Modeling Combined Travel Choices of Electric Vehicle Drivers with Variational Inequality Formulation

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ABSTRACT: Plug-in electric vehicles become more attractive with government incentives and policies. Tractable modeling techniques that can incorporate electric vehicles are critical for developing realistic and effective travel behaviors models. It is shown in this paper that the special features of electric vehicles and the limited charging infrastructures lead to different travel behavior of electric vehicle drivers from gasoline vehicle drivers. This paper investigates both the temporal and spatial travel choices behavior of electric vehicles drivers. A multi-class quasi-dynamic model formulation is presented in the form of variational inequality. An optimization-based heuristic method is adopted to solve the model. Finally, numerical experiments are conducted to show that the model generates a network equilibrium solution of the combined choices.

INTRODUCTION

Battery electric vehicle (BEV) is a typical type of plug-in electric vehicle (PEV), which uses a battery to store the electrical energy that powers the motor. Unlike traditional gasoline vehicles (GVs) refilled by gas, BEVs are recharged by plugging a cord into an electric power source. The usage of BEVs will diversify the U.S. national energy usage, increase the energy independence from petroleum, and reduce harmful emissions from transportation fuels to the environment. Because of these opportunities, BEVs have received tremendous attention in recent energy policy discussions.

While BEVs have many benefits on improving environments and reducing gas usage, they are facing some concerns as well. First, there are a limited number of public charging infrastructures for BEV drivers to recharge their vehicle, and the range limits are usually lower than that of GVs. Therefore, most BEV drivers will fully charge their vehicle at home before traveling and will tend to finish their whole trip chain before their car running out of electricity. If the whole trip chain cannot be finished with one full charged battery, the BEV drivers will have to recharging their vehicles at one of the intermediate destinations. These facts lead to the *range anxiety* and the *park-and-charge* concept. Second, with provided public charging infrastructures at work, shopping centers, or parking garages, the charging time for electric vehicles is usually up to a number of hours under the current technologies. For example, a midsize BEV with a 20 kWh battery pack may require 6 to 8 hours for a full charge with a level II charger (providing the 240 volt AC charging), and up to 20 hours with a level I charger (providing the 120 volt AC charging) [1] .

The concerns on BEVs' range limitation and on the availability of public charging infrastructures will affect the drivers' travel behaviour, such as route choice and parking choice (as a result of the park-and-ride charging concept). In addition, the long-time recharging requirement for electric vehicles at an intermediate destination will affect the driver's time spending at that destination. With the above facts, the electric vehicle drivers' behaviour will be different from the GV driver's behaviour. However, at this early stage with few electric vehicles on the road, the electric vehicle drivers' behaviour has not been incorporated in the network based travel behaviour models. With the increasing adoption of BEVs, the existing network models for travel choices without considering the special recharging requirement and limitations of BEVs will lead to inaccurate predictions for transportation system conditions. Therefore, this paper focuses on building a network based travel behaviour model taking into account the recharging activities of individual BEV driver.

The impacts of recharging activates on BEV drivers' behaviour is both temporal and spatial. Temporally, BEV drivers' responses to the recharging requirements may include travel choices such as departure time choice and duration of stay choice at destination. Spatially, the responses may include route choice and charging/parking location choice, etc. Knowing electric vehicle drivers' responses and behaviours will help decision makers and agencies to obtain new traffic pattern information and set transportation policies accordingly. The overarching theme of this work is the need to build a model for quantifying the electric vehicle drivers' above travel choices behavior. As the travel choices include several aspects, a combined model for joint choices modeling is required [2].

In this paper, a time-dependent joint choices model is presented to identify the traveling behavior of electric vehicle users on a network level with mix flows (including BEVs and GVs) on the road. Multiple user classes are considered in this problem and the user class is defined by the duration of stay, purpose of trips and the types of vehicles used together. The problem examined in this paper is not only timely and important on its own, but also can be mingled within the consideration of a network design problem for public charging infrastructure.

The rest of this paper is structured as follows. The next section reviews the literature of related modelling techniques and solution methods for the research tasks we mentioned above. Section 3 discusses the problem settings and the network model for joint travel choices of electric vehicle drivers. Section 4 presents a solution algorithm for the model. Section 5 discusses the experimental results from applying the model and solution methods. Finally, Section 6 concludes the paper.

LITERATURE REVIEW

The network equilibrium models are useful tools for long term transportation planning. Many researchers proposed network equilibrium models which combine different choices together. Such combined choices include the mode and routes choices, destination choices and route choices, departure time and route choices, etc. [2-5].

The combined choices have been proposed for dozens of years on different levels of choice combinations. Sheffi (1985) presented a hypernetwork approach to accommodate the joint travel choices[6]. The hypernetwork consists of hyperlinks for trip generation, mode choice, destination choice and the route choice. There are also works for combining the trip distribution and traffic assignment into one model, where the two steps in planning are solve simultaneously to obtain more consistent results [7, 8]. Later, the convex optimization formulation is extended by introducing origin and/or destination constraints [8-10] and is also used for combining trip distribution, mode choice and traffic assignment models with a set of hierarchical choices[11, 12]. Lam and Huang (1992) and Boyce and Bar-Gera (2001) Some work formulated multimode, multiclass network equilibrium models to incorporate trip distribution with traffic assignment [13, 14]. In addition, the logit model has been adopted in many research for investigating many choices such as mode choices, destination choices, route choices, departure time choices and the joint/combined choices [11, 15-19].

