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An Evaluation Framework for High-Occupancy/Toll (HOT) Lanes

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1 *Abstract*

2 A high-occupancy/toll (HOT) lane is an increasingly popular form of traffic management which

3 reserves a set of freeway lanes for HOVs and transit users, while allowing low-occupancy

4 vehicles (LOV) to enter for a fee. In turn HOT lanes must maintain a minimal level of service

5 which is accomplished by regulating the volume of entering LOVs. Modeling the choice process

6 which dictates the volume of LOVs that choose to pay and take the HOT lane is integral in the

7 toll setting process, and the focus of this paper. Two simple formulations (an all-or-nothing

8 assignment and an additive logit model) are compared with a formulation based on the

9 population VOT distribution which is shown to be superior through numerical analysis. Both

10 static and dynamic toll setting algorithms are developed based on the proposed lane choice

11 model, and their performance is compared (under deterministic traffic behavior) in regards to the

12 performance of an HOT facility.

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14

1 1. INTRODUCTION

2 High-occupancy/toll (HOT) lanes refer to high-occupancy vehicle (HOV) facilities that allow
3 lower-occupancy vehicles (LOV) to pay a toll to enter. HOT lanes represent one way to utilize
4 the remaining capacity of HOV lanes, and are becoming an increasingly prevalent form of
5 congestion pricing in the U.S. and internationally. The first HOT lane was implemented in 1995
6 on State Route 91 in Orange County, California. The 1997 FasTrak project on I-15 in San-Diego
7 and the 1998 QuickRide project on I-10 and US290 in Houston in 1998 (1) converted HOV lanes
8 to HOT lanes by adding a toll. Additional HOT projects are under consideration in Illinois,
9 Washington D.C. (I-495 & I-95/395), and Atlanta (I-75 & I-85) (2).

10 HOT lane operation policies have multiple objectives –to maximize throughput of the
11 entire freeway (both general purpose (GP) and toll lanes), and to provide free-flow traffic service
12 on the toll lane. Because HOV lanes are designed “first and foremost to provide less congested
13 conditions for carpoolers and transit users” (3) priority is often given to the second objective

14 In order to guarantee a minimal level of service on the HOT lane the volume of entering
15 LOVs must be regulated. The volume of LOVs that will choose to pay and take the HOT lane is
16 dependent on the toll value, the user’s value of time (VOT), and the travel time savings gained
17 from taking the HOT lane relative to the GP lane. Modeling this choice process is crucial to
18 understanding how tolls should be set, and forms the focus of this paper. Two simple
19 formulations (an all-or-nothing assignment and an additive logit model) are first discussed,
20 followed by a formulation we believe to be superior, based on the population VOT distribution.
21 Both static and dynamic tolls are evaluated, and the proposed model is compared with alternative
22 lane choice models in regards to the performance of an HOT facility. The contributions of this
23 paper are (1) a comparison of three choice models; (2) development of a toll-setting algorithm

1 based on the preferred choice model; and (3) numerical comparison of these choice models and
2 various toll algorithms.

3 The remainder of this paper is organized as follows: Section 2 reviews related literature,
4 Section 3 describes the problem methodology, Section 4 provides numerical results for a
5 demonstrative facility, and Section 5 concludes the paper.

6 **2. LITERATURE REVIEW**

7 In practice, HOT tolling strategies vary significantly. The toll in HOT facilities may be fixed in
8 value (I-15, Salt Lake City, Utah; US 290, Houston, Texas); it may be pre-scheduled by time of
9 day (SR-91, Orange County, California; I-25, Denver, Colorado; I-10W, Houston, Texas); or it
10 may be determined in real time (I-15, San Diego, California; I-95 Miami, FL; SR-167, Seattle,
11 WA; I-394, Minneapolis, MN; I-35W, Minneapolis, MN) (4). The plan for the I-30 Tom Landry
12 Freeway in Dallas was to use pre-scheduled tolls (updated monthly) during the first six months
13 of operation, and dynamic tolls thereafter (5).

14 The toll rates for I-394 HOT lanes in Minnesota can be adjusted as often as every three
15 minutes. When a change in density on the HOT lane occurs, the rate is adjusted according to a
16 predefined lookup table (6). In contrast to pre-defined look-up tables various optimization based
17 approaches have been proposed in the literature. Michalaka, Lou, and Yin (7) developed a
18 scenario-based robust toll optimization model, and used simulation to compare their model
19 results with network performance under a density lookup table. The results showed substantially
20 less toll fluctuations, and better performance for the proposed model. Morgul and Ozbay (8)
21 proposed a methodology to extend the application of dynamic tolling to two neighboring tolled
22 facilities. A tolling algorithm was developed to calculate toll rates and define route decisions of
23 users of two parallel routes. Microscopic simulation of the Holland and Lincoln tunnels from NJ
24 to NYC was conducted to compare static and dynamic density-based tolls. The simulation results

1 showed that average occupancies decreased 36% for Holland Tunnel and 11% for Lincoln
2 Tunnel, and average speeds increased by 24% and 4%, respectively, as a result of the dynamic
3 pricing case compared with the static pricing case. Ozbay et al (9) conducted mesoscopic
4 simulation of bridges in Manhattan, comparing prevailing static tolls with density-based PID
5 dynamic tolls using TransModeler. Simulation results again support the use of dynamic pricing
6 for reducing congestion, especially at peak periods, when the occupancies at some of the tolled
7 crossings are high. In addition, 16% higher toll revenues are collected in the hypothetical
8 dynamic pricing scenario compared to the simulated static tolled scenario.

