**Strategic User Equilibrium Assignment under Trip Variability**

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**Abstract**

One common criticism of traditional traffic network assignment is the lack of observable equilibrium. It is easy to confirm that traffic networks vary continually due to uncertain travel demand, traffic capacity, or individual behaviour. However, in the traditional deterministic user equilibrium assignment, users select paths to minimize their travel time based on a single known demand value. Even with such issues, equilibrium models have persisted in a planning context due to their important mathematical properties (e.g., solution consistency and convergence). What is needed are new network models which directly account for existing variability while still maintaining the beneficial properties of traditional traffic equilibrium models; ideally, new models which better explain variation but are still as comparably simple and practical as existing methods. To address the aforementioned need, this paper examines strategic network assignment models which ensure that users recognize the variations in travel time to their destination and rationally choose routes while considering all possible demand scenarios in a known distribution. The set of chosen routes are then followed regardless of the specific travel demand on any given day. The strategic assignment model therefore produces link flows which will not result in a state of network equilibrium under specific demand realizations. The proposed strategic assignment problem is analytically formulated, and the link proportions and the variance in travel time for the links are analytically derived. Numerical analysis is conducted, and results are compared with two deterministic user equilibrium assignment models, all evaluated with variable demand.

## 1. INTRODUCTION AND LITERATURE REVIEW

Traffic network equilibrium remains a critical component in the transportation planning process. This is in spite of the commonly noted fact that network equilibrium is not routinely observed in practice. Even with the noted issues of equilibrium, it remains a relatively simple model that captures the inherent self-centred nature of travellers in a mathematically consistent manner with critical convergence properties. Numerous attempts have been made for improved behavioural traffic assignment models including Stochastic User Equilibrium and many others. Often, though, the resulting models are either substantially more time consuming, mathematically complex, or neglect critical elements of the underlying causal uncertainty (i.e., demand or capacity uncertainty).

It is well known that the most observable stochastic element, link travel time reliability, is an important measure of network performance; preferences over variability of travel time have been verified Small et al. *(1)*. But, more broadly, variability of travel time in a network can be attributed to variability in demand *(2-6)*, supply *(7-9)*, route choice behaviour *(10-14)* and departure time choice behaviour *(15-17)*.

It is due to these underlying variations that equilibrium traffic flows are rarely observed, which as noted, has led to criticism of existing traffic assignment models. Watling and Hazelmen *(18)* provide an in-depth discussion on the definition of equilibrium, and also identify this issue, and the existence of observable equilibrium. In addition, the new concept of dis-equilibrium has become an important area of research, in spite of the mathematical and computational complexity of the problem when stated in the most general (and abstract) terms.

Various reasons have been provided in response to the lack of observed equilibrium conditions. Claims were made that travel decisions are affected by learning from previous days, leading to potentially unstable conditions; users adapt on a day-to-day basis, and their adapting mechanism may undermine the notions of a stable equilibrium *(19)*. Another source of daily variation may be a result of supply side reductions in capacity, like that resulting from traffic incidents or adverse weather conditions *(20-21)*.

Traditionally, as noted earlier, the uncertainty in user perception in travel time has been addressed extensively through Stochastic User Equilibrium models *(22-24)*. This paper instead proposes strategic based assignment models which fundamentally veer away from this notion, and describes equilibrium as travellers assigning themselves proportionally to routes so as to reduce their expected travel time, where the uncertainty can be a manifestation of demand or capacity (for purposes of demonstration constrained to only demand in this paper). In strategic assignment approaches, the resulting model does not attempt to optimize path (or link) flow directly, but rather to discern strategies (i.e., path flow proportions) that are applied across the realizations of some uncertain variable.

Variations of strategic approaches have been applied successfully by Chriqui and Robillard *(25)*, Nguyen and Pallottino *(26)* as well as Spiess and Florian *(27)* albeit to the modelling of urban transit networks and often in an adaptive manner (whereas this paper adopts a simpler non-adaptive view to maintain ease of implementation). In the transit approach, users select a line from a subset of the available lines and board the first incoming vehicle from the selected line. Marcotte and Nguyen *(28)* considered this strategy framework non-ideal for transit, since the sets of attractive lines in the transit application are unordered. They used the mixed strategy approach to solve equilibrium for traffic networks with rigid finite capacities. Hamdouch et al *(19)* expanded this method to a dynamic formulation in which again users follow strategies based on preferences, while meeting the first in first out constraints.

The goal of the proposed strategic approach is to equilibrate based on an expected condition as opposed to a deterministic cost. To capture this behaviour a strategic route assignment model is proposed which ensures that users recognize the variations in travel time to their destination and rationally choose routes while considering all possible demand scenarios in a known distribution. The set of chosen routes are then followed regardless of the realized travel demand on any given day. The strategic assignment model therefore produces link flows which will not result in a state of network equilibrium under particular demand realizations. The proposed strategic assignment problem is mathematically formulated, and the link proportions and the variance in travel time for the links are analytically derived. For the current analysis, we focus on a strategic equilibrium in the presence of day-to-day fluctuations in travel demand. This work does not consider demand fluctuations that result from incidents, degraded links, or other non-recurring phenomena.