Variational Inequality (VI) is widely used for the combined choices [20-23], especially for the time-dependent combined travel choices [24-26].Yang et al. (1998) proposed a space-time expended network (STEN) for the departure time and route choice in a queuing network with elastic demand to determine the optimal variable congestion tolls [27]. Friesz et. al. (1993) first formulated an infinite-dimensional VI model for the combined choices of departure time and route choice without providing the solution approaches[28]. Later, Wei et.al.(1995) developed a discretized VI formulation for simultaneous route and departure time choice equilibrium problem[29]. In addition, they presented a heuristic algorithm but with no convergence established. Zhou et. al. (2007) developed a VI model and a heuristic procedure to describe and to solve the combined mode, departure time and route choices in multimodal urban transportation network[30]. Zhang (2007) considered simultaneous departure time and route choices using VI formulation [31]. Florian et.al. (2002) formulated a VI formulation for a multi-class multi-mode variable demand equilibrium, in which the joint choices model was a hierarchical Logit function [19]. Wu and Lam (2003) proposed a network equilibrium model that predicts the mode choice and route choice simultaneously in the VI form [32]. Lam et al. (2006) proposed a time-dependent network equilibrium VI formulation for simultaneously departure time, route, parking location and parking duration choice in deterministic user equilibrium [33].

It should be noted that the time-dependent model in this paper is actually a quasi-dynamic model for long-term planning purposes. Quasi-dynamic model for strategic planning purposes have been proposed in some previous research as well [33]. It means that in the model developed, the travel demand in each time interval is in steady-state equilibrium. The connection between successive time intervals is represented by the charging stations occupancy that is carried over to the next interval, which is similar to the problem setting in [34].

PROBLEM STATEMENT AND NOTATION

It is assumed that drivers have a preferred time window for their arrivals at destinations and they will encounter a schedule delay cost if arriving at the destination out of the time window. During different time of the day, congestion levels are different. Therefore, different departure times will result in different travel time on a certain route. The departure time and the network conditions together will affect the drivers' arrival time at destinations. Moreover, the electricity-charging time at destinations plays an important role in the duration of stay for the BEVs drivers. Before traveling, BEV drivers will have an expected duration of stay at their destinations. However, this expected duration of stay might be different from their actual duration of stay at destinations in the end. For example, if a BEV driver needs to recharge his/her vehicle at the destination for 2 hours to get enough electricity for his/her following trip, while his/her expected duration of stay at destination is 1 hours, his/her actual duration of stay at destination will be longer than the expected duration of stay. We define a cost for this extra duration of stay time (which is 1 hour in the above example) at the destination for BEV drivers, named *extra-charging time cost*.

A combined problem of departure time choice, duration of stay choice and route choice for electric vehicle drivers is considered in this paper. The problem is modeled using a nested-logit (NL) structure and formulated as an equivalent variational inequality formulation in this paper. The NL structure is shown in Figure 1: In the upper-level, the drivers make choices on the departure time and the expected duration of stay at destination. The alternative of departure time and duration of stay combination is a generic alternative in this level. In the lower-level, the drivers choose the perceived cheapest route directing to the destination. This hierarchical choice structure has been adopted by many related studies [25, 33, 35].

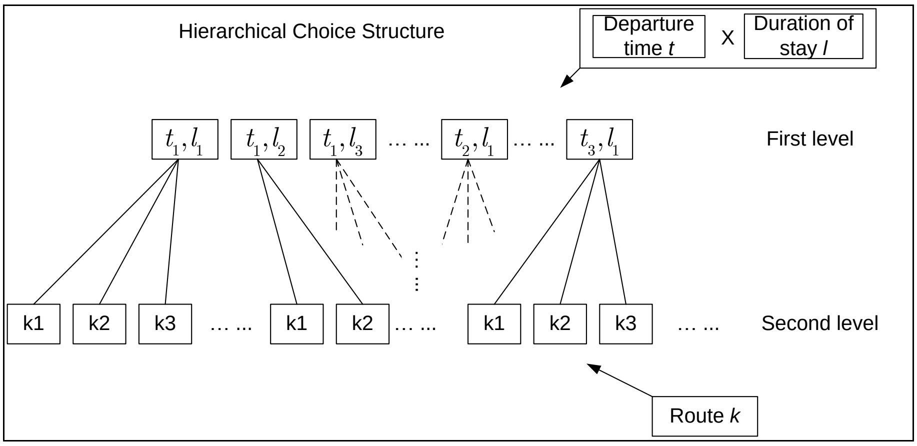


Figure 1. Combined Choices Structure.

Nested-Logit Structure

The departure time choice set is predefined as a set of discrete time intervals. Hypothesizing the whole day splitting into several periods, let the whole study period be $[0,\overline{T}]$, the whole study period is divided into several equal time intervals $t \in T = \{0,1,...,\overline{T}\}$. It is assumed that $\overline{T}$ is large enough for all travelers to complete their journeys [33, 34, 36].