9 Lou, Yin and Laval (10) extended the work by Yin and Lou (11) and further developed a
10 reactive self-learning approach for determining time-varying tolls in response to the detected
11 traffic arrivals with a more realistic representation of traffic dynamics and an explicit
12 formulation for toll optimization. The approach learns motorists' willingness to pay and
13 determines toll rates to maximize the freeway's throughput while ensuring superior travel service
14 to the users of the toll lanes. The logit parameters in this paper were based off those calibrated by
15 Lou et al (10).

16 Previous studies have also examined time-varying tolls for bottlenecks under conditions in
17 which optimal solutions can be analytically derived (12;13;14;15;16). A driver's VOT is central
18 to all toll-related research, and plays a major role in the choice models and algorithms described
19 below. VOTs are often assumed known, or to follow a known distribution. Various studies have
20 attempted to infer actual VOTs through the use of stated preference surveys (17) or examination
21 of travel patterns which provide the revealed preferences of commuters (18;19;20;21;10).
22 Results are highly variable, with calculated VOT's ranging from \$7-\$12/hr (17) to \$40-\$60/hr,
23 higher than the average \$36/hr wage (Prince, 2011). Goodall and Smith (19) found that the actual

1 proportion of HOT users was determined mainly by time of day, and the impact of toll was rather
2 modest. Cho (22) discovered a limited correlation between time saved and proportion of travelers
3 using HOT. In this work the VOT distribution is used to determine lane choice behavior.

4 Multiple region- specific case studies have been conducted to investigate the impact of
5 implementing HOT lanes and other types of managed lanes (23-31). In addition, current HOT
6 facilities have been evaluated in attempts to identify user preferences and improve future
7 performance (32;33;34).

8 3. MODEL

9 This section describes the model used for the experiments in this paper, first discussing the
10 traffic flow model, lane choice model, and toll algorithms and then describing the simulation
11 process. The framework for this investigation is a freeway with two lane groups: an untolled
12 group of GP lanes, and a managed HOT lane which is free to HOVs, but accessible to LOVs
13 which are willing to pay a toll. Both lane groups are spatially homogeneous, with bottlenecks at
14 the downstream end of capacity \bar{q}_{GP} and \bar{q}_{HOT} . The free-flow time on the two lane groups are
15 denoted τ_{GP}^0 and τ_{HOT}^0 , both assumed to be integers in the unit system chosen. We assume there
16 are no onramps or offramps in the region of interest to focus our investigation on the issue of the
17 lane choice model. We discretize time into $T + 1$ intervals of equal length, and index these
18 intervals with $t \in \{0, 1, \dots, T\}$, where the index t refers to the time at the start of the t -th interval.

19 Let V be the set of vehicle classes (for instance, single-occupant vehicles, HOVs, and
20 transit) and $\bar{V} \subseteq V$ the set of vehicle classes which must pay the toll. The median value of time
21 for class v is ζ_v . Let d_v^t be the number of vehicles of class v arriving at the GP/HOT diverge

1 point during time interval t . The mean occupancy of class v is o_v . The toll on the HOT lane in
 2 time interval t is c^t .

3 The following quantities are endogenous: the travel times by lane group over time (τ_{GP}^t
 4 and τ_{HOT}^t are the travel times of a vehicle entering the system at the start of time t), the
 5 proportion of vehicles of class v choosing the HOT lane over time (p_v^t), the average vehicle
 6 travel time ($AVTT$), and the average passenger travel time ($APTT$). The latter two are calculated
 7 by:

$$AVTT = \frac{1}{\bar{d}} \cdot \sum_t \sum_v d_v^t (p_v^t \tau_{HOT}^t + (1 - p_v^t) \tau_{GP}^t) \quad (1)$$

$$APTT = \frac{1}{\bar{o}} \sum_t \sum_v o_v d_v^t (p_v^t \tau_{HOT}^t + (1 - p_v^t) \tau_{GP}^t) \quad (2)$$

8 where $\bar{d} = \sum_t \sum_v d_v^t$ is the total number of travelling vehicles and $\bar{o} = \sum_t \sum_v o_v d_v^t$ is the total
 9 number of travelling persons (occupants). The toll revenue collected is given by

$$R = \sum_t \sum_{v \in \bar{V}} d_v^t p_v^t c^t \quad (3)$$

10 3.1 Traffic Flow Model

11 Our traffic flow model is simple, consisting of facilities with a single bottleneck at the
 12 downstream end, and a higher capacity upstream. We implicitly assume that the total demand
 13 never exceeds this upstream capacity – this is reasonable if the freeway immediately upstream of
 14 the modeling area has the same characteristics as the section in the model, since the flow across
 15 the boundary can never exceed the capacity. We further assume that the bottleneck capacity and
 16 study area are large enough that the entering flows are never restricted by a queue at the
 17 downstream bottleneck.

