PREDICTING DISRUPTED NETWORK BEHAVIOUR INCORPORATING USER EQUILIBRIUM WITH RECOURSE

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ABSTRACT

Disruptions to a network create uncertainty which affects the rationality of a user as well their familiarity of the road network. Accordingly traditional equilibrium concepts are not applicable when disruptions are present on a network, especially when developing models for the purposes of incident mitigation or disaster planning. To address this need, this work presents the disrupted equilibrium assignment with recourse (DEAR) model that incorporates a decision-making process in which users gain information about the uncertainty created by a disruption as they travel though the network. The core of the work builds on an existing static, Disrupted Network Assignment Model (DNAM) by incorporating and adaptation of the user equilibrium with recourse (UER) model. Demonstration of the model methodology is presented on a simple network with results indicating the necessity to account for the adaptive behaviour of users in light of a disruption.

Keywords: Disrupted Network, User equilibrium with Recourse, Perceptions, Adaptive Routing

1. INTRODUCTION

Traditional transport planning models portray travel behaviour under normal day-to-day conditions. However, in the event of disruptions, ranging from minor events such as breakdowns or accidents to major catastrophic events such as flooding or earthquakes, user perceptions change and therefore traditional models can no longer be used to accurately evaluate network performance. Nevertheless, it is crucial to form network models to quantify disrupted conditions for the purpose of incident mitigation or disaster planning in order to support city planning and growth management. This work focuses on new techniques for modelling disrupted network conditions.

Additionally, an important difference in disrupted network user behaviour results from the information cues a traveller gains throughout travel. Using knowledge learned from day-to-day experiences, users may recognize the “severity” of a disruption on a link (i.e., long queue length) upon arrival, and react by changing to a more suitable route that is less disrupted. Traditional methods that do not account for adaptive routing will not be able to represent this scenario. Therefore in order to capture the impact of information gained through perception on route choice behaviour, this work proposes the disrupted equilibrium assignment with recourse (DEAR) model, a novel framework that adapts a user equilibrium with recourse (UER) model (Unnikrishnan and Waller, 2009) with a method of accounting for the change in user perception due to a disruption that has a foundation in real experimental data.

The focus of the study is to understand the impact on network flows when users not only transform costs in light of a disruption but also learn about the information of a disruption en route to their destination. This work incorporates an existing method called the Disrupted Network Assignment Model (DNAM) that applied the concept of travel time perception transformation in light of a disruption (Dixit et al., 2012). DNAM provided a framework for predicting driver reactions to network disruptions and presented trends and patterns relating to infrastructure and severity of the incident. However, DNAM does not consider the uncertainty of the network resulting from a disruption and how users adapt to knowledge of this uncertainty. Therefore, this research extends DNAM by incorporating an adapted UER model that accounts for re-evaluation of route choice by users in disrupted conditions.

2. BACKGROUND

Road users’ behavioural response to disruptions of a transport network is essential to understand when formulating traffic assignment models which account for disruptions. Numerous empirical studies have been completed presenting the impact of travel time uncertainty on route choice (Lam and Small, 2001, Danczyk et al., 2010, Wu and Nie, 2011). Danczyk et al. (2010) presents the study of the changes in travel demand post the I-35W bridge collapse in Minneapolis, Minnesota in 2007. The paper indicates that following a disruption an avoidance phenomenon is observed resulting in travel time variability. In addition there is a change in route choice behaviour following the re-establishment of the pre-collapse routes potentially due to the memory of the event or the impact of the change in route following the event. However the I-35W bridge collapse studies consider the long term impact of disruptions whereas this research focuses on more immediate and short term impacts of a disruption on driver behaviour.

Limited research has been conducted on understanding the perception of travel time and how it is impacted by disruptions and incidents. A majority of the research regarding driving perception is related to the context of reliability. There have been several studies that have investigated the perception of travel time comparing different modes (Chapman et al., 2006, Taylor et al., 2009, van Exel and Rietveld, 2010) and their willingness to pay for it. Several of these studies show that perceived travel times significantly differ from actual travel times (Wu et al., 2009). Moreover, Montello (1997) found that the perceived travel time is affected by a number of environmental features including perception of distance, route, and travel effort. The presence of an incident will affect all these features and accordingly affect the perception of travel time. These studies highlight the importance of accounting for user perceptions within transport models. However there is a gap in knowledge in the application of perceptions within transport models. One such model that does incorporate perceptions is discussed in Dixit et al. (2012) and detailed further in the next section. Though this model accounts for the change in user perceptions under disrupted conditions it does not account for the adaptive behaviour of users when faced with a disruption which is the primary goal for this model formulation.

