A LINEAR PROGRAM NETWORK DESIGN MODEL INCORPORATING SYSTEM OPTIMAL STRATEGIC DYNAMIC TRAFFIC ASSIGNMENT BEHAVIOUR

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ABSTRACT

The transport road network design problem examines the expansion or addition of link capacity in a network. While this topic has a rich history in the literature, it becomes particularly challenging when complexities such as dynamics or inherent network uncertainties are considered. This work introduces a dynamic, system optimal flows approach which includes demand uncertainty and for which a globally optimal solution can be found due to the linear programming model at its foundation. In the proposed model, stochastic demand scenarios are accounted for using a strategic approach, in which optimal flow proportions are identified to minimize total system travel time in all demand scenarios. This approach results in non-optimal flows for any particular demand realization. The network design linear program model proposed here does not add significant computational complexity to the base model. Results are demonstrated on a sample cell network.

Keywords: network design problem, dynamic traffic assignment, linear programming

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1. INTRODUCTION

Traditionally, the traffic network design problem (NDP) is one of the more challenging issues in the transportation research. Robust modeling approaches that provide systematic methods to determine the optimal distribution of a budget over a range of possible projects and quantify the corresponding system impact are vital for the success of transportation planning agencies around the world. However, introducing complexities such as dynamics or inherent network uncertainties, e.g., stochastic demand, results in a challenging mathematical problem to solve.

This work introduces the strategic system optimal dynamic traffic assignment (StrSO DTA) approach for which a globally optimal solution can be found due to the linear programming model at its foundation. The proposed work incorporates an enhanced version of a system optimal linear programming model proposed by Ziliaskopoulos (2000) that embeds the cell transmission model to realistically propagate traffic through a network. Waller et al (2013) propose the StrSODTA LP model that incorporates strategic route choice behavior, demand scenarios based on a discrete distribution, and path based proportions. Strategic behavior implies that users choose routes in a way to minimize travel time over a range of stochastic demand scenarios, instead of a single deterministic value. This results in a set of flows that are not an optimal solution to any single demand scenario, and thus display a day to day volatility that is commonly observed in traffic. The research in this work proposes simple modifications to apply the StrSO DTA LP to the NDP.

This approach overcomes many issues associated with the traditional dynamic traffic assignment approach to the network design problem (i.e., is not simulation based or reliant on a link cost function), although it faces challenges in terms of its computational complexity. This work first presents the model formulation, and then demonstrates results on a sample network. These results are analyzed for a range of potential budgetary applications. Furthermore, this work examines the impact of the strategic approach as compared to the results that would be obtained using a single scenario expected demand approach. Finally, possibilities to address the issue of computational complexity are discussed.

2. BACKGROUND

This work focuses on a novel approach to solving the transport road network design problem, in which the optimal links in a network are identified in order to achieve some objective. Traditional approaches to the NDP that utilize a link cost function to represent vehicle movement become challenging nonconvex mathematical problems when a variable is added to the capacity term. Therefore, heuristic solution methods have been developed by other researchers to solve the bi-level traffic NDP for a number of applications including multi-objective signal timing (Sun et al., 2003), accounting for demand uncertainty (Ukkusuri et al., 2006), optimal toll pricing strategies (Gardner et al., 2008), examining the impact of environmental justice considerations (Duthie and Waller, 2008), and minimizing emissions (Sharma and Mathew, 2011; Ferguson et al., 2012).

This work presents an application of the strategic system optimal dynamic traffic assignment model introduced by Waller et al. (2013). This work incorporates the linear programming SODTA model presented by Ziliaskopoulos (2000), which further draws on Daganzo's cell transmission model (Daganzo, 1995; Daganzo, 1994) to present a simple formulation of traffic flow that captures flow variability inside the link while avoiding the drawbacks associated with link performance functions. Waller and Ziliaskopoulos (2001) first uses a variation of this approach to examine the network design problem while accounting for stochastic demand. Additionally, Waller et al. (2006) use the LP formulation to optimally solve for the continuous network design problem, a result which would not be possible given the usual non-convex formulations for the NDP. Ukkusuri and Waller (2008) formulates a user-optimal version of this problem, which Karoonsoontawong and Waller (2005) use to compare results in the network design application accounting for stochastic demand. The problem examined in this research differs from that in the literature in that it incorporates the concept of a strategic approach to equilibrium within a linear programming framework, and further captures the finer grain resolution phenomena observable through the use of CTM, and then uses this approach to investigate the impact on the network design problem. This work differs from previous approaches in the strategic approach to accounting for the impact of multiple demand scenarios, not just expected demand, when identifying optimal path proportions (instead of expected flows).