In section 2 the three assignment models are described and mathematically formulated. Section 3 provides numerical results comparing the models' performance for a test network, and the results are further validated for a medium size network. Extensions to the multiple OD version of the model are also discussed. Section 4 concludes the paper.

## 2. PROBLEM FORMULATION

Three assignment models are considered; the proposed strategic user equilibrium (StrUE) assignment model is compared against two deterministic user equilibrium based assignment models: i) Deterministic User Equilibrium with Perfect Information (DUE PI) and ii) Deterministic User Equilibrium with Fixed Proportions (DUE FP). Each model is described in detail and mathematically formulated in the following section.

### 2.1 DUE with Perfect Information (DUE PI)

The first model, included for comparison purposes, is a traditional deterministic user equilibrium assignment model which assumes users have perfect information on the state of the network (i.e. the day-to-day demand value) and select routes to minimize their travel time based on this knowledge. For the DUE PI model the assignment patterns result in a state of network equilibrium each day. Because the actual demand varies day-to-day the resultant assignment pattern (i.e. link flow volumes and link travel times) will also vary day-to-day.

Consider a stochastic transportation network $G = (N, A, \mathbf{D}, \Psi)$ consisting of a set of nodes $N$; a set of directed arcs $A$; a demand matrix **D** with $|N|$ rows and columns, mapping the demand for travel from every node to every other node. In this work a single origin destination case was analysed, where, $R$ and $S$ represent the origin and destination respectively. Let $\Psi$ denote the demand distribution and $\omega \in \Psi$ be a realization of one particular demand from the distribution. $d_{RS}^{\omega}$ denotes the value of one particular demand realization between origin $R$ and destination $S$. Let $K_{RS}$ represent the set of paths connecting origin $R$ and destination $S$ and $i \in K_{RS}$ is an index for one path. Let $A$ denote the set of all arcs and $a \in A$ is an index for one particular arc in the network. $f_{RS}^{i\omega}$ represents the total flow on path $k$ connecting origin $R$ and destination $S$ in demand realization $\omega \in \Psi$. Let $v_a^{\omega}$ represent the total link flow on link $a \in A$ under demand realization $\omega \in \Psi$ and $\delta_a^{i\omega}$ is the link path incidence variable. Let $V^{\omega}$ represent the vector set of feasible link flows for demand realization $\omega \in \Psi$.

$$V^{\omega} = \{v_a^{\omega} \ \forall a \in A\colon\ v_a^{\omega} = \textstyle\sum_{i \in K_{RS}} \delta_a^{i\omega} f_{RS}^{i\omega}, \sum_{i \in K_{RS}} f_{RS}^{i\omega} = d_{RS}^{\omega} \forall RS\} \qquad [1]$$

Let $T(.)$ represent the vector of link cost functions for all links in the network. The link cost function may be any function that defines the relationship between the number of users traveling a particular link and the cost to travel that particular link (cost can be travel time, money, etc). While any link cost function could be substituted, a common link-cost function used in transportation literature and practice is the Bureau of Public Roads (BPR) formulation (U.S. Department of Commerce, 1964), and is the function used in this paper for demonstration purposes. The BPR function is defined below:

$$T_a(v) = t_f \left[1 + \alpha \left(\frac{v_a}{C}\right)^k\right] \qquad [2]$$

Where $t$ is link travel time, $t_f$ is free-flow travel time, $v$ is hourly volume, $C$ is hourly capacity, and $\alpha$ and $k$ are parameters that depend on link geometry. In this work, we seek a collection of flow vectors $V^{\omega*}$ for user equilibrium link flow that depend on the demand realization, and satisfy the following inequality:

$$\left(T(V^{\omega*})\right)^T (Y^{\omega} - V^{\omega*}) \geq 0 \quad \forall Y^{\omega} \in V^{\omega}, \omega \in \Psi \qquad [3]$$

The constraint represents the set of equilibrium link flows given demand realization $\omega$ and link cost functions *T(.)***.** The model output specifies the link level flows $v_a^{\omega}$, and link travel times $T_a^{\omega}$ for each demand realization. Monte Carlo sampling was implemented to select an origin-destination (O-D) specific demand realization chosen from a lognormal travel demand distribution with a known mean and variance. For each demand realization the link flows for a DUE were computed using a traditional Frank-Wolfe algorithm. Based on the resultant assignment pattern for each demand realization, system performance measures are calculated. $F_{\omega}(\Lambda(V^{\omega*}))$ represents a function of total system travel time for every realization $\omega \in \Psi$ which is computed based on the resultant link travel flows and link travel costs. This was repeated for multiple iterations, and the expected value and variance of the system performance functions computed.