The duration of stay choice set is also a set of discrete time durations $l \in L = \{l_1, l_2, ...\}$. However, the feasible choices of durations of stay in this set depend on BEV drivers' route choice (see Figure 2). We assume that the BEV drivers will recharge their vehicles only if the remaining electrical is lower than a certain "threshold". In other words, let d be the distance a BEV driver has traversed, let D be the effective range limit of the vehicle, and let sd represent the above threshold, the driver will need to recharge his/her vehicle when the distance he/she has traveled is greater than ($D-sd$).For example, let l be the duration of stay for a BEV driver , if the route the driver chose has a length less than $D-sd$ (which means the driver doesn't need to recharge the vehicle), the duration of stay will be a set of time duration ranging from zero, that is $l \in L = \{0, \Delta t, 2\Delta t ...\}$, where Δt is a time interval within which a steady-state equilibrium is assumed. Otherwise, if the BEV driver needs to recharge the vehicle at the destination, the duration of stay choice set may not range from zero. Specifically, let $l_{\min}$ be the minimum of the duration stay choice for a BEV driver, the duration of stay choice set for the driver will be $l = L = \{l_{\min}, l_{\min} + \Delta t, l_{\min} + 2\Delta t ...\}$ .This $l_{\min}$ is calculated based on,

$$l_{\min} = \min\{\text{electricity-charging time, minimun expected duration of stay}\} .$$

The electricity-charging time is related to the length of the path the driver took. It should be long enough for the BEV to get enough electricity to go back to the origin or to the next destination.

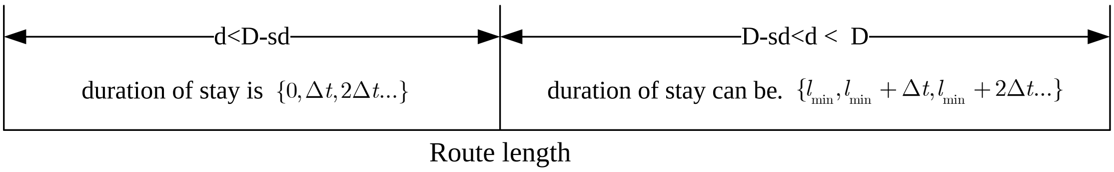


Figure 2 Duration of Stay Choice Set

The route choices are probabilistic choices over a subset of all routes between each O-D pair. Moreover, the choice of charging station can also be accommodated into the route choices (see Figure 3). For example, let $p = \{p_1, p_2, p_3, p_4\}$ be charging locations (which are usually located at a parking garage/surface), and let index k be a route from an origin r to a destination s, then the generic choice (p, k) is (charging location, route). As is presented in Figure 3, a dummy destination s' is adjacent to all the charging locations available at destination s. An "extended network" can be used to represent the charging location/route choice. In this case, the alternative (p, k) discussed above corresponds to an extended path, made up of the original path k and a sequence of links corresponding to the choice of charging location p. Thus, the generic alternative (p, k) described could be represented on the extended network by a path, say k instead [34].

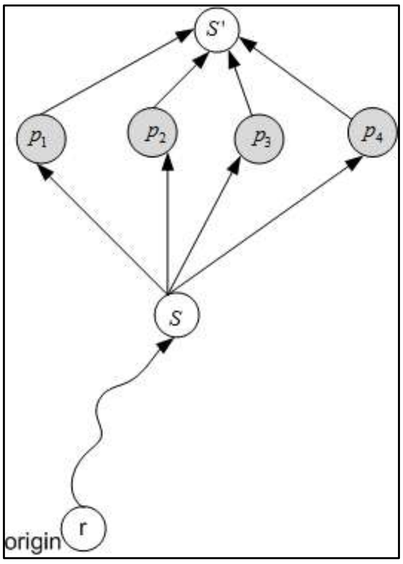


Figure 3 Extended Network for Charging Location and Route Choice

We consider a traffic network $G=(N,A)$, where N is a finite set of all nodes and A is a finite set of all directed arcs. Let B be the set of vehicle types, let J be the set trip purposes, and let L be the set of durations of stay for motorists at destinations, then $M=\{m:(b,j,l)\in B\times J\times L\}$ represents the set of motorist class, where "×" denotes the Cartesian product. Let k be the generic choice of route (or route/charging station combination) for BEV drivers. Let K_c^{rs} be the subset of route between O-D pair (r,s) by taken which the BEV drivers need to recharge their vehicles at destination s. In addition, we have the following notations:

Parameters

d_a	Length/distance of link a
d_k^{rs}	Length/distance of route k between O-D pair (r,s)
$t^{rs,m}$	Expected duration of stay for drivers of class m between O-D pair (r,s)
$\delta_{a,k}^{rs}$	Link-path indicator for link a and route k between O-D pair (r,s)
e_h	Travel time-equivalent cost per unit distance for electricity charging at origin h
e_s	Travel time-equivalent cost per unit distance for electricity charging at destinations s
θ	Dispersion parameter, which is a positive scaling factor related to the variance of the perceived travel cost
σ	Charging time needed for per unit distance traveled
$q^{rs,b,j}$	total demand of motorists of vehicle type b with trip purpose j traveling between O-D pair (r,s)