1 To describe the evolution of congestion and vehicle flows, we use upstream and
 2 downstream cumulative counts: $N_i^\uparrow(t)$ and $N_i^\downarrow(t)$ are the arrival and departure curves,
 3 respectively representing the total number of vehicles that have passed the upstream and
 4 downstream end of lane group $l \in \{GP, HOT\}$ at the start of time interval t . These values
 5 completely define the state of our traffic model. Our model is discrete, so we calculate these
 6 cumulative counts only at times $t \in \{0, 1, \dots, T\}$ and linearly interpolate to obtain intermediate
 7 counts. The upstream count equations are:

$$N_{GP}^\uparrow(t) = N_{GP}^\uparrow(t-1) + \sum_v d_v^t (1 - p_v^t) \quad (4)$$

$$N_{HOT}^\uparrow(t) = N_{HOT}^\uparrow(t-1) + \sum_v d_v^t p_v^t \quad (5)$$

8 for $t > 0$ with $N(0) = 0$ for all lane groups. If there were no bottlenecks downstream, we would
 9 have $N_i^\downarrow(t) = N_i^\uparrow(t - \tau_i^0)$; however, the bottleneck constrains the exiting flow in any time interval
 10 to be no greater than \bar{q}_l . Thus

$$N_i^\downarrow(t) = \min\{N_i^\downarrow(t-1) + \bar{q}_l, N_i^\uparrow(t - \tau_i^0)\} \quad (6)$$

11 for $l \in \{GP, HOT\}$. To reflect system conditions at times not corresponding to one of the
 12 discretization points t , linear interpolation is used:

$$N(t) = (t - \lfloor t \rfloor)N(\lfloor t \rfloor) + (\lceil t \rceil - t)N(\lceil t \rceil) \quad \text{if } t \in (0, T) \text{ is not an integer} \quad (7)$$

13 The same formula applies to both upstream and downstream counts of all lane groups. Using
 14 this continuous formula, an inverse function can be meaningfully defined:

$$T(n) = \min\{t : N(t) \geq n\} \quad (8)$$

1 representing the time at which the n -th vehicle passes the point where N is measured (omitting
 2 subscripts and superscripts for brevity). Thus, the travel time for a vehicle entering lane group l
 3 at time t is

$$\tau_l(t) = T_l^\downarrow(N_l^\uparrow(t)) - T_l^\uparrow(N_l^\uparrow(t)) \quad (9)$$

4 3.2 Lane Choice Model

5 The calculation of p_v^t is the major object of study in this paper, and three rules are presented
 6 here. The first two (all-or-nothing and additive logit) are discussed briefly, along with reasons
 7 we believe them both to be lacking. This leads to discussion of the third rule (based on the VOT
 8 distribution itself), which we believe to be superior.

9 All of these rules are based on two quantities: the travel time savings from the HOT lane
 10 $\Delta\tau = \tau_{GP} - \tau_{HOT}$ and the toll c for paying vehicles. We seek a mapping $P_v : \mathfrak{R}^2 \rightarrow [0,1]$ which
 11 gives the proportion of vehicles choosing the HOT lane as a function of the travel time
 12 differential and cost: $p_v^t = P_v(\Delta\tau, c)$. Time superscripts are omitted in this section for brevity.

13 The simplest possible rule is an all-or-nothing assignment

$$P_v^{AON}(\Delta\tau, c) = \begin{cases} 1 & \zeta_v \Delta\tau > c \\ \bar{q}_{HOT} / (\bar{q}_{HOT} + \bar{q}_{GP}) & \zeta_v \Delta\tau = c \\ 0 & \zeta_v \Delta\tau < c \end{cases} \quad (10)$$

14 assigning all vehicles to the lane group with lower generalized cost. In the results reported here
 15 the “tiebreaking” rule in all choice models is to follow capacity proportions. This mapping is far
 16 too simplistic: it assumes there is literally no variation in VOT across vehicles of class v , and it is
 17 not continuous in $\Delta\tau$ or c , making it highly unlikely that a stable solution will emerge if there is
 18 any congestion at all. The one situation where this rule is plausible is with vehicle classes which

1 do not pay the toll – in this case, the model reduces to simply choosing the lane group with lower
 2 travel time (which should be the HOT lane).

3 A simple improvement to the all-or-nothing assignment is a logit model, with the utility
 4 of each lane being given by $U_l^v = -\zeta_v \tau_l - c + \varepsilon$, adding independent and identically distributed
 5 Gumbel disturbance terms to the generalized cost, and calculating $p_v = \Pr(U_{HOT}^v > U_{GP}^v)$. The
 6 resulting mapping is

$$P_v^{LOGIT}(\Delta\tau, c) = \frac{1}{1 + \exp(\theta(c - \zeta_v \Delta\tau))} \quad (11)$$

7 where θ reflects the variance in the disturbance terms. Unlike P_v^{AON} , this mapping is continuous
 8 in its parameters. However, it suffers from several deficiencies as well: adding the disturbance
 9 term to the generalized cost suggests that the variation in population preferences is due to factors
 10 independent of cost and travel time. Since the HOT lane and general purpose lanes are parallel
 11 and presumably similar facilities, there does not seem to be much room for unobserved variation.
 12 Furthermore, this specification leads to counterintuitive results (if both lane groups are at free
 13 flow, single-occupant vehicles will use both lanes even if the HOT lane is tolled).

14 Instead, we feel that the primary variation in lane choice preferences is due to the
 15 distribution of VOT across the population: even within a single vehicle class, different drivers
 16 will have different values of time due to demographic factors, trip purposes, and individual
 17 heterogeneity. In this case, the proportion of travelers choosing the HOT lane is exactly the
 18 proportion of travelers whose VOT exceeds $c/\Delta\tau$. If F_v denotes the cumulative distribution
 19 function (CDF) of VOT for vehicle class v , then we have

$$P_v^{VOT}(\Delta\tau, c) = 1 - F_v(c/\Delta\tau) \quad (12)$$

1 with the convention that $F_v(c/\Delta\tau) = 1$ if $\Delta\tau$ is zero. Assuming that F_v is continuous and strictly
 2 increasing, this mapping is continuous and has the advantage of directly reflecting variation in
 3 VOT across the population. Notice that if the travel times of both lanes are equal, no travelers
 4 will choose the HOT lane, alleviating the problem seen with the simple logit model.