3. MODEL FORMULATION

The model presented in this work is the accumulation of a number of previous works. The most relevant of this research is the StrSODTA model proposed by Waller et al (2013). This model utilizes a two-stage approach; in the first stage, system optimal route *proportions* are determined so as to minimize *expected* total system travel time accounting for a specified range of possible discrete demand scenarios. In the second stage, the actual travel demand is realized, and the model outputs scenario-dependent flows. However, these flows will not represent a system optimal solution for any of the realized demand scenarios, thus representing the changing nature of traffic observed in reality and additionally, introducing a variance in expected total system travel time for an optimal model that has not previously been present. Notation is presented in Table 2.

Table 2: Notation for the StrSODTA LP model

C	Set of all cells
E	Set of all cell connectors
C_R , C_S	Set of origin (source) cells and destination (sink) cells, respectively
τ	Departure time index
t	Time period index
T	Set of discrete time intervals, i.e., $T = \{\pi, 2\pi, 3\pi, \ldots, \|T\|\pi\}$, and with no loss of generality, assume that $\pi = 1$
Ξ	Set of demand scenarios.
ξ	Demand scenario index.
$d_{\tau}^{rs,\xi}$	Demand between OD pair (r, s) at departure time τ in demand scenario ξ
RS	Set of all OD pairs $\{(r,s): r \in C_R, s \in C_S, \sum_{\tau=0}^{T} d_{\tau}^{rs} > 0\}$
$\Phi(rs)$	Set of all paths , $\phi_1^{rs} \ldots , \phi_i^{rs}$ connecting OD pair (r, s)
$x_{i,\phi,\tau}^{t,rs,\xi}$	Number of vehicles at time interval t on cell i which departed at time τ, following path ϕ between origin r and destination s in demand scenario ξ
$x_i^{t,\xi}$	Number of vehicles contained in cell i at time interval t in demand scenario ξ
$y_{ij,\phi,\tau}^{t,rs,\xi}$	Flow from cell i to cell j at time interval t for OD pair rs with departure time τ and travelling along path ϕ in demand scenario ξ
$\omega_i^{t,\xi}$	Total flow into cell i at time t in demand scenario ξ
$\psi_i^{t,\xi}$	Total flow out of cell i at time t in demand scenario ξ
N_i^t	Maximum number of vehicles that can be present in cell i at time interval t
Q_i^t	Maximum number of vehicles that can flow into or out of cell i during time interval t
$P(i)$	Set of predecessor cells to cell i
$S(i)$	Set of successor cells to i
δ_{ϕ}^{ij}	Indicator equal to 1 if cell connector (i, j) is included along path ϕ, and 0 otherwise
$p_{\phi\tau}^{rs}$	Probability of using path ϕ at departure time τ for OD pair rs
p^{ξ}	Probability associated with discrete demand scenario ξ $\{\boldsymbol{p}: \sum_{\xi} p^{i} = 1\}$

First this model discusses the StrSODTA LP model, and then the enhancements necessary for the NDP are explored. The model proposed in this work uses a linear program to realistically propagate traffic according to the cell transmission model (Daganzo, 1994; 1995). As a result of the underlying CTM model, the objective function for this model is the expected total system travel time. This becomes simply the aggregate density of each cell $i \in C$ for each time period $t \in T$ for each demand scenario $\xi \in \Xi$, multiplied by the probability of that demand scenario p^{ξ}.

$$Minimize \sum_{\forall \xi \in \Xi} \sum_{\forall t \in T} \sum_{\forall i \in C \backslash C_S} p^{\xi} x_i^{t\xi} \qquad (1)$$

The first four constraints for the LP model represent the propagation of traffic through the network. Constraint (2) conserves the density of each cell and between all cells that are not source cells or sink cells, while constraint (3) conserves the flow into sink cells (which are assumed to have infinite capacity). Constraint (4) limits the flow out of a cell to the number of vehicles (the density) of that cell. Note that the path structure of the network in introduced using the indictor variable δ_{ϕ}^{ij}.