## 2.2 Strategic User Equilibrium (StrUE)

In the proposed strategic user equilibrium (StrUE) assignment users choose routes so as to minimize their expected travel time between an origin and destination. Using similar notation to the DUE PI model, we define path proportions $\xi_i$ on path *i*, where path *i* belongs to the set $K_{RS}$, as the percent of OD demand between *R* and *S* which choose to travel on route *i.*

$$\xi = \{\xi_i\}_{i\epsilon K_{RS}} = \left\{\xi : \textstyle\sum_{i\epsilon K_{RS}} \xi_i = 1, \forall \xi_i \geq 0\right\} \qquad [4]$$

The proportion of flow on a link is the sum of the path proportions that are incident on the link, and are described below.

$$p = \{p_a\ \forall a \in A : p_a = \textstyle\sum_{i\epsilon K_{RS}} \delta_a^i \xi_i , \sum_{i\epsilon K_{RS}} \xi_i = 1\ \forall R, S\} \qquad [5]$$

StrUE involves each user choosing a path so as to minimize their expected cost. A Wardrop's first principle states that equilibrium is reached when the expected travel costs are equal on all used paths, and this common cost is less than the actual cost on any unused path. An equilibrium strategy *p* is then a solution of the nonlinear complementarity problem:

$$\mathrm{p}\ [E(T_i(pD)) - \lambda] = 0 \text{ Where, } p\epsilon P \text{ and } E(T_i(pD)) - \lambda \geq 0 \qquad [6]$$

Let $T(.)$ represent the vector of link cost functions for all links in the network. $T_i(pD)$ denotes the link cost function along path *i*. The BPR function (equation 2) applied in the StrUE model is defined in equation 7:

$$T_a(pD) = t_f\left[1 + \alpha\left(\frac{p_a D}{C}\right)^k\right] \qquad [7]$$

The BPR function can be written in the reduced form:

$$T_i(pD) = \gamma_i + \delta_i p^k D^k \qquad [8]$$

Where $\gamma_i = t_f$, $\delta_i = \alpha t_f$ and $p = \frac{p_a}{C}$. Therefore, the expected travel time is

$$E(F_i(pT)) = \int_{-\infty}^{\infty} (\gamma_i + \delta_i p^k D^k)\ \varphi(D) dD \quad [9]$$

$$E(T_i(pD)) = \gamma_i + \delta_i p^k M_k, \quad [10]$$

Where $M_k$ is the $k^{th}$ moment of the demand distribution which has a pdf $\varphi(D)$. Replacing Equation 10 in Equation 6 we get

$$p_i\ [\gamma_i + \delta_i p^k M_k - \lambda] = 0 \quad \text{Where } p \epsilon P \text{ and } \gamma_i + \delta_i p^k M_k - \lambda \geq 0 \quad [11]$$

Equation 11 can be re-written as a variational inequality

$$\langle \gamma + \delta\ p^k M_k, p - q \rangle \leq 0 \quad \forall q \epsilon P, p \epsilon P \quad [12]$$

The novelty of the proposed model is the assumption that the users' behaviour is dictated by travel strategies to minimize their expected travel time over the demand distribution, where the demand distribution captures the randomness associated with day-to-day variation.

Given the strategies it is also possible to analytically characterize the variance of travel time on a route. The variance on links is due to uncertainty in demand, and can be used to measure reliability. The variance on a link can be written as

$$Var(T_i(pD)) = E((T_i(pD))^2) - E(T_i(pD))^2 \quad [13]$$

Using Equation 8 and equation 13 we get:

$$E((T_i(pD))^2) = \gamma_i^2 + \delta_i^2 p^{2k} M_{2k} + 2\gamma_i \delta_i p^k M_k \quad [14]$$

$$E(T_i(pD))^2 = \gamma_i^2 + \delta_i^2 p^{2k} M_k^2 + 2\gamma_i \delta_i p^k M_k \quad [15]$$

$$Var(T_i(pD)) = \delta_i^2 p^{2k}(M_{2k} - M_k^2) \qquad [16]$$

Equation 16 characterizes the variance that is observed in the network due to day-to-day fluctuations in travel demand, assuming users behave based on a predetermined strategy.

In the construct of the StrUE model path proportions are dependent on the distribution of the demand, and not any particular demand realization. For a given demand distribution the expected cost of travelling on a link is given by Equation 10. These link cost functions are used to solve for the proportion of flows travelling on each link. Once again the Frank-Wolfe algorithm is used with the modified link cost function. For any given demand realization, the demand is assigned based on the *a priori* computed link proportions. The same set of demand realizations sampled in the DUE PI evaluation was used to evaluate the expected network performance and variance under the StrUE model.