Variables

$T_k^{rs,m}(t)$	the in-vehicle travel time cost between O-D pair (r,s), for the travelling purpose j, using vehicle type b, departure at interval t stay at destination of duration of l, via generic route k. Alternatively represented by $T_k^{rs,b,j,l}(t)$
$z^{rs,m}(t)$	waiting/congestion costs at charging for user class m, arrival at interval t at charging at destination s.
$e_k^{rs,m}$	charging cost for BEV drivers using route k between O-D pair (r,s) for user class m.
$pe_k^{rs,m}$	charging time penalty cost for motorists using route k between O-D pair (r,s) for traveler class m.
$SD_k^{rs,m}(t)$	schedule delay costs for motorists of class m between O-D pair (r,s), via route k, arrival at time interval t.
$f_k^{rs,m}(t)$	vehicle flows on route k of user class m, between O-D pair (r,s), departure at time interval t.
$q^{rs,m}(t)$	the portion of demand using vehicle type b that departs at interval t and stays at destination for duration of l between O-D pair (r,s). Alternatively represented as $q^{rs,b,j,l}(t)$.
$C_k^{rs,m}(t)$	the systematic disutility component (measured in time unit) for motorists class m departing at time interval t via generic route k between O-D pair (r,s).
$G_k^{rs,m}(t)$	perceived disutility for motorists class m departing at time interval t via generic route k between O-D pair (r,s).
$\varepsilon_k^{rs,m}(t)$	random residual for the perceived disutility $G_k^{rs,m}(t)$.
$P^{rs,b,j}(t,l)$	the probability of choosing departure time and duration of stay alternative (t,l) for traveling between O-D pair (r,s) for travelers using type b vehicles and purpose of travel j.
$P^{rs,m}(t)$	an alternative representation of $P^{rs,b,j}(t,l)$, where m is defined by b,j,l.
$P^{rs,b,j}(k/t,l)$	the conditional probability of choosing path k after choosing departure time and duration

of stay alternative (t,l) between O-D pair (r,s) for travelers using type b vehicles and purpose of travel j. Alternatively represented as $P_k^{rs,m}(t)$.

The proposed formulation for the travel choices is a generalization of the classical random utility model [6, 37]. The O-D trip demand data for all purposes of travel and all vehicle types are considered given. The disutility for user class m departing at interval t and though generic route k between O-D pair (r,s) is the summation: (1) on board cost $T_k^{rs,m}(t)/T_k^{rs,j,l}(t)$; (2) charging station waiting/congestion costs $z^{rs,m}(t)$; (3) electricity-charging cost $e_k^{rs,m}$; (4) extra-charging time cost $ctp_k^{rs,m}$. (5) schedule delay cost $SD_k^{rs,m}(t)$. The charging cost can be considered as the "operating cost" for BEVs, similar to the gas cost for driving GVs. However, the electricity-charging price at different places might vary a lot. For example, the electricity-charging price could be much cheaper at home than at a shopping center. This price difference makes the electricity-charging cost functions more complicated and affects the BEV drivers charging choices. The measured travel disutility $C_k^{rs,m}(t)$, is formulated as the summation of the all costs from origin to the destination. The perceived travel disutility, $G_k^{rs,m}(t)$ is summation of the measured travel disutility and the random residual $\varepsilon_k^{rs,m}(t)$, where the random residual $\varepsilon_k^{rs,m}(t)$ is independent and identically Gumbel distributed. We have the total travel disutility of choosing route k, departing at time interval t and staying for a duration of l as follows,

$$\begin{aligned} C_k^{rs,m}(t) &= \alpha_1^m T_k^{rs,m}(t) + \alpha_2^m z^{rs,m}(t + T_k^{rs,m}(t)) + \alpha_3^m e_k^{rs.m} + \alpha_4^m pe_k^{rs,m} \\ &+ \alpha_5^m SD^{rs,m}(t + T_k^{rs,m}(t) + z_s^{rs,m}(t + T_k^{rs,m}(t))) \end{aligned} \tag{1}$$

Let W_{lt} represents the expected utility of choosing a (departure time, duration of stay) alternative. This expected utility could be calculated from the subsequent choice - route choices, as follows,

$$W_{lt} = \frac{1}{\theta} \ln \sum_k \exp\{-\theta C_k^{rs,m}(t)\} \tag{2}$$

The probability of motorist driving vehicle type b, for the purpose of traveling j, departing at time interval t and staying at destination for the duration of l, between O-D pair (r,s) as follows,

$$P^{rs,m}(t) = \frac{\exp\{\beta W_{lt}\}}{\sum_{(l,t)} \exp\{\beta W_{lt}\}} \tag{3}$$

where the values of parameters β reflects the accuracy of drivers' perception of the variation of the disutility of the (departure time, duration of stay) alternative. Therefore, we have,

$$q^{rs,b,j,l}(t) = q^{rs,m}(t) = q^{rs,b,j} \cdot P^{rs,m}(t) \tag{4}$$

The conditional probability of user class m traveling between O-D pair (r,s) at time interval t and staying at the destination for duration l via generic route k, can be given by the following equation

$$P_k^{rs,m}(t) = \frac{\exp\{-\theta C_k^{rs,m}(t)\}}{\sum_{k'} \exp\{-\theta C_{k'}^{rs,m}(t)\}} \tag{5}$$

The trip demand that uses route k, which is the route flow of user class m departing at time interval t between O-D pair (r,s), is as follows,

$$f_k^{rs,m}(t) = q^{rs,m}(t) \cdot P_k^{rs,m}(t) \tag{6}$$

The above equations (Eq. 2-6) give the NL model for the travelers' departure time, duration of stay and route choices.