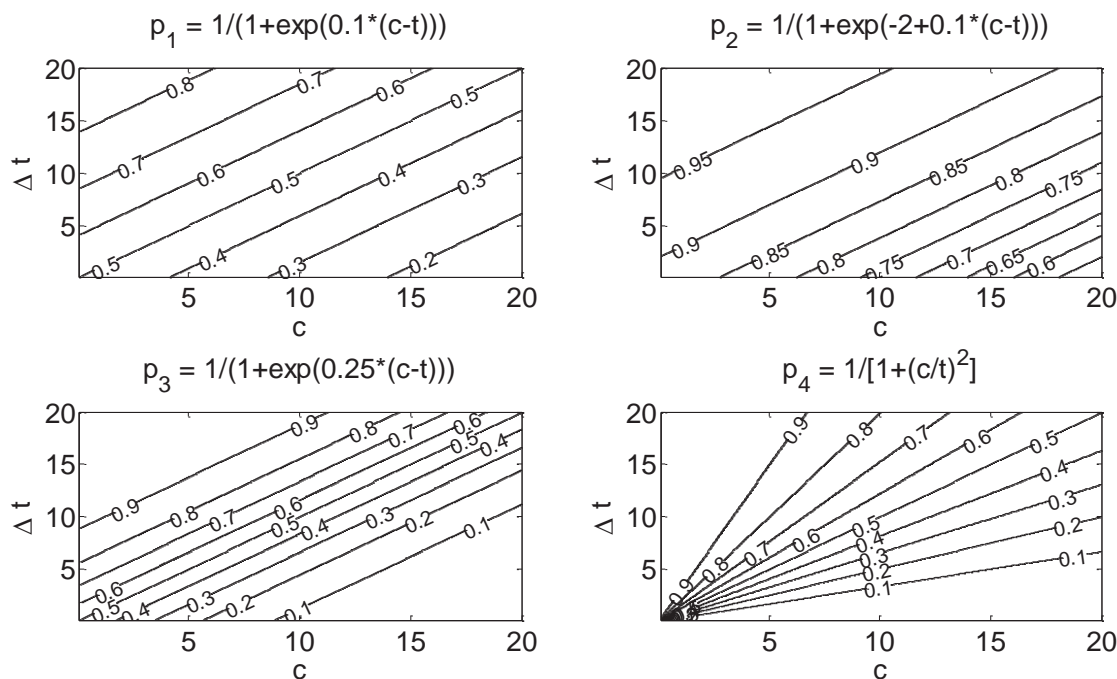
5 A natural question with this mapping is how the VOT distribution F_v should be
 6 calibrated. Kleiber and Kots (2003) review several families of distributions used to model
 7 income, including the four-parameter general beta distribution of the second kind (GB2), as well
 8 as its special case three-parameter Singh-Maddala distribution, a.k.a. Burr XII or simply the Burr
 9 distribution. The CDF of the latter is $F(x) = 1 - [1 + (x/b)^a]^{-q}$ (ibid.; p. 198, eq. 6.43). In this
 10 paper we use a further simplified variant:

$$F\left(\frac{c}{\Delta\tau}; \zeta, \gamma\right) = 1 - \frac{1}{\left[1 + \left(\frac{c}{\zeta \cdot \Delta\tau}\right)^\gamma\right]} \quad (13)$$

11 where γ is a shape parameter affecting the relative width of the VOT distribution. Using the US
 12 income distribution in 2008, the 25th percentile to median ratio suggests $\gamma \sim 1.5$, and the 75th
 13 percentile to median ratio suggests $\gamma \sim 2$. The experiments reported here show results for both
 14 values.

15 Figure 1 illustrates the fundamental difference between P_v^{LOGIT} and P_v^{VOT} graphically,
 16 showing contour plots of p in $(c, \Delta\tau)$ space. The lower-right panel (p_4) represents P_v^{VOT} , while
 17 the other panels represent P_v^{LOGIT} with different specifications of the logit parameters. The
 18 diagonal proportion contour line in p_1 is reasonable, suggesting that half of drivers will use the
 19 HOT lane if the travel time difference is equivalent to the toll in terms of median VOT.

1 However, the diagram also says that even with zero travel time difference, p declines very
 2 gradually with c . It also implies that with zero toll, p increases only gradually as $\Delta\tau$ increases.
 3 Calibrating the parameters of the logit function does not change anything fundamental. Adding a
 4 constant (mode bias) in p_2 shifts the diagram to the right, but we still get the same problematic
 5 parallel lines, and now even for the proportion of 0.5 observed VOT will not meet the median
 6 VOT. Changing the cost sensitivity in p_3 changes the density of the lines, but otherwise still
 7 nothing much has changed. By contrast, if the most critical stochastic feature is VOT variability
 8 among travelers, the cost and time observations at a given facility for a constant toll with a given
 9 capacity ratio (and therefore a given HOT proportion) should correspond to the VOT of the same
 10 proportion in the population. This immediately suggests that the contour lines of the choice
 11 function should form a fan of rays, as in p_4 (the P_v^{VOT} model).