It is important to recognize that for StrUE model the path (and link) proportions will not change day-to-day. However, the actual link flow volumes will vary as a function of the realized demand (since the link flows would be the product of the realized demand and the link proportions), meaning equilibrium conditions are unlikely to be met for many demand realizations. This outcome is consistent with real world traffic networks where equilibrium conditions are not observed on a day-to-day basis. One of the strengths of this approach is that the uncertainty in travel times and flows can be analytically tied back to the demand uncertainty.

## 2.3 DUE with Fixed Proportions (DUE FP)

The final assignment model evaluated in this work is a Deterministic Equilibrium Model with fixed proportions (DUE FP). This model is a hybrid of the previous two models. In the DUE FP users are assumed to know only the *expected demand* (average demand) value and do not care about the distribution, compared with the StrUE model where the users are assumed to know the distribution of the demand and make their choices based on *expected costs*, while in the DUE PI model users are assumed to know the *actual demand* value on any given day.

In DUE FP users equilibrate once simply based on a single deterministic demand value equal to the expected demand and then determine the resultant path proportions based on the assignment. These path proportions are used to determine the flows based on day-to-day realized demand. Like the StrUE model, the DUE FP path proportions will not change day-to-day, but the actual path flows and link flows will vary under different demand realizations. It is trivial to show that the variance in cost for a DUE FP is the same as equation 16, but the path proportions could be significantly different, because the proportions are determined using a different link cost. Qualitatively it should be expected that in uncongested conditions or tight demand distributions with low variance the DUE FP should result in similar results to StrUE, but would diverge when there is congestion and the variances are high.

The same set of demand realizations sampled in the DUE PI and StrUE evaluation was used to evaluate the expected network performance and variance under the DUE FP model. Once again user equilibrium conditions will likely not be met on any given day.

In summary the three equilibrium-based assignment models evaluated in this work are differentiated as follows:

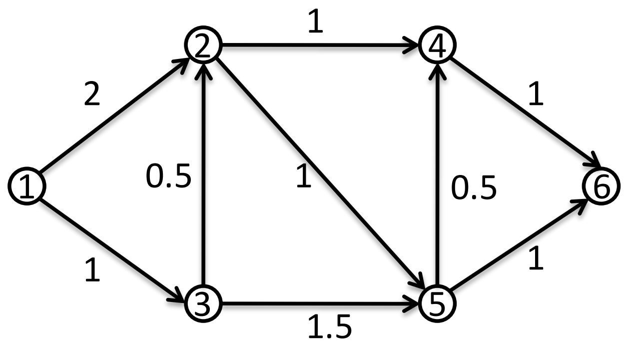
1. **DUE PI**: Users equilibrate each day to minimize their travel time based on the actual realized demand. A given user's path may change day-to-day.
2. **DUE FP:** Users equilibrate to minimize their travel time based on a single value of demand equal to the expected demand, resulting in a set of path assignments. These path assignments are used to determine the path proportions. For a given demand realization, the traffic is distributed based on the path proportions. Each user follows this assigned path strategy day-to-day independent of the demand realization.
3. **StrUE:** Users equilibrate to minimize their *expected* travel time based on a known demand distribution curve, resulting in a set of path proportion assignments. Each user follows this assigned path strategy day-to-day independent of the demand realization.

To evaluate each model three performance measures were considered: i) total system travel time (TSTT), ii) link flows, and iii) link travel times (link TT). For a given demand distribution curve 1000 randomly selected demand scenarios generated through Monte Carlo sampling were used to compute the expected value and variance for each performance measure. In addition the link TT variance computed from the StrUE simulation model is compared with the analytical link TT variances calculated for the same demand sample set.

## 3. NUMERICAL RESULTS

This section compares the three assignment models in terms of expected system level and link level performance measures. The results are based on the sample network depicted in figure 1. The network has 6 nodes and 9 links, with a single O-D - node 1 is the origin and node 6 is the destination. There are 7 possible paths from the origin to the destination. The numbers next to the links are the link lengths in miles. The capacity for each link is 50 and the free flow speed for each link is 60 mph. The BPR parameters $\alpha$ and $\beta$ are 0.15 and 4, respectively.

For the test network shown in Figure 1 the resultant UE network conditions for a deterministic demand of 100 are provided. The flow units are vehicles and the travel time is in minutes. The same set of link flows and total system travel times are computed by all three assignment models under deterministic demand.




| Deterministic Case, D=100 | | |
|---|---|---|
| Link | Flow | TT |
| (1,2) | 30.7561 | 2.043 |
| (1,3) | 69.2439 | 1.5517 |
| (2,4) | 45.0514 | 1.0989 |
| (2,5) | 18.3592 | 1.0027 |
| (3,2) | 32.6545 | 0.5136 |
| (3,5) | 36.5895 | 1.5645 |
| (4,6) | 45.0514 | 1.0989 |
| (5,4) | 0 | 0.5 |
| (5,6) | 54.9486 | 1.2188 |
| TSTT : 429 | | |

(a) (b)