Cost functions for BEV drivers

In this paper, an origin-destination-origin trip chain is considered as the basic travel analysis unit for constructing the travel cost functions for BEV drivers. In addition, we assume that the origin-to-destination trip and the destination-to-origin trip have equivalent length. According to the above settings, we first construct an electricity-charging cost function for BEV drivers. In the case of an origin-destination-origin trip chain and equivalent of route length, it is easy to set $sd = D/2$. That is, for a BEV driver choosing path k between O-D pair (r,s), if $D/2 < d_k^{rs} < D$, where $d_k^{rs} = \sum_a d_a \delta_{a,k}^{rs}$, the BEV needs to be recharged with at least an equivalent amount of electricity to $(2d_k^{rs} - D)$ (in terms of distance) for the return trip. For modeling convenience, the electricity-charging cost is represented by its time-equivalent travel cost here. If a BEV driver do not recharge the vehicle at destination, the total charging cost spent for the tour will be $2e_h d_k^{rs}$.If the driver needs to recharge the vehicle at destination, the additional electricity-charging cost incurred is $e_s(2d_k^{rs} - D)$ and the total electricity-charging cost for the tour is $e_h D + e_s(2d_k^{rs} - D)$.We can simply derive that the electricity-charging cost for the origin-destination trip is half of the above total electricity-charging cost derived,

$$e_k^{rs} = \begin{cases} e_h \sum_a d_a \delta_{a,k}^{rs}, & \sum_a d_a \delta_{a,k}^{rs} \le \frac{D}{2} \\ \frac{1}{2}[e_s \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) + e_h D], & \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D \end{cases} \quad \forall k,a,r,s \tag{7}$$

Second, we construct the extra-charging time cost. We define a constant rate of electricity-charging speed σ, in the unit of time/distance. That is, for recharging electricity for one unit distance travel, the recharging time need is σ. As discussed above, the extra-charging time costs occurs if an only if the required electric-recharging time is greater than the driver's expected duration of stay. Let $t^{rs,m}$ be the expected duration of stay for drivers of class m between O-D pair (r,s), the extra-charging time costs is as follows,

$$pe_k^{rs,m} = \begin{cases} 0\ , & \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) \le t^{rs}, \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ 0\ , & \sum_a d_a \delta_{a,k}^{rs} \le \frac{D}{2}, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs,m}, & \left(\begin{array}{l} \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) > t^{rs,m} \end{array} \right. \end{cases} \tag{8}$$

The travel time cost through a route is the summation of the travel time cost on different links constituting the route, which is also referred as the on board cost in this paper,

$$T_k^{rs,m}(t) = \sum_a t_a(x_a(t)) \delta_{a,k}^{rs} \tag{9}$$

The number of charging infrastructures is finite at each destination. This may cause congestion or waiting for the BEVs that need to recharge. The waiting or searching time for a free charging spot is considered as waiting/congestion cost for BEV drivers. It should be noted that only those BEV drivers who need to recharge their vehicles will encounter this cost. We use a well-defined parking searching function as our charging waiting/congestion function because we believe the searching/waiting for charging infrastructure is similar to that at parking garages [33, 38]. It is a function of the current number of BEVs that at recharging when a new BEV comes for waiting.

$$z^{rs,m}(t + T_k^{rs,m}(t)) = z_s^{0,m} + 0.31 \times \left(\frac{D_s(t + T_k^{rs,m}(t))}{C_s}\right)^{4.03} \tag{10}$$

where $z_s^{0,m}$ is the access time for a BEV driver who needs to recharge his/her vehicle when there are no other BEVs using charging infrastructures, so called the free-flow charging access time. C_s is the number of charging infrastructures at the destination s (which is the capacity of the charging infrastructure). $D_s(t)$ is the number of existing BEVs that are recharging at time interval t. For a driver departing at time interval t traveling between O-D pair (r,s) via route k, the arrival time at the charging is $t+T_k^{rs,m}(t)$. The number of existing BEVs recharging at destination s at time interval t equals the total number of BEVs recharging electricity that arrives before time interval t (also represented as the accumulating recharging arrival flow $A_s^m(t)$) minus the total number of BEVs recharging electricity that leaves before time interval t (also represented as the accumulating recharging departure flow $B_s^m(t)$), i.e., $D_s^m(t)=A_s^m(t)-B_s^m(t)$,where $A_s^m(t)$ and $B_s^m(t)$ are given in [33] as follows,

$$A_s^m(t)=\sum_{r:(r,s)}\sum_{\xi-1}^{t-1}\sum_{tt:tt+T_k^{rs,m}(t)=\xi}\sum_{k\in K_c^{rs}} f_k^{rs,m}(tt) \tag{11}$$

$$B_s^m(t)=\sum_{r:(r,s)}\sum_{\xi-1}^{t-1-l}\sum_{tt:tt+T_k^{rs,m}(t)=\xi}\sum_{k\in K_c^{rs}} f_k^{rs,m}(tt) \tag{12}$$

Let the preferred time of arrival at destination s for user class m traveling between O-D pair (r,s) be represented by $[t_{rs}^{m,lb},t_{rs}^{m,ub}]$. We assume either early or late arrival at the destination result in a cost, which is named as the schedule delay cost. Similar to [39], the schedule delay cost could be defined as a piecewise linear cost function [27].

$$SD(t)=\begin{cases}\lambda_e^m(t_{rs}^{m,lb}-t), & t_{rs}^{m,lb}>t\\ 0, & t_{rs}^{m,lb}\le t\le t_{rs}^{m,ub}\\ \lambda_l^m(t-t_{rs}^{m,ub}), & t_{rs}^{m,ub}<t\end{cases} \tag{13}$$

Where λ_e^m is the value of early schedule delay and λ_l^m is the value of late schedule delay for user class m. This schedule delay cost function assumes that if travelers arrive at destination within the time window, there is no schedule delay cost.