12
 13 **FIGURE 1 Choice probability contour plots for additive Logit (p_1 , p_2 & p_3)**
 14 **and for Burr VOT distribution (p_4)**

1 3.3 Toll Algorithms

2 In this paper, we compare two toll-setting methods: a constant toll, and a time-varying toll
 3 adjusted to maximize inflow to the HOT lane without exceeding its bottleneck capacity (the “full
 4 utilization” algorithm). In a deterministic setting, the full utilization algorithm minimizes the
 5 total occupancy-weighted travel time: any solution that does not fully utilize the capacity of the
 6 HOT lane cannot be optimal, because it is possible to move some SOVs from the GP to the HOT
 7 lane, while keeping the travel time of the HOT lane at free flow. This means that vehicles that
 8 previously used the HOT lane are not affected, and all others gain.

9 In all the solutions that fully utilize the capacity of all lanes, the total duration of
 10 congestion will be the same, and the non-weighted AVTT will be the same. It is just a queue, and
 11 if some get ahead in the queue their gain is identical to the loss of others. In terms of occupancy-
 12 weighted APTT, it is clear that any switch allowing an HOV to get a head in the queue at the
 13 expense of an SOV improves the outcome. So a solution where HOVs travel at free flow is
 14 optimal. To find the toll corresponding to full utilization (hereby referred to as FU toll), we solve
 15 the following equation for c :

$$\sum_{v \in \bar{V}} d_v P_v^{VOT}(\Delta\tau, c) + \sum_{v \in V \setminus \bar{V}} d_v = \min \left\{ \bar{q}_{HOT}, \sum_v d_v \right\} \quad (14)$$

16 Time superscripts are omitted for clarity, and this equation is solved at the start of each time
 17 interval to determine the dynamic toll for that period. For the special case when only a single
 18 class of vehicles \bar{v} must pay the toll, and when this class has a VOT distribution described by the
 19 Burr distribution, the solution to this equation can be expressed analytically:

$$c = \Delta\tau \sqrt[\gamma]{\frac{d_{\bar{v}}}{\min \left\{ \bar{q}_{HOT}, \sum_v d_v \right\} - \sum_{v \neq \bar{v}} d_v} - 1}} \quad (15)$$

1 This is the formula used in the experiments reported in this paper.

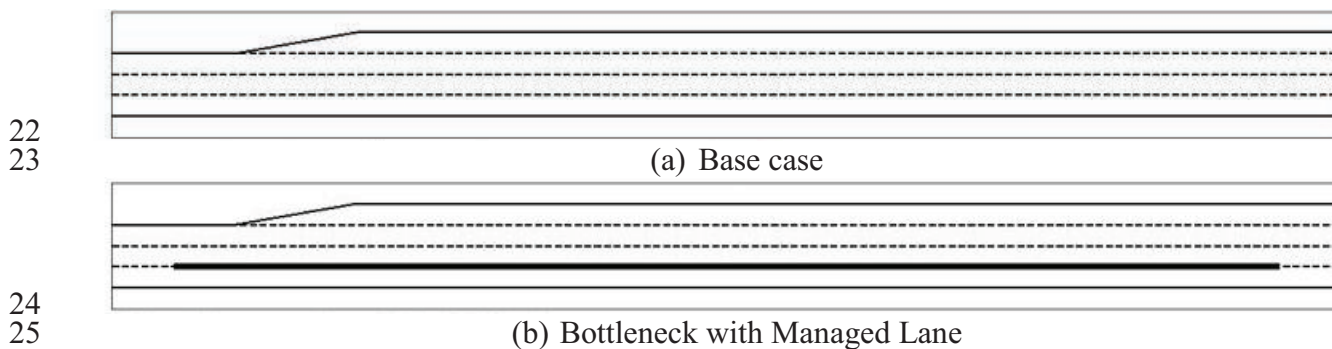
2 3.4 Simulation Algorithm

3 The system described above is implemented in a simulation program written in C. This
4 simulation performs the following steps:

- 5 1. Initialize: set $t = 0$, travel times on GP and HOT lanes to free flow, and all N values to
6 zero.
- 7 2. Calculate toll c (either constant or based on the full-utilization formula)
- 8 3. Calculate lane choice probability p'_v using one of the lane choice mappings described
9 above.
- 10 4. Propagate flow, updating upstream counts based on lane choice parameters and demand
11 during interval t , and updating downstream counts as described above.
- 12 5. Update statistics (lane group travel time, revenue, and other metrics)
- 13 6. If $t < T$, increment t and return to step 2. Otherwise, terminate.

15 4. NUMERICAL ANALYSIS

16 The results presented in this section are based on the facility depicted in Figure 2. The facility is
17 comprised of a single managed lane and three GP lanes which merge to two at the downstream
18 bottleneck. The HOT lane has a capacity of 1800 vph and each GP lane has a capacity of 2100
19 vph. The length is 10 km and the free flow speed is 100 km/h. The demand profile (provided in
20 Table 1) is chosen such that the LOV demand exceeds the GP lane capacity and a queue is
21 formed at the bottleneck.