**FIGURE 1 (a) Test network and (b) system performance under deterministic demand of 100, including link flows, link costs and TSTT**

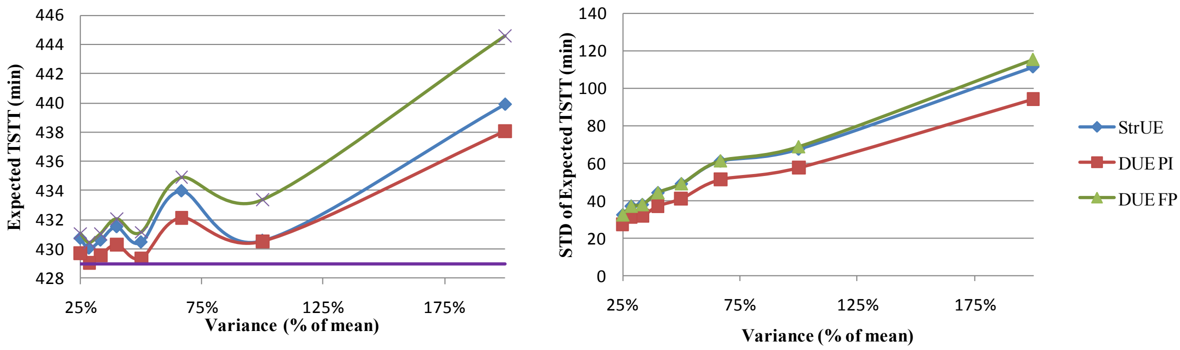
When demand variability is introduced the resultant link flow volumes (and travel times) may deviate from the deterministic set of flows (and travel times). The proposed StrUE model incorporate demand uncertainty into the user's decision making process, therefore the remainder of the analysis considers only stochastic demand conditions. Specifically, the demand follows a lognormal distribution with a prescribed mean and variance. The mean, $\mu$, is set to one of four values, [25, 50, 75, 100], and the set of possible variances for each expected demand value is [$1/2\mu$, $\mu$, $3/2\mu$, $2\mu$, $5/2\mu$, $3\mu$, $7/2\mu$, $4\mu$], representing 32 possible demand distribution curves. For each distribution 1000 demand realizations were sampled from each curve for the analysis.

In the following section the expected value and standard deviations of the three performance measures are illustrated. For the link-level performance measures the results are presented so as to illustrate the behaviour of the StrUE model relative to each of the DUE-based model.

### 3.1 Expected TSTT and Variance of TSTT

In figure 2.a and 2.b the expected TSTT (E[TSTT]) and standard deviation of TSTT (STD[TSTT]) computed for each model are illustrated for an expected demand of 100, and the 8 variance levels listed above. In figure 2.a the straight purple line marks the deterministic TSTT, which is less than E[TSTT] for all three models, suggesting that using a single deterministic demand value for planning purposes underestimates system performance. This finding is supported by previous research by Waller (2001) *(29)*. Figure 2.a also illustrates a lower E[TSTT] under StrUE and DUE PI models, relative to the DUE FP model. The lower E[TSTT] should be expected under the DUE PI model because people equilibrate to minimize their experienced travel time based on the actual demand. The lower E[TSTT] in the StrUE can be explained by the path proportion assignment which is based on minimizing expect cost. In contrast the DUE FP proportions are assigned based on a single demand value, without accounting for potential variability in demand, therefore they can be expected to underperform compared with a model that incorporates the possible demand variability into routing decisions. This is precisely the reason why the expected TSTT is lower for StrUE as compared to DUE FP highly uncertain demand values, and why the E[TSTT] for the three models is closer under lower variance.

Figure 2.b illustrates roughly an 18% increase in STD[TSTT]under the StrUE and DUE FP model relative to the DUE PI model. As expected the STD[TSTT] for the StrUE and DUE PI models diverge as the demand variability increases. The explanation is a function of the information available to the users. In the DUE PI model users react to the actual demand each day, while the StrUE and DUE FP models restrict users to their *a priori* defined strategy. Under these restrictions the strategies are likely to result in highly sub-optimal link flows under low and high demand realizations, increasing the STD[TSTT].



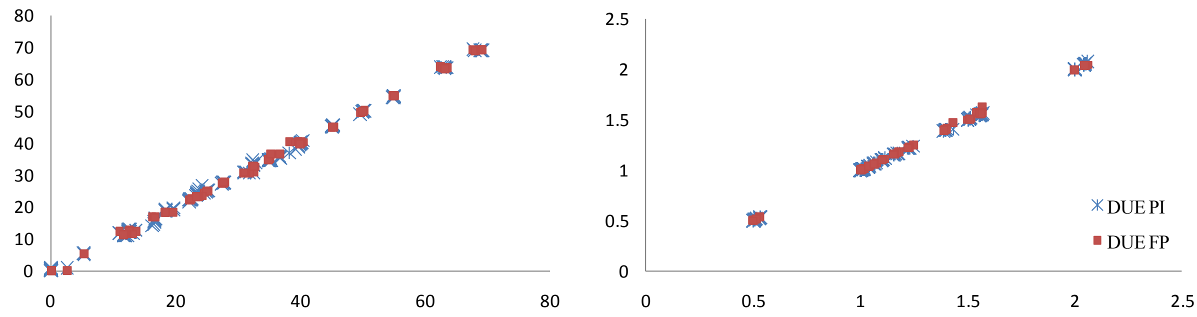

(a) (b)