Network assignment models that account for adaptive en-route behaviour have primarily focused on transit assignment and capacitated networks (Nguyen and Pallottino, 1989, Marcotte et al., 2004). There have also been simulation based approaches, for example Gao and Chabini (2006) presents dynamic user optimal traffic assignment in which a user is assigned to links dependent on the state of the link. Possibility theory (Henn and Ottomanelli, 2006) and fuzzy logic theory (Koutsopoulos et al., 1994) have been used to model the impact of information and driving behaviour perceptions on equilibrium flows. This particular study incorporates User Equilibrium with Recourse (UER) presented by Unnikrishnan and Waller (2009). UER is a static equilibrium that accounts for en-route route choice in the presence of information. The static nature of the model creates a tractable mathematical solution methodology allowing an extension to this specific problem investigating adaptive routing in the presence of a disruption.

2.1 Disrupted Network Assignment Model (DNAM)

The method proposed in this research incorporates the DNAM, which was introduced by Dixit et al. (2012) . DNAM is a novel approach to accounting for the impacts of traffic disruptions in transport models. While previous research by Sheu et al. (2001) used stochastic assignment methods to account for the uncertainty created by the presence of a disruption, the DNAM methodology is based on the results from psychological testing of road users to determine how the perceptions of travel time distributions change for different types of incidents.

The DNAM approach was separated into three stages. The first stage determined equilibrium travel times or costs using deterministic user equilibrium (Sheffi, 1985). These flow patterns are then assumed to be the mean link travel times prior to the disruption. The associated standard deviation of each link was determined based on the linear mean-standard deviation relationship of travel times (Sirivadidurage et al., 2009) based on relevant field data. The second stage DNAM transforms these travel costs based on the results of the travel time perception experiments. In addition to transforming the travel times the data collected was used to determine the severity of the disruption. The third stage of the DNAM model uses these transformed travel times to assign the traffic using logit model to determine the transformed flows.

Although this model highlighted the importance in accounting for travel time perceptions in uncertain conditions as well as the impact of transforming travel times in light of a disruption, it does not consider the adaptive behaviour of users when faced with a disruption. This study investigates users' experience of disruptions and how this affects route choice by implementing user equilibrium with recourse to account for en-route re-evaluation of users.

2.2 User equilibrium with recourse (UER)

The DEAR model introduced in this work seeks to account for the impact of information gained *during travel* on decision-making in disrupted networks by incorporating the UER assignment model. Traditional equilibrium models assume that users deterministically choose minimum cost paths, and then remain on that path regardless of realized network conditions; probabilistic link costs are not considered. However, UER incorporates the way that road users react to information regarding uncertain system states gathered from network indicators and make decisions en-route as a result of this information. The reader is referred to Unnikrishnan and Waller (2009), where UER is introduced and discussed in detail, including mathematical formulations; this work contains only the description that is relevant to the application presented in the DEAR model.

UER is a static equilibrium model that accounts for one-step local information and user recourse on account of gaining that information. Thus, in a UER scenario, each link on the network could possess multiple states with a certain probability of occurrence. Depending on the state of the link, a user would choose the next link to travel on to reach his or her destination. To account for users' response to network conditions, UER considers not the least cost path but instead a selection of possible hyper-paths known as routing policies. For example, a user may have two possible paths in a network, an arterial and freeway path with two possible states, state 1 and state 2. The user will use the arterial path in state 1 and freeway path in state 2 and when information is provided mid-path (for example, due to the presence of ITS technology such as a VMS board) about the state of the network, then the user will change en-route according to the state and preferences. An advantage of UER is that it accounts for the presence of information in the user route decision-making process. A network considering user recourse in is equilibrium when the expected cost of all used routing policies is minimum and equal. This work introduces a novel application and interpretation of the UER network as applied in the DEAR model. The details of how UER is applied are discussed in Section 3.3.

3. MODEL FORMULATION

This section describes the methodology of the DEAR model proposed in this work. Figure 1 contains a summary of the DEAR framework. While the modelling procedure is similar to Dixit et al. (2012), this work evaluates the impact of en route information by implementing UER to assign traffic following the transformation of the travel times to account for users adaptive route choice behaviour.

The DEAR approach is separated into three stages. The first stage accounts for the "normal conditions" in a network, which may be interpreted as the foundation of a user's knowledge. The second stage transforms these conditions to account for the presence of a disruption based on the travel time perceptions of users. The final stage re-assigns traffic using UER to account for the information users' gain during travel, where the states of traffic are defined by the transformed conditions derived from the perceptions of the user. Therefore, the first two stages are used to define the knowledge that users have of the network and their perception of the disruption, while the final stage accounts for how users will react to knowledge gained throughout travel (i.e., visual cues).