$s.t.$

$$x_{i,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t-1,rs,\xi} - \sum_{k \in P(i)} \delta_{\phi}^{ki} y_{ki,\phi,\tau}^{t-1,rs,\xi} + \sum_{j \in S(i)} \delta_{\phi}^{ij} y_{ij,\phi,\tau}^{t-1,rs,\xi} = 0, \qquad \forall i \in C \backslash \{C_R, C_S\}, \forall t \in T, \forall \phi \in \Phi(rs), \forall rs \in RS, \forall \xi \in \Xi \qquad (2)$$

$$x_{i,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t-1,rs,\xi} - \sum_{k \in P(i)} \delta_{\phi}^{ki} y_{ki,\phi,\tau}^{t-1,rs,\xi} = 0 \qquad \forall s \in C_S, \forall t \in T, \forall \phi \in \Phi(rs), \forall rs \in RS, \forall \xi \in \Xi \qquad (3)$$

$$\sum_{j\in S(i)} \delta_{\phi}^{ij} y_{ij,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t,rs,\xi} \leq 0, \qquad \begin{array}{l}\forall i \in C\backslash C_S, \forall t \in T, \forall \xi \in \Xi \\ \forall \phi \in \Phi(rs), \forall (rs) \in RS, \forall \xi \in \Xi\end{array} \tag{4}$$

Constraints (5), (6), and (7) aggregate the density in a cell, the total flow into a cell, and the total flow out of a cell. These constraints are included in order to more clearly express the meaning of the following constraints.

$$x_i^{t,\xi} = \sum_{RS} \sum_{\phi\in\Phi} \sum_{\tau=0}^{t} x_{i,\phi,\tau}^{t,rs,\xi} \qquad \forall i \in C, \forall t \in T, \forall \xi \in \Xi \tag{5}$$

$$\omega_i^{t,\xi} = \sum_{RS} \sum_{\phi\in\Phi} \sum_{\tau=0}^{t-1} \sum_{k\in P(i)} \delta_{\phi}^{ki} y_{ki,\phi,\tau}^{t,rs,\xi} \qquad \forall i \in C, \forall t \in T, \forall \xi \in \Xi \tag{6}$$

$$\psi_i^{t,\xi} = \sum_{RS} \sum_{\phi\in\Phi} \sum_{\tau=0}^{t-1} \sum_{j\in S(i)} \delta_{\phi}^{ij} y_{ij,\phi,\tau}^{t,rs,\xi} \qquad \forall i \in C, \forall t \in T, \forall \xi \in \Xi \tag{7}$$

The following set of constraints define the limitations of vehicle movement created by the physical characteristics of the links and their representation in the cell transmission model. Constraint (8) represents the jam density of a cell, which is the maximum number of units a cell can contain, representing the "capacity" of a link. Constraints (9) and (10) represent the saturation flow rate into and out of a cell respectively. The saturation flow rate places a limitation on the amount of flow that can move between cells. These two parameters create a realistic traffic movement, in particular shockwave propagation.

$$\omega_i^{t,\xi} + x_i^{t,\xi} \leq N_i^t, \qquad \forall i \in C\backslash\{C_R, C_S\}, \forall t \in T, \forall \xi \in \Xi \tag{8}$$

$$\psi_i^{t,\xi} \leq Q_i^t, \qquad \forall i \in C\backslash C_S, \forall t \in T, \forall \xi \in \Xi \tag{9}$$

$$\psi_i^{t,\xi} \leq Q_i^t, \qquad \forall i \in C\backslash C_S, \forall t \in T, \forall \xi \in \Xi \tag{10}$$

Finally, the proportions of the origin-destination demand enter the model through constraints (11) and (12). Constraint (11) loads flow into the network and the proportions of the demand on each path, while constraint (12) represents the law to total probability.