**FIGURE 2 (a) Expected TSTT and (b) Standard Deviation of TSTT for Sample Network**

Results for the expected link flows and expected link travel times (and respective standard deviation) are provided in figures 3.a and 3.b for each of the three assignment models. In each plot a single performance measure is evaluated for each of the 32 demand distribution curves (defined by a mean and variance) and each assignment model (StrUE, DUE PI, or DUE FP). The performance measure is plotted for each link in the test network.

## 3.2 Expected Link Behavior

Figure 3 illustrates that all three models predict the same set of expected link flows and link travel times averaged over the 1000 demand realizations. The expected link flows and link travel times for DUE PI, DUE FP and StrUE line up perfectly. This result is expected from the DUE PI model because the average assignment patterns over a large sample size should converge to the StrUE, for which the assignment is based on expected costs. Similarly, the average assignment patterns for a DUE FP which is based on expected demand should converge to StrUE over a large sample as well. However the same linear behavior will not hold true for the standard deviations of link-level flows and travel times.

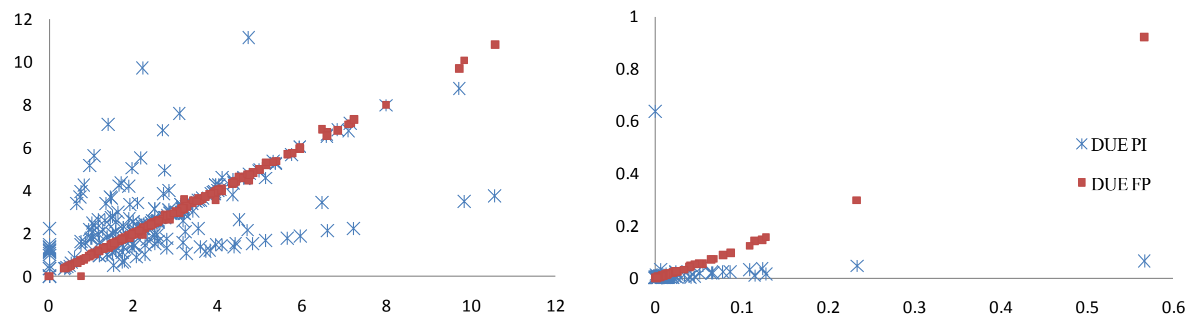



(a) (b)

**FIGURE 3 Expected (a) Link Flow Volumes and (b) Link Travel Times. The x-axis represents the StrUE model results and the y-axis represents the DUE PI and DUE FP model results.**

### 3.3 Variance of Link Behavior

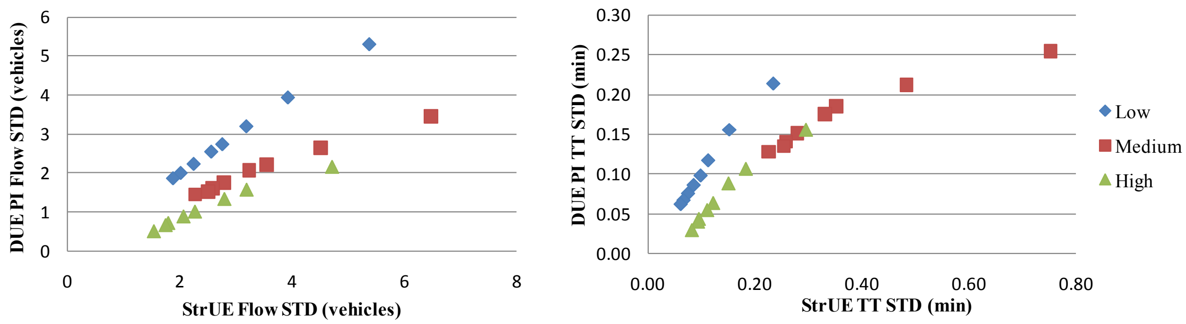
In contrast to the expected link behaviour, the three models do not result in the same variability in link performance (see figure 4). The StrUE and DUE FP models predict similar link level variability. On the other hand link level variability for the DUE PI model differs from the StrUE model. The standard deviation of the link flow from the DUE PI model is often significantly higher or lower than the in the StrUE model (see figure 4.a). The inconsistency between models is a function of the specific link and demand distribution. From figure 4.b it is evident that the predicted link travel time variability is almost always lower for the DUE PI than the StrUE model. These discrepancies in link level variability between the StrUE and DUE PI support system level results shown in figure 2.b.