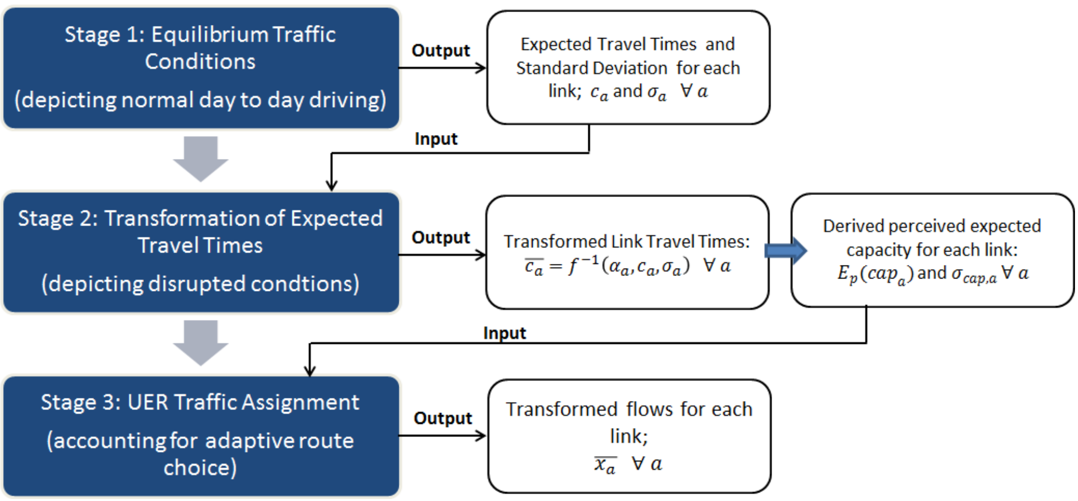


Figure 1: Summary of DEAR model framework

3.1 Stage 1 – Equilibrium Traffic Conditions

Stage 1 of the DEAR model is similar to that of the DNAM model where equilibrium traffic conditions are determined to depict normal driving conditions. Deterministic User Equilibrium (UE) assignment of flows is applied to determine equilibrium conditions. This is a well-established assignment model, and the reader is referred to Sheffi (1985) for the convex mathematical programming formulation. This work uses the traditional Franke-Wolfe algorithm to compute the equilibrium flow assignment patterns. In this method, users' knowledge of the network is based on the traditional equilibrium link travel times.

Correspondingly, users also have knowledge of the volatility of travel times in a network under normal conditions. This work assumes that link travel times are normally distributed (Herman and Lam, 1974) and accordingly the volatility on link a can be described by the standard deviation of travel time, σ_a. This is approximated for each link based on the linear mean-standard deviation relationship empirically derived from travel time data (Sirivadidurage et al., 2009) in similar fashion to Dixit et al. (2012).

3.2 Stage 2 – Travel Time Transformation

In order to account for the conditions in a disrupted network, the DEAR model incorporates a method of transforming link costs based on user perception experiments. Experiments found that the change in user perception of link travel costs can best be described using the following expression: $\overline{c_a} = f^{-1}(\mathrm{\alpha}_a, c_a, \sigma_a)$ (Dixit et al., 2012). In this formula, $\overline{c_a}$ is the transformed link travel time, and $f^{-1}(\alpha_a, c_a, \sigma_a)$ is the inverse cumulative function of the Normal distribution, where c_a represent the equilibrium travel time and σ_a found in Stage 1. The transforms applied represent quantified shifts from the mean link costs obtained from Stage 1 to reflect the presence of disrupted conditions in Stage 2 of the modelling.

The term α_a represents the severity of a network disruption, and corresponds to the "Z" value (standard score) in the inverse cumulative calculation. Average travel conditions are assigned α = 0.5 and as α increases the severity of the disruption increases where α = 1 depicts catastrophic network conditions. This is consistent with the findings from Dixit et al. (2012) that indicated the following: α = 0.45 for sunny, α = 0.59 for rainy, and α = 0.91 for a chemical spill, a reflection of the above trend. For demonstration of the model, it is assumed that the standard deviation of the link travel times remains unchanged in disrupted conditions.

3.3 Stage 3 – UER assignment

The third stage of the DEAR model finds the expected link flows based on the disrupted network conditions. In the DEAR interpretation, the source of information experienced by users is created by the disruption. In other words, users are assumed to have knowledge of the network and this knowledge provides "information" through perception. For example, upon arriving at a link, a traveller may observe increased queue lengths, brake lights, or even general increases in traffic volumes in closer proximity – this information may then be used to alter the route to the destination.