27 **FIGURE 2 Case study facility. (a) base case - all lanes are for general purpose**
28 **(GP); and (b) one lane is managed.**

1 **TABLE 1 Peak Period Travel demand Profile**

	Average Occ.	7:00-8:00	8:00-9:00	9:00- 10:00
LOV	1.2	6300	5100	3900
HOV	4	600	600	600
Transit	40	300	300	300
Total		7200	6000	4800

2

3 **4.1 Facility Performance Measures**

4 The facility performance is evaluated based on three measures: APTT, AVTT, and revenue.

5 Results are shown for the base case where all lanes are GP, the HOV case where only HOV and

6 transit users use the managed lane, and each of the lane choice models considered under the

7 HOT case. Facility performance results are provided in figure 3. In addition, a more detailed

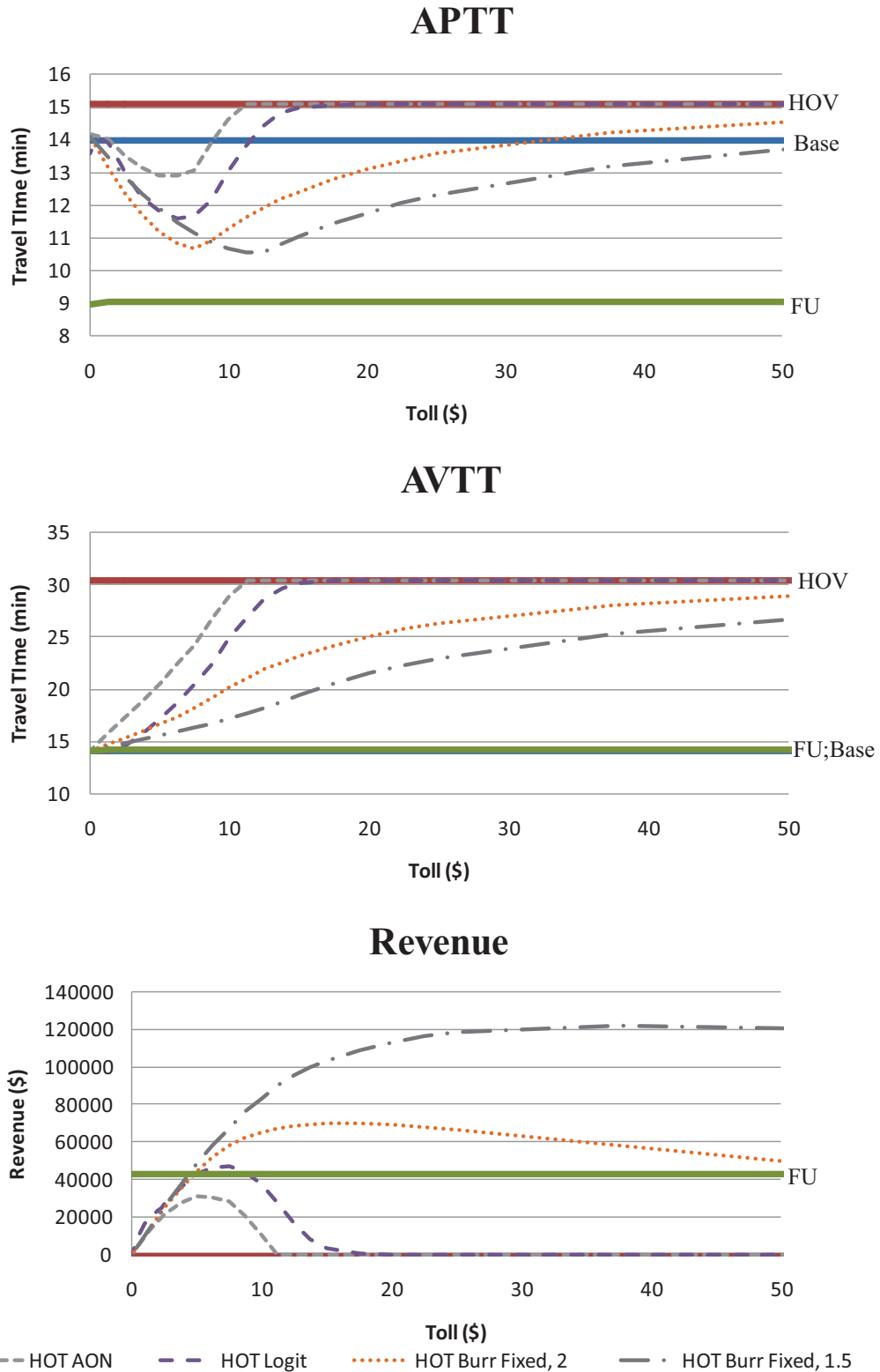
8 level of analysis is provided to depict lane choice behavior and lane properties temporally over

9 the entire peak period, including entering flow volumes, percent of LOVs choosing to take the

10 HOT lane, and travel time differences between the HOT and GP lane as a function of time.

11 For the analysis presented, median VOT is 15\$/hour, and the Logit parameter is set at $\theta =$ 12 0.25, as in Lou (2011). The Burr distribution was evaluated for both $\gamma=1.5$ and $\gamma=2$.

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FIGURE 3 APTT, AVTT, and revenue as a function of fixed toll value