Variational Inequality formulation

The departure time and the duration of stay choice are represented using the logit model. The route choice is modeled as logit based stochastic traffic assignment problem. Therefore, these two-level choices form a NL structure. In the following, an equivalent VI formulation of the joint choices problem is given. The feasible set, Ω, of all the constraints is defined by the following equations (14)-(17), where $q^{rs,m}(t)=q^{rs,b,j,l}(t)$,

$$\sum_k f_k^{rs,m}(t)=q^{rs,m}(t), \quad \forall rs,m,l,t \tag{14}$$

$$\sum_{(l,t)} q^{rs,b,j,l}(t)=q^{rs,b,j}, \quad \forall rs,b,j \tag{15}$$

$$f_k^{rs,m}(t)>0, \quad \forall rs,m,l,t,k \tag{16}$$

$$q^{rs,m}(t)\ge 0, \quad \forall rs,m,t,k \tag{17}$$

Equation (14) is the route flow conservation constraint. Equation (15) is the departure time and duration of stay choice demand conservation constraint. Constraints (16) and (17) are the nonnegative and positive requirements of route flows, departure time and duration of stay demands. This equivalent VI formulation (Eq.14-16, Eq.20) of the quasi-dynamic model will be adopted to capture all the components of the proposed NL structure in an integrated form,

$$\sum_{rs}\sum_{m}\sum_{t}\sum_{k}C_k^{rs,m*}(t)\cdot[f_k^{rs,m}(t)-f_k^{rs,m*}(t)]+\sum_{rs}\sum_{m}\sum_{t}\sum_{k}\frac{1}{\theta}\ln\frac{f_k^{rs,m*}(t)}{q^{rs,m*}(t)}[f_k^{rs,m}(t)-f_k^{rs,m*}(t)]$$
$$+\sum_{rs}\sum_{m}\sum_{t}\frac{1}{\beta}\ln q^{rs,m*}(t)[q^{rs,m}(t)-q^{rs,m*}(t)]\geq 0 \tag{18}$$

subject to the feasible space $(f_k^{rs,m}(t),q^{rs,m}(t))\in\Omega$. For the model to be internally consistent, $\theta\geq\beta>0$ must hold [40].

In the following, we prove that the above VI formulation is equivalent to the NL structure of the joint choices. The Karush–Kuhn–Tucker (KKT) conditions of the VI formulation are derived and it is shown that the KKT conditions recover the NL model.

Let $L_t^{rs,m}$ be the dual variable associated with the constraint (14), and let $L^{rs,b,j}$ be the dual variable associated with the constraint (15). let $\lambda_{t,k}^{rs,m}$ be the dual variable associated with the constraint (16) and (17). The KKT conditions are as follows,

$$f_k^{rs,m}(t): \qquad C_k^{rs,m}(t)+\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}-L_t^{rs,m}-\lambda_{t,k}^{rs,m}=0 \tag{19}$$

$$q^{rs,m}(t): \qquad \frac{1}{\beta}\ln q^{rs,m}(t)-L^{rs,b,j}+L_t^{rs,m}=0 \tag{20}$$

The complementarity conditions

$$\lambda_{t,k}^{rs,m}\cdot f_k^{rs,m}(t)=0 \tag{21}$$

$$\lambda_{t,k}^{rs,m}\geq 0 \tag{22}$$

From the complementarity conditions we know that if $f_k^{rs,m}(t)\neq 0$ then $\lambda_{t,k}^{rs,m}=0$. For stochastic traffic assignment problem, we know that the route flow $f_k^{rs,m}(t)>0$, therefore, we get the following transformation from Eq.(23),

$$\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}=-C_k^{rs,m}(t)+L_t^{rs,m}\Rightarrow\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}=\exp(-\theta C_k^{rs,m}(t)+\theta L_t^{rs,m}) \tag{23}$$

Sum both sides of the above equation over all routes k, together with the route flow conservation constraint (14), we get the following equations,

$$\sum_k\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}=\sum_k\exp(-\theta C_k^{rs,m}(t)+\theta L_t^{rs,m})=\exp(-\theta L_t^{rs,m})\cdot\sum_k\exp(-\theta C_k^{rs,m}(t))=1 \tag{24}$$

$$\Rightarrow\exp(\theta L_t^{rs,m})=\frac{1}{\sum_k\exp(-\theta C_k^{rs,m}(t))} \tag{25}$$

Therefore, the route choice model is,

$$f_k^{rs,m}(t)=q^{rs,m}(t)\cdot\frac{\exp\{-\theta C_k^{rs,m}(t)\}}{\sum_{k'}\exp\{-\theta C_{k'}^{rs,m}(t)\}} \tag{26}$$

We can see it follows the Logit choice model as shown in equation (5) and (6). From Eq.(20), we have

$$q^{rs,m}(t)=\exp(\beta L^{rs,b,j}-\beta L_t^{rs,m}) \tag{27}$$

Sum both sides of the Eq. (27) over all combination of departure time t and duration of stay l,

$$\sum_{(t,l)}q^{rs,m}(t)=\exp(\beta L^{rs,b,j})\cdot\sum_{(t,l)}\exp(-\beta L_t^{rs,m}) \tag{28}$$

Divide Eq. (28) over Eq. (27) on both sides,

$$\frac{q^{rs,m}(t)}{\sum_{(t,l)}q^{rs,m}(t)}=\frac{\exp(\beta L^{rs,b,j})\cdot\exp(-\beta L_t^{rs,m})}{\exp(\beta L^{rs,b,j})\cdot\sum_{(t,l)}\exp(-\beta L_t^{rs,m})} \tag{29}$$

Therefore, we get the following departure time choice and duration of stay choice model, presented by the Logit choice model,

$$q^{rs,m}(t) = q^{rs,b,j} \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} \tag{30}$$

In a hierarchical Logit structure, the expected upper-level disutility is obtained from utility of lower-level choices, as shown in Eq. (2-4). From Eq.(25), the costs of choosing an alternative of departure time t and duration of stay l is the propagated cost from the route choices, in the logsum format, as follows:

$$L_t^{rs,m} = -\frac{1}{\theta} \ln \sum_k \exp(-\theta C_k^{rs,m}(t)) \tag{31}$$

Therefore, the proposed VI formulation leads to the joint choices of departure time, duration of stay and the stochastic route choice. That is, the hierarchical logit model representing the joint probabilities of simultaneous choices is equivalent to the VI formulation (18) subject to Ω.