Based on these indicators people will re-evaluate their route choice decisions based on their past experiences and perceptions of travel time for all the possible routes at that specific decision point. This behaviour is captured using the UER assignment method (Unnikrishnan and Waller, 2009). Note that this work incorporates a version of UER in which the probabilistic link states are the same for all users. Using the transformed link travel times that are output from Stage 2 of the model, corresponding link traffic states are calculated as input to the UER model.

However, Stage 2 directly outputs transformed *travel times*; this method then infers *link states* by assuming that capacity and travel time are direct products of one another. Therefore, perceived expected capacity ($E_p(cap_a)$) of the disrupted conditions are derived through the BPR link cost function using the transformed travel times obtained in Stage 2. $E_p(cap_a)$ defines one possible link state that a user could encounter. Additional link states are found using the coefficient of variation relationship derived from DEAR Stage 1. The model presented is a static network planning model which utilises static link cost functions which results in the link capacity state experiencing the same coefficient of variation as the travel time. Using this result, the standard deviation of capacity ($\sigma_{cap,a}$) is estimated.

Using parameters $E_p(cap_a)$ and $\sigma_{cap,a}$, the three traffic states used in this application of the DEAR model are presented in Table 1. These states depict variations of the perceived conditions with states being qualitatively interpreted as "expected", "better than expected", and "worse than expected". The probability of occurrence is obtained from the normal distribution as it is assumed that travel times are normally distributed.

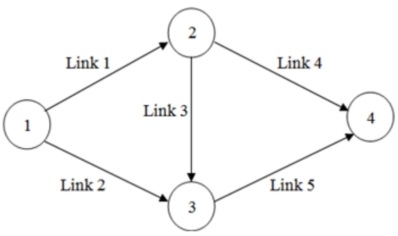
Table 1: Network States used for Modelling

	State	**Link State**	**Probability of Occurrence**
1	Perceived Conditions	$E_p(cap_a)$	0.682
2	Better than Perceived Conditions	$E_p(cap_a) + \sigma_{cap,a}$	0.159
3	Worse than Perceived Conditions	$E_p(cap_a) - \sigma_{cap,a}$	0.159

DEAR-based UER assignment results in transformed flows accounting for the perception of users in disrupted conditions as well as the potential for adaptive routing as a result of the disruption.

4. RESULTS AND DISCUSSION

This section demonstrates the proposed DEAR modelling framework that accounts for the impact of en route information on user behaviour in disrupted network conditions. The sample network is similar to the well-known Braess's paradox network. Figure 2 contains the sample network and associated network parameters. There are three possible paths that users can utilise; travelling along Link 1 and Link 4 (defined as Path 1), travelling along Link 1, Link 3 and Link 5 (defined as Path 2) and travelling along Link 2 and Link 5 (defined as Path 3). The model was assessed at a fixed demand of 4000 vehicles with varying levels of alpha to understand the impact of the severity of a disruption on the traffic flow conditions.



	Link Cost Function
Link 1	$c_1 = 7.5[1 + 0.15(x_1/1{,}800)^4]$
Link 2	$c_2 = 13.3[1 + 0.15(x_2/1{,}800)^4]$
Link 3	$c_3 = 2[1 + 0.15(x_3/1{,}800)^4]$
Link 4	$c_4 = 3.75[1 + 0.15(x_4/1{,}800)^4]$
Link 5	$c_5 = 6.67[1 + 0.15(x_5/1{,}800)^4]$

(a) (b)

Figure 2: (a) Sample Network (b) Link Costs associated with the network

Figure 3 presents a comparison of the path travel costs of the DEAR model to that of DUE assignment. The horizontal axis indicates the level of severity (represented by the alpha value in the DEAR model Stage 2), and the vertical axis shows the travel time of a path in minutes. DEAR does not assign costs to minimize path flow, and so all three paths have slightly different expected costs. Figure 3 also shows the deterministic path costs in the situation that does not account for information or adaptive routing. Both models clearly indicate that perceived travel costs increase as the severity of the disruption increases. The DEAR model also presents a further inflation of travel costs consistent with the findings of Unnikrishnan and Waller (2009) indicating that the information of a disruption affects travel costs.