(a) (b)

**FIGURE 4 Standard Deviation of (a) Link Flows and (b) Link Travel Times. The x-axis represents the StrUE model results and the y-axis represents the DUE PI and DUE FP model results.**

The scatter behavior of the link level standard deviation for the DUE PI model relative to the StrUE model is illustrated for link 5 in the test network in figure 5. Each series corresponds to a different expected demand value (low=50, medium=75 and high=100), and each point corresponds to a specific demand curve. While the two models predict the same set of link flows and travel times on average, the StrUE model and DUE PI models predict similar link level variability for uncongested networks, but are inconsistent for congested networks.




(a) (b)

**FIGURE 5 Comparison of the standard deviation of Link 5 flow and travel times for the StrUE and DUE PI models**

In figure 6 the standard deviation of link travel times computed from the StrUE simulation model are compared with the derived analytical values (Equation 16). From figure it is evident that the analytically derived link travel time variance closely approximates the link travel time variability observed in the simulation (as evident by the fit of Y=X with an $R^2$=0.99).

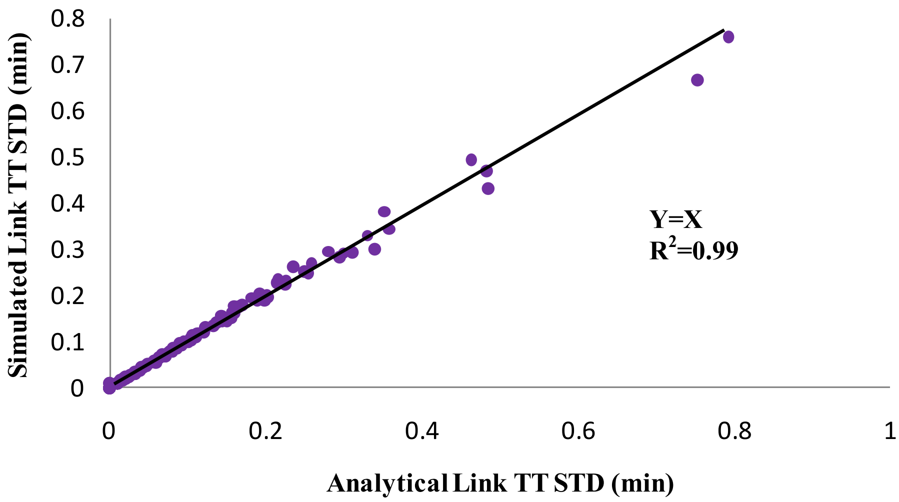



**FIGURE 6 Comparison of simulated and analytical link travel time standard deviations for the StrUE model, for the test network**

## 3.4 Anaheim Network Analysis

To demonstrate the scalability of results for StrUE to larger more realistic networks, StrUE was tested on the Anaheim planning network that was acquired from Bargera's website on Transportation Network Test Problems *(30)*. The network was used to compare the analytical link travel time variance predicted for StrUE with those observed through simulation.

Figure 7 depicts the Anaheim planning network which contains 416 nodes and 914 links. The original network had 38 nodes which were either origins or destinations. The network was converted to a single OD network through the use of super nodes. A super-source node (node 1) was created and connected to all original origin nodes (nodes 3-38), with an aggregate demand equal to the sum of the original set of demands generated by each origin, which was 104,694 ($\mu$). Similarly a super-sink node (node 2) was created and all original demand nodes (nodes 3-38) were connected to it. This increased the total number of links to 986. The network was run with eight different variances [$1/2\mu$, $\mu$, $3/2\mu$, $2\mu$, $5/2\mu$, $3\mu$, $7/2\mu$, $4\mu$] cases. For each distribution curve 100 demand realizations were sampled and evaluated.

The modified Franke-Wolfe algorithm that was used on the smaller test network earlier was used to determine the link proportions for StrUE in the larger Anaheim case. Based on the equilibrium proportions identified the link flows were determined as the product of the link proportions and the total demand. The sample variance across all flows and costs were compiled for all the cases and compared with the analytically predicted variances (Equation 16).