$$x_{r,\phi,\tau}^{t,rs,\xi} - x_{r,\phi,\tau}^{t-1,rs,\xi} + \sum_{j\in S(r)} \delta_{\phi}^{rj} y_{rj,\phi,\tau}^{t-1,rs,\xi} = p_{\phi\tau}^{rs} d_{t-1}^{rs,\xi} \qquad \begin{array}{l}\forall r \in C_R, \forall t \in T, \\ \forall \phi \in \Phi(rs), rs \in RS, \forall \xi \in \Xi\end{array} \tag{11}$$

$$\sum_{\forall \phi\in\Phi(r,s)} p_{\phi t}^{rs} = 1 \qquad \forall rs \in RS, \forall t \in T, \forall \xi \in \Xi \tag{12}$$

Additionally, the StrSODTA LP model has non-negativity constraints for the decision variables and specifies that the network contains no flow prior to the first time period.

$$x_{i,\phi,\tau}^{0,rs,\xi} = 0, x_{i,\phi,\tau}^{t,rs,\xi} \geq 0, \qquad \forall i \in C\backslash\{C_R, C_S\}, \tau \in T, \forall rs \in RS, \forall \phi \in \Phi(rs), \forall \xi \in \Xi \tag{13}$$

$$y_{ij,\phi,\tau}^{0,rs,\xi} = 0, y_{ij,\phi,\tau}^{t,rs,\xi} \geq 0 \qquad \forall (i,j) \in E, \tau \in T, \forall rs \in RS, \forall \phi \in \Phi(rs), \forall \xi \in \Xi \qquad (14)$$

$$p_{\phi,t}^{rs} \geq 0 \qquad \forall t \in T, \forall \phi \in \Phi(rs), \forall\, rs \in RS \qquad (15)$$

Although the model introduced in equations (1) – (15) is complex, it has an important advantage: due to the LP approach, the model can be solved for globally optimal flows using any commercial solver. Additionally, this approach is advantageous because of the relatively simple extensions to important transport problems like network design and infrastructure evaluation.

The continuous NDP application of the LP StrSO model requires the addition of a budgetary constraint, and the modification of the constraints representing the physical characteristics of network links. The budgetary constraint simply specifies the cost for expanding the capacity of a link. For demonstration purposes, this work assumes a unit cost of β to add g units to the jam density of cell i, and that this amount will proportionally add $\alpha_i = Q_i/N_i$ units to the saturation flow of cell i.

$$\sum_{\forall i \in C \backslash C_s} \beta_i g_i \leq G \qquad (16)$$

Additionally, the NDP approach in this paper alters constraints (8)-(10) above to show the amount of capacity and flow that are added to a cell.

$$\omega_i^{t,\xi} + x_i^{t,\xi} \leq N_i^t + g_i, \qquad \forall i \in C \backslash \{C_R, C_S\}, \forall t \in T, \forall \xi \in \Xi \qquad (17)$$

$$\psi_i^{t,\xi} \leq Q_i^t + \alpha_i g_i, \qquad \forall i \in C \backslash C_S, \forall t \in T, \forall \xi \in \Xi \qquad (18)$$

$$\psi_i^{t,\xi} \leq Q_i^t + \alpha_i g_i, \qquad \forall i \in C \backslash C_S, \forall t \in T, \forall \xi \in \Xi \qquad (19)$$

As previously stated, unlike traditional approaches to the NDP, the application presented in this work does not require significant additional computational complexity as compared to the base model. Instead, the addition presented here consists of only a single constraint and one decision variable for each cell. However, as Waller et al (2013) note, the base StrSODTA LP model grows significantly with the number of demand scenarios and the number of paths considered, resulting in additional computational time.

4. MODEL DEMONSTRATION

This section demonstrates the LP NDP model described in the previous section. The model was solved using the GAMS programming interface and the optimization software package CPLEX. A network consisting of 15 cells, 2 origins, and 2 destinations was selected in order to isolate the impact of the strategic approach on infrastructure expansion decisions. The origin cells are 1 and 2, while the destination cells are 14 and 15. This network consists of two arterial corridors and a small “highway” segment with greater flow and capacity.

This network contains three paths for each of the four possible origin-destination pairs. Table 2 contains the demand parameters for this demonstration, including the aggregate demand in each demand scenario, the proportions of the total demand for each OD pair, and then the proportions of the OD demand that leave at each of the four included departure times; for simplicity, the departure time proportions are assumed to be the same for each demand scenario, following a "peak" pattern, but the total demand in each scenario as well as the proportions of the demand for each OD pair for each scenario are changing. This demonstration includes three demand scenarios representing the average congestion case, the lightly congested case, and the heavily congested case respectively. Forty time periods were simulated to ensure all demand was able to exit the network even in the heavily congested case.