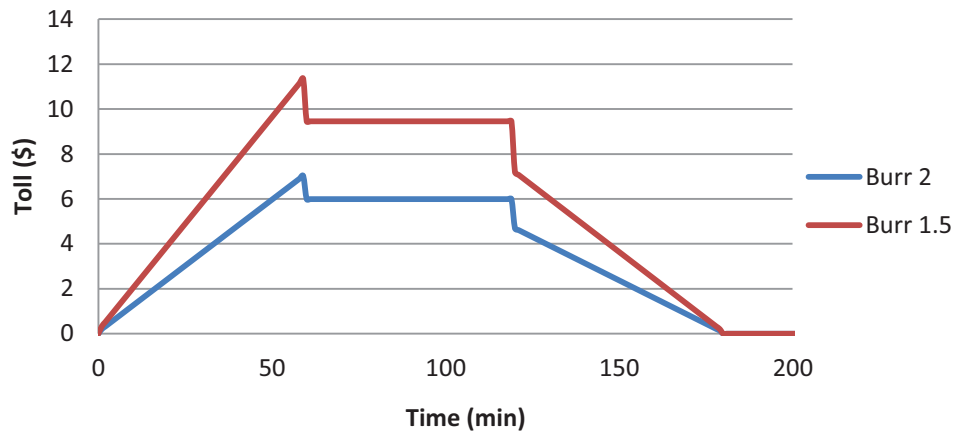


FIGURE 4 FU tolls over the peak period

Figure 3 illustrates the three performance metrics as a function of the fixed toll value for the base case, HOV case and each of the lane choice models. Of note, the FU tolls result in the same APTTs for both Burr parameters, and a 36% savings relative to the base case, because the FU tolls maximize the entering HOT lane volume subject to its capacity, as shown in Figure 5.a. For both $\gamma=1.5$ and $\gamma=2$, the FU toll values follow a similar pattern over the peak period (see figure 4) because γ is a shape parameter, resulting in an inflated toll value when $\gamma=1.5$ to accommodate the wider distribution of VOT. In addition the AVTT for both Burr parameters under FU tolls is equal to the base case AVTT, which makes the most optimal use of the facility in terms of vehicle travel. In contrast the AVTT increases as the fixed toll value increases for all other models, though at a slower rate for the Burr model. For a high enough toll value the AVTT converges with the HOV case under all fixed toll models, at which point no LOV users will opt to pay. The maximum toll value some users are willing to pay is highest for the Burr models and lowest for the AON model.

Similar APTT and AVTT behavior under Burr fixed tolls is illustrated for both parameters, with the main difference being a higher optimal fixed toll value for $\gamma=1.5$. The impact of these

1 inflated tolls is evident in the increased facility revenue for the $\gamma=1.5$. Based on Figure 3, a static
2 toll has the potential to save a significant portion of the average person travel time savings
3 achieved by a dynamic FU toll (i.e. the difference between the base case and the full utilization
4 toll). For example, a static toll of \$7.5 saves 3.3 minutes out of a potential APTT saving of 5
5 minutes, i.e. two thirds of the benefit.

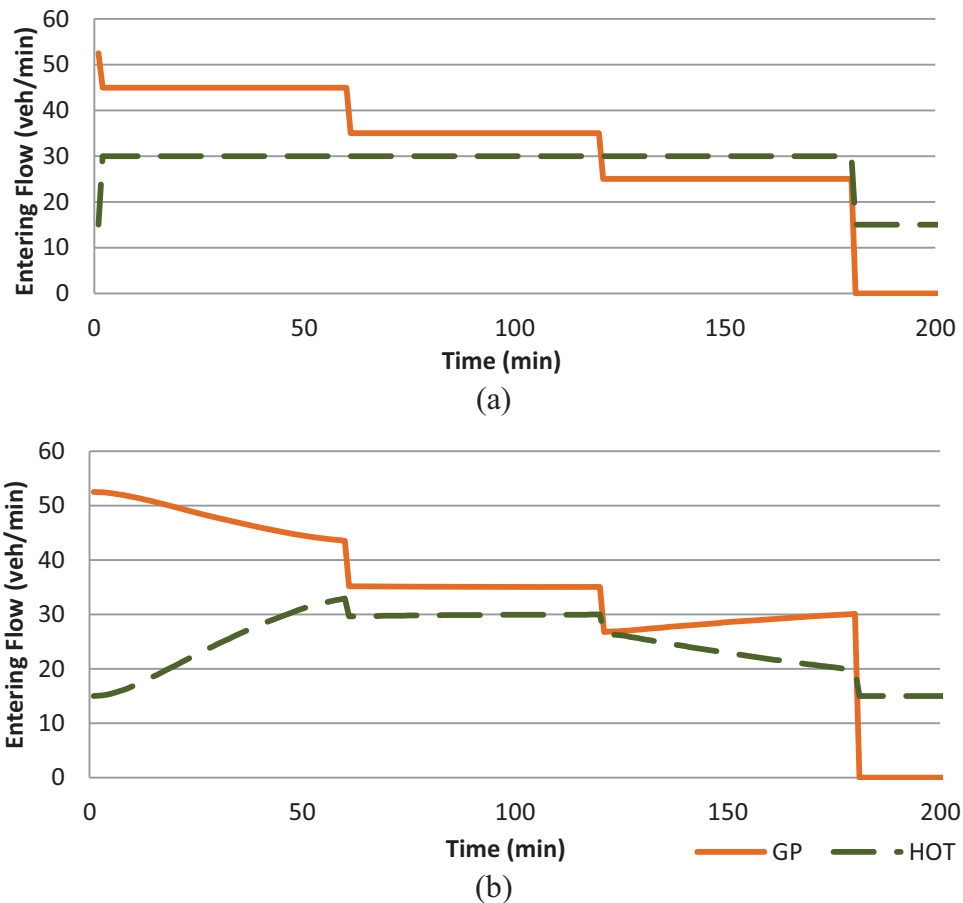
6 It should be mentioned that with zero toll the Logit model predicts that more LOVs will
7 use the HOT than required to equalize travel time, as a result GP travel time is shorter than the
8 HOT travel time, and all HOVs use the GP lane. This unrealistic behavior is a direct result of the
9 fundamental structure of the additive Logit choice model.

