|$ (binary) assignment variables y_j
- $|A| * |K|$ (continuous) auxiliary variables β_{ijk}

Hence, the order of the number of variables is of $\mathcal{O}(|A| * |K|)$. The maximum number of constraints is equally limited. Using the above trip identification, the number of links in the network is

$$|A| = |N_s| * |D| + 2(|D| * |C|) + |C| * |N_e|$$

In contrast, using time restrictions on the vehicle schedule and sub-node break down would require $|C|$ to be replaced by $|D|S^{D}_{max}$ and $|C|$ to be replaced by $|C|S^{C}_{max}$, as defined in Section 3. It is clear that when time considerations can be omitted, the proposed formulation yields a significantly economic model for the solving of this RMC dispatching problem.

On the other hand, and in comparison with similar formulations such as (Asbach et al., 2009, Maghrebi et al., 2014), it is obvious that the number of side constraints in the proposed formulation is less than those in the formulation. Furthermore, because the proposed formulation deals with customers rather than deliveries, the decision variables in this formulation would be less or equal to a similar formulation. If, and only if, all the customers need only one delivery then the number of decision variables in the proposed formulation is equal to the number of decision variables of similar formulations; however, in practice, the chance of having such circumstances is very low. Therefore, using the proposed formulation is resulted in a reduction in the number of decision variables as well.

5. Conclusion

Several mathematical formulations have been introduced for optimally solving Ready Mixed Concrete (RMC) problems. Most of these have focused on deliveries. The main drawback of these models is computing time, which in practice cannot be used even for small instances. In this paper, a novel formulation was presented which deals with customers rather than deliveries. This results in a reduction in the number of constraints as well as in the decision variables. Therefore, less computing effort is required compared with similar formulations. It also has the potential to be used in the solving of small RMC instances.

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