SOLUTION ALGORITHM

Since the feasible set of the VI model is defined by linear constraints, positive and nonnegative constraint, it is a compact set. In addition, all the functions in the VI formulation are continuous; therefore, there is at least one solution to the VI model. We build an optimization-based heuristic which consists of two sequential steps to solve the above model, and the numerical experiments show that this algorithm is able to obtain a satisfactory solution.

The demand matrices are computed at every successive iteration by using the method of successive averages (MSA). The feasible duration of stay choice set for BEVs is updated at every successive iteration based on the utilized route length and the vehicle flows. The diagram of the solution approach is shown in Fig. 4.

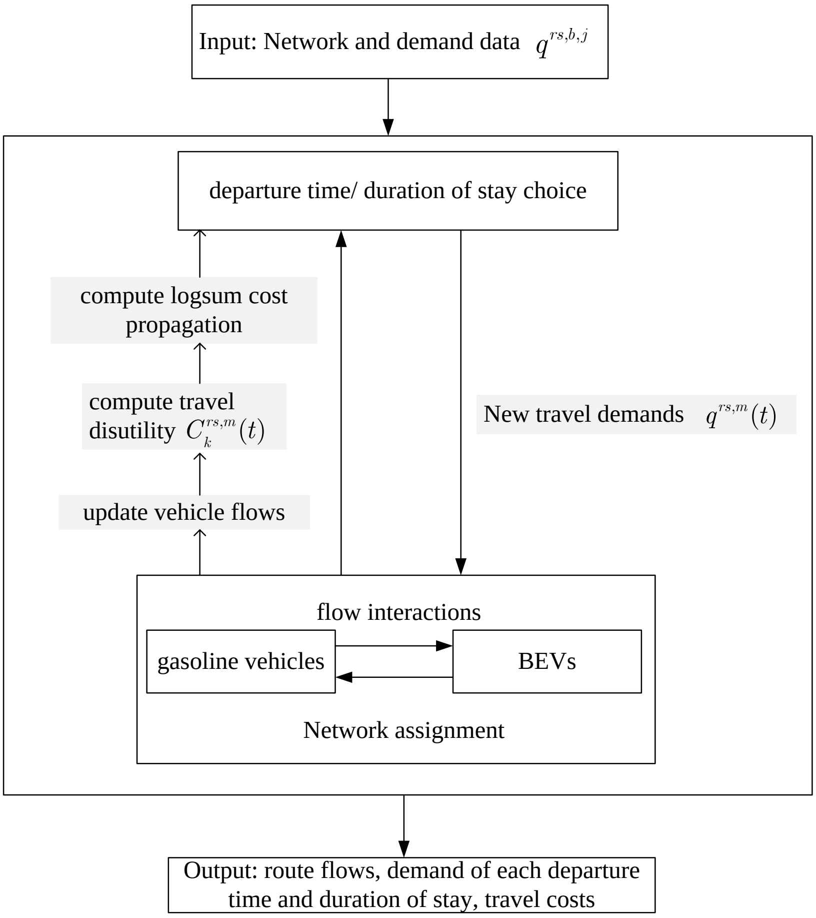


Figure 4 Solution Approach

In solving the route choices part, we assume the motorists are homogenous on value of time. In this paper, two distinct types of vehicles are considered, the GVs and the BEVs. The network assignment for all types of vehicles requires the solution for the following VI problem.

$$\sum_{rs}\sum_{m}\sum_{t}\sum_{k} C_k^{rs,m*}(t)\cdot[f_k^{rs,m}(t)-f_k^{rs,m*}(t)] \\ +\sum_{rs}\sum_{m}\sum_{t}\sum_{k}\frac{1}{\theta}\ln\frac{f_k^{rs,m*}(t)}{q^{rs,m*}(t)}[f_k^{rs,m}(t)-f_k^{rs,m*}(t)]\geq 0 \tag{32}$$

Subject to (14) and (16). It is easy to prove that the solution of the above VI model (32) with constraint (14) and (16) is equivalent to the logit stochastic user equilibrium condition (26). The disaggregated simplicial decomposition (DSD) algorithm is used to solve the above stochastic traffic assignment model. This method was developed by Larsson and Patriksson for the user equilibrium problem [41], and later extended by Damberg et. al. for the logit-based stochastic traffic assignment problem [42]. The algorithm works on a restricted route choice set for achieving equilibrium conditions (restricted master problem phase), combined with a path generation (column generation phase) to find the augmenting route (usually, this is the shortest route). The two phases are solved iteratively until the stopping criterion is achieved. More details about the algorithm can be found in [2, 41, 42]. The paper adopt this algorithm with some necessary modifications in the path generation phase to obtain feasible path alternatives for BEV drivers. In particular , in order to taking into account both the electricity-charging cost and the extra-charging time cost for BEV drivers, three constrained shortest path problems need to be solved to find the augmenting shortest path. The augmenting path for GVs drivers is the path with the shortest travel time cost; while the augmenting path for BEV drivers is generated based on the following three constrained shortest path (CSP) problems.