10 **4.2 Comparison of Fixed and FU Tolling Schemes**

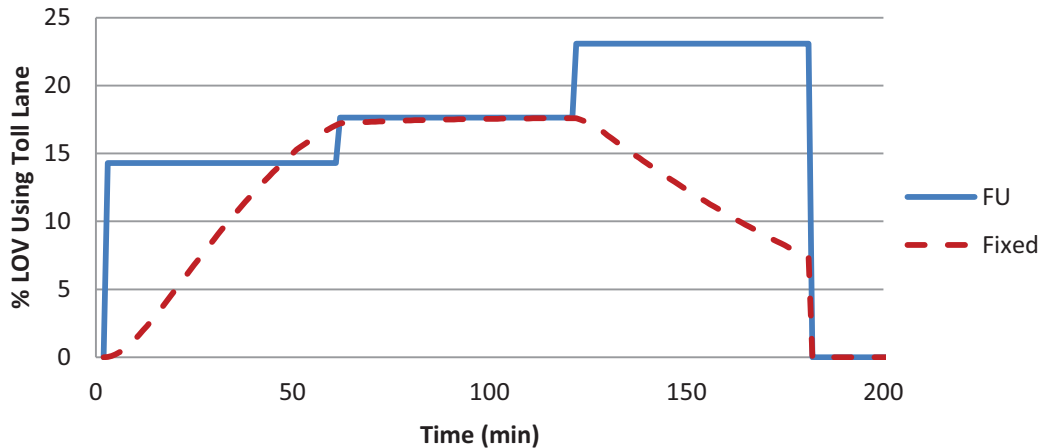
11 For the remainder of the analysis presented the Burr parameter is set at $\gamma=2$ and the fixed toll is
12 set to \$7.50, which is the fixed toll resulting in the lowest APTT for this lane choice map. Figure
13 5 depicts total volume entering each lane throughout the peak period for the Burr model with FU
14 and fixed tolls. For both cases, the maximum volume entering the HOT lane is equal to the lane
15 capacity, 30 vpm.

16 The increase in percent of LOVs using the HOT lane (see Figure 6) is due to a combined
17 impact of decreasing demand over the peak period, and the FU toll objective of keeping the HOT
18 lane at capacity. The increase in LOV usage occurs despite a decrease in travel time savings,
19 because of the falling toll. For the Burr fixed toll this is not the case, and the percent of LOV
20 users follows a similar profile to the travel time savings. The travel time savings are greater for
21 the fixed toll, meaning a larger discrepancy in lane performance between the HOT and GP lanes,
22 therefore a less equitable tolling policy.

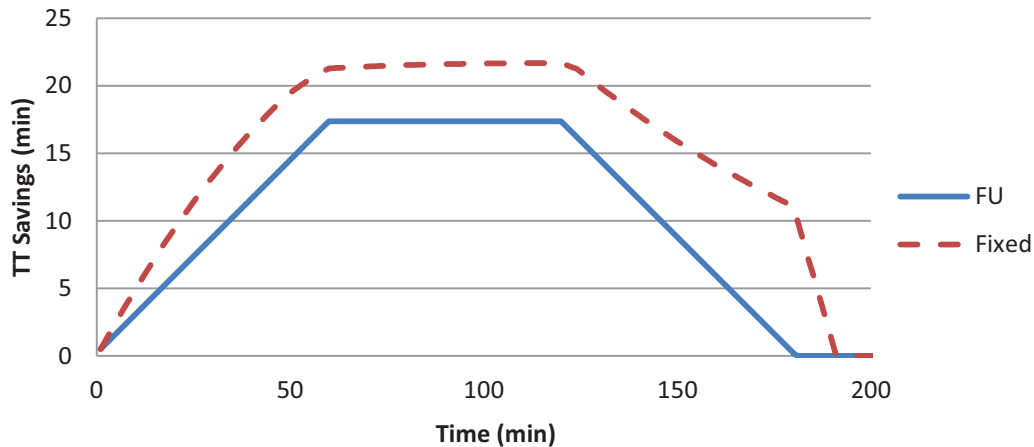
1 Figure 6 also indicates that 17% of SOV users pay to use the HOT lane during the middle
 2 of the peak period under both tolling schemes. Figure 7 illustrates travel time savings achieved
 3 by taking the HOT lane. Under a fixed toll of \$7.50 users save an average of 21.6 minutes
 4 whereas the FU toll value is \$6 over this same time interval and users save an average of 17
 5 minutes. Both tolling schemes equate to a willingness to pay \$0.35/min of travel time savings, or
 6 \$21/hr which is interestingly 40% higher than the median VOT of \$15/hr assumed in this work.



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11 **FIGURE 5 Entering Flow per Lane for (a) FU Tolls and (b) Fixed Toll=\$7.50**
 12 **during the peak period, assuming VOT distribution is Burr with $\gamma=2$**
 13



1
2 **FIGURE 6 Percent of LOVs choosing HOT lane during peak period**



3
4 **FIGURE 7 Travel time savings by taking HOT lane during the peak period**

5 **5. CONCLUSIONS**

6 This paper studied the impact of various lane choice models and toll algorithms in a
7 deterministic context. Based on this investigation, we believe that the lane choice model should
8 be directly linked to the VOT distribution in the population: it is a continuous mapping,
9 capturing the essential stochastic feature in the model (VOT). Numerical experiments comparing
10 the proposed choice model to other alternatives demonstrate its superior plausibility.

11 We also showed that the modeling framework enables the evaluation of "full utilization"
12 tolls and fixed tolls in comparison to the reference options of regular HOV lane, and base case

1 (no managed lanes). Still, there are many ways to build upon this research in the future: to name
2 only a few examples, exploring stochasticity in arrival rates and capacities, travel time reliability,
3 and more sophisticated networks involving on-ramps and off-ramps would all be of great
4 practical and theoretical interest.

5

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