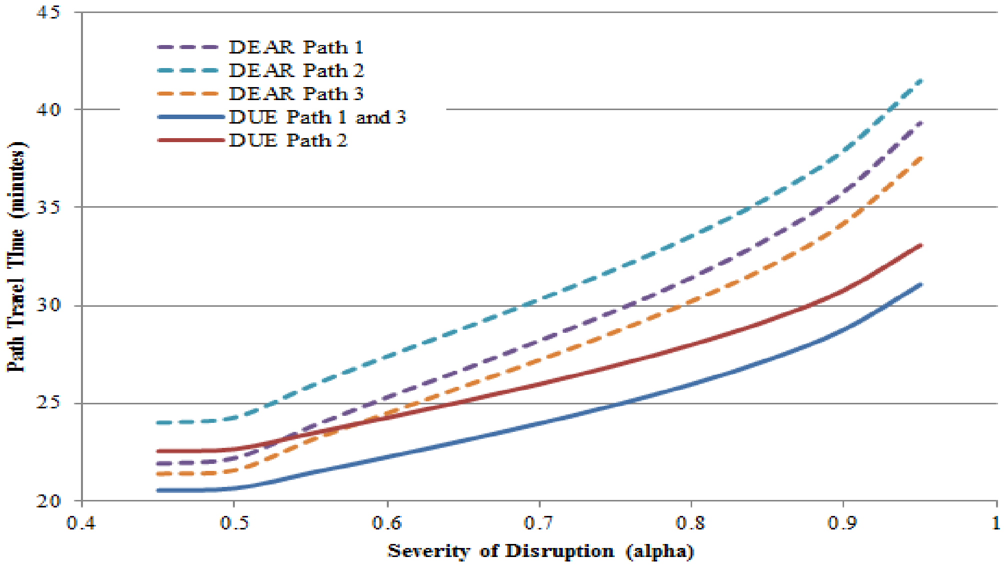


Figure 3: Comparison of travel costs between DEAR and DUE models

Figure 4 presents a comparison of path flows between the DEAR and DUE model with increasing severity of disruption. Again, the horizontal axis shows an increasing level of severity. However, the vertical axis of Figure 4 represents path flows. The DUE model results in identical flows regardless of the incident severity as users are assigned to the least cost path. The DEAR model accounts for the user perceptions of disruptions as well as allows for en-route route choice and accordingly shows a variation in path flow as the severity of the disruption increases with a reduction in flows for path 2 and 3 and an increase in flow for path 1. Furthermore, the peaks presented in the DEAR model path flow graphs indicate shift from better than average travel conditions ($\alpha < 0.5$) where users deflate their expected travel times to disrupted conditions ($\alpha > 0.5$) where users inflate expected travel times thus resulting in the variation of path flows.

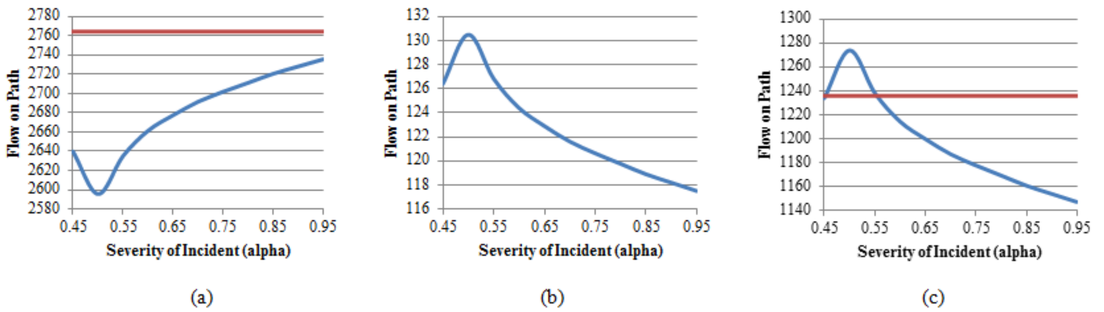


Figure 4: Comparison of variation in flows disruptions between the DEAR(blue line) and DUE models (red line); (a) Flow on Path 1, (b) Flow on Path 2 , (c) Flow on Path 3

5. CONCLUSION

This research proposed a novel modelling framework called the disrupted equilibrium assignment with recourse (DEAR) model that accounts for the impact of information learned throughout travel on en route decision making in disrupted conditions. The DEAR model includes an adapted version of two previous models; DNAM transforms user perception based on experimental data, and UER accounts for adapting routing behaviour in network users. The DEAR model considers people’s perceptions and experiences within disrupted conditions as the source of information which dictates route choice when conducting the adaptive assignment. The results of a demonstration of the model indicate the necessity to not only account for the transformation of perceptions in disrupted conditions but also to also include for the ability of users to adapt their routes based on information gained during travel.

Future work will concentrate on incorporating additional behavioural realism related to adaptive route choice and scaling the implementation to more realistic sized networks that incorporate field data.

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