Results are demonstrated under a varying budget, where the cost is $\beta_i = \$10{,}000$ add one unit of capacity to a cell. The budget was varied from 0-200,000, or the equivalent of adding 20 units of capacity. Figure 2 displays the results for total system travel time (including all demand scenarios) corresponding with the varying budget for the cases of considering one demand scenario (equivalent to the expected demand case), considering the average and lightly congested demand scenarios, and considering all three demand scenarios.

Table 2. Demand parameters for the demonstration network

Demand Scenario	Total Demand	Proportion of demand for OD pair RS			
		(1,14)	(1,15)	(2,14)	(2,15)
Average congestion case	210	0.3	0.2	0.2	0.3
Light congestion case	190	0.4	0.25	0.2	0.15
Heavy congestion case	230	0.43	0.2	0.17	0.2

OD Pair	Proportion of OD demand at departure time			
	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$
(1,14)	0.35	0.35	0.12	0.18
(1,15)	0.15	0.55	0.2	0.1
(2,14)	0.3	0.5	0.15	0.05
(2,15)	0.25	0.45	0.15	0.15

Figure 2 shows a nonlinear decrease in total travel time with increasing budget. Additional analysis shows that including one, or two, or three, demand scenarios in order to solve for the optimal capacity additions in the demonstration network results in selecting generally (but not always) the same set of links. The largest percentage of the budget is added to cell 8 in all cases. However, the exact amount to be added to each cell differs, indicating that accounting for strategic behaviour may result in a different set of project rankings.

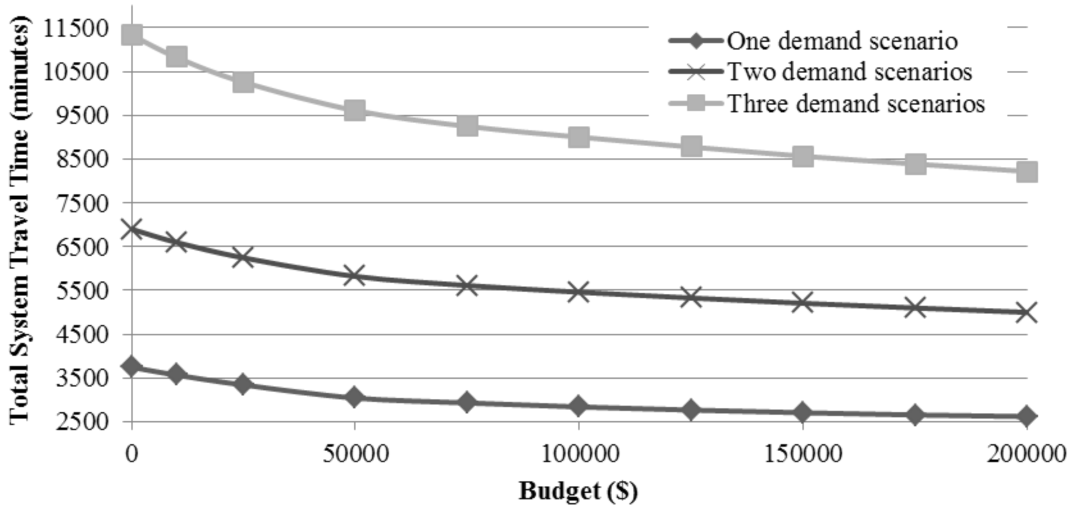

Figure 2. Total system travel time for the StrSO DTA LP model under a varying budget

5. CONCLUSION

This work proposed a system optimal network design model that accounts for stochastic demand using a strategic approach. In the strategic approach, the optimal path *proportions* are assigned so as to minimize the expected total system travel time in all demand scenarios. However, the actual path flows that will be realized for a given demand realization will not consist of a system optimal solution for that individual demand realization. This work has presented an application of the StrSO DTA LP model to the network design problem, and results demonstrate the differences between accounting for the strategic approach and simply the expected demand case. The system optimal model in this work represents a lower bound of the user equilibrium problem, but future work will examine a user optimal approach to representing strategic flows using the linear programming approach.

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