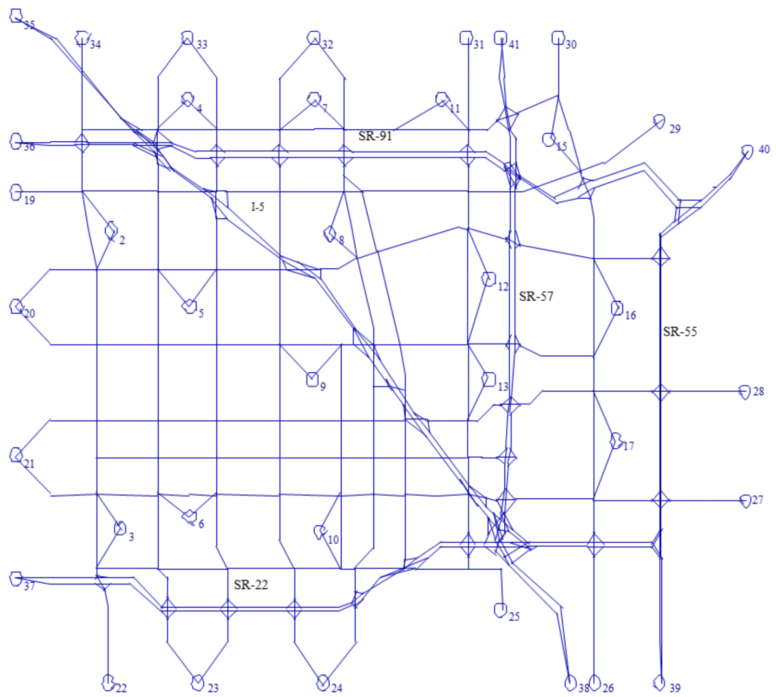



**FIGURE 7 Anaheim Network**

In figure 8 the standard deviation of link travel times computed from the StrUE simulation model are plotted against with the analytically derived values for the Anaheim network. Similarly to the test network the analytically derived link travel time variance closely approximates the individual link travel time variability computed from the simulation (as evident by the fit of Y=X with an $R^2$=0.99). The results further validate the scalability of the proposed StrUE model and demonstrate the model's ability to capture variability in travel time as a consequence of demand variability.

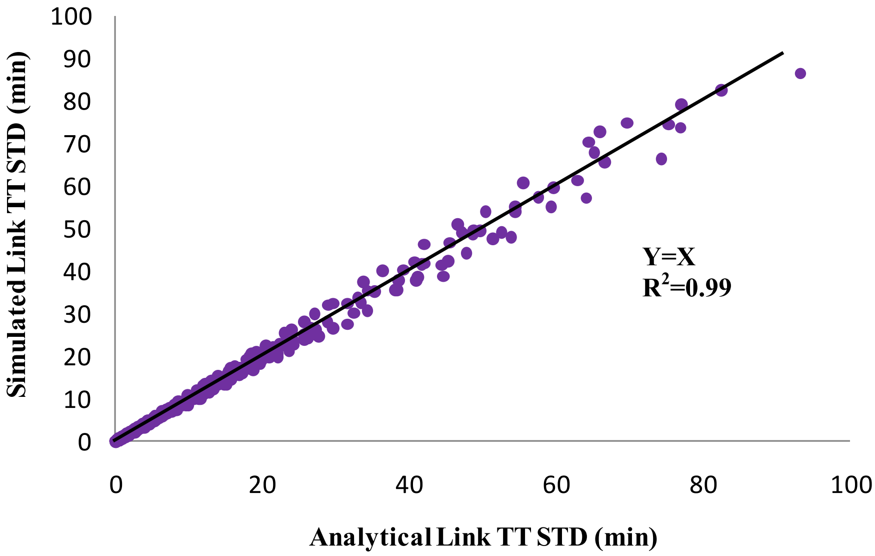



**FIGURE 8 Comparison of simulated and analytical link travel time standard deviations for the StrUE model, for the Anaheim Network**

## 4. CONCLUSIONS

In this work a new strategic network assignment model is proposed which incorporates some elements of day-to-day travel demand variation. The StrUE model ensures that users recognize the variations in travel time to their destination and rationally choose routes while considering all possible demand scenarios in a known distribution. The set of chosen routes are then followed regardless of the specific travel demand on any given day. The strategic assignment model therefore produces a range of non-equilibrated link flows which can be different for each demand realization; an outcome consistent with the lack of equilibrium observed day-to-day. However, the non-equilibrated flows result from a rational strategic-level equilibrium (which aligns with the concepts of consistency and convergence for the expected flow/cost case, thereby still highly suitable for planning applications). Further, the proposed strategic assignment problem was analytically formulated, and empirically evaluated using a simulation model. To gain insights into the practical differences in this new model's behaviour, numerical analysis was conducted comparing the StrUE model with two DUE-based assignment models, all subject to variable demand. Results from the test network demonstrated StrUE's ability to capture network variability in terms of link and system level performance. In addition the analytically derived variance was further confirmed using the simulation model for a more realistic sub-network, Anaheim. For both the test and Anaheim network the analytical variance was validated through simulation, as evident by the $R^2$=0.99.

The advances presented in this paper facilitate a potentially robust area of network equilibrium research which has the potential of alleviating one of the most criticized issues in the field (the lack of observable equilibrium) while still maintaining important mathematical properties and consistent system-wide expected flows. While this paper presents the formulation for StrUE for a single OD the solution method (using a Frank-Wolfe algorithm with modified cost functions), it can be easily generalized to a multi-OD network; however theoretical properties of link proportions as related to solution consistency is beyond the scope of this paper and remains for future research.

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