[CSP1]

$$z^1 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv}x_{uv} + e_s \sum_{(u,v)\in A} d_{uv}x_{uv} - \frac{1}{2}(e_s - e_h)D \tag{33}$$

subject to

$$\frac{1}{2}D < \sum_{(u,v)\in A} d_{uv}x_{uv} \le \frac{D}{2} + \frac{t^{rs}}{2\sigma} \tag{34}$$

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{35}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{36}$$

[CSP2]

$$z^2 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv}x_{uv} + e_h \sum_{(u,v)\in A} d_{uv}x_{uv} \tag{37}$$

subject to Eq.(35-36) and,

$$\sum_{(u,v)\in A} d_{uv}x_{uv} \le \frac{D}{2} \tag{38}$$

[CSP3]

$$z^3 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv}x_{uv} + e_s \sum_{(u,v)\in A} d_{uv}x_{uv} - \frac{1}{2}(e_s - e_h)D + \sigma[(2\sum_{(u,v)\in A} d_{uv}x_{uv} - D) - t^{rs}] \tag{39}$$

subject to Eq.(35-36) and

$$\frac{D}{2} + \frac{t^{rs}}{2\sigma} < \sum_{(u,v)\in A} d_{uv}x_{uv} \le D \tag{40}$$

These above three CSP problems are solved separately, and their objective values of the original objective function z^1, z^2, z^3 are compared. The solution path with the least objective values among $\{z^1, z^2, z^3\}$ is selected and augmented to the restricted master problem set.

The step-by-step procedure for solving the joint choices problem is described in the following:

Step 0. Initialization. Set $i = 1$, given the demand data, choose an initial O-D travel pattern $q^{rs,m}(t)$.

Step 1. DSD algorithm for stochastic assignment (see [2] for more details).

Step 1.1. Find initial subset of routes, compute the initial route flow. Set $j = 1$

Step 1.2. Restricted master problem phase. Assign $q^{rs,m}(t)$ to the subset of routes, obtain the travel time cost $T_k^{rs,m}(t)$ and route flows $f_k^{rs,m}$

Step 1.3. Column generation phase. Find the augmenting paths obtained from the shortest path for GVs and from solving the CSP1 ~CSP3 problems for BEVs.

Step 1.4. At convergence, compute the electricity-charging cost $e_k^{rs,m}$, charging time penalty cost $pe_k^{rs,m}$. Compute the charging/parking congestion cost, the schedule delay cost, and finally the travel disutility $C_k^{rs,m}(t)$.

Step 2. In accordance to the above travel disutility, calculate the logsum cost $L_t^{rs,m}$ Update the feasible duration of stay set.

Step 2.1. Perform the departure time and duration of stay choice according to the Logit choice model, get the auxiliary O-D travel pattern $q'^{rs,m(i)}(t)$.

Step 2.2. Update the O-D travel demand using MSA

$$q^{rs,m(i+1)}(t) = q^{rs,m(i)}(t) + \frac{1}{i}[q'^{rs,m(i)}(t) - q^{rs,m(i)}(t)]$$

Step 3. Convergence check. If convergent, terminate. Otherwise, set $i = i + 1$, and go to Step 1.

NUMERICAL EXPERIMENTS

The numerical experiments are conducted on the network in Figure 5. In this test network, there are two O-D pairs $(1,12)$ and $(5,13)$, 13 nodes, and 24 road links. The dummy links (dash links) represent the charging links, where BEVs get recharged. One charging station is considered associated with each destination. The parameters of all the link travel time functions are given in Table 1. The charging capacities at $s1$ is 2400, at $s2$ is 1680. There are a total demand of 10000 vehicles (including both BEVs and GVs) between O-D pair $(1,12)$ and a total demand of 7000 vehicles between O-D pair $(5,13)$. BEV penetration rate is 30%. The BEV effective range $D = 102$.The study period runs from 6:00 am to 7:00 pm, divided into hourly intervals.. Both GV drivers and BEV drivers are further categorized into classes according to different duration of stay (1h, 2h, or 3h) and trip type (type A and B). All drivers between O-D pair $(1,12)$ conduct A type of trip, and drivers between (5,13) conduct B trip. The perception coefficients for all drivers between O-D pair $(1,12)$ for trip purpose A are $\theta_1 = 0.8$, $\beta_1 = 0.4$ and for all drivers between O-D pair $(5,13)$ for trip purpose B are $\theta_2 = 0.7$, $\beta_2 = 0.3$ .The number of classes in the example is 12 (2 vehicle types × 3 duration of stay × 2 type of trip purpose). The scheduled arrival time window for all drivers of trip A between O-D pair (1,12) is $[t_{(1,12)}^{1,lb}, t_{(1,12)}^{1,ub}] = [12pm, 2pm]$ and the parameters for the schedule delay functions are $\lambda_e^1 = 0.5, \lambda_l^1 = 1.5$. The scheduled arrival time window for all drivers of trip purpose B between O-D pair (5,13) is $[t_{(5,13)}^{2,lb}, t_{(5,13)}^{2,ub}] = [8:30am, 9:30am]$ and the parameters used for the schedule delay cost function are $\lambda_e^2 = 0.5, \lambda_l^2 = 1.5$. Parameter values for GVs and BEVs in the total cost functions are $\alpha_1^{BEV} = 1$, $\alpha_2^{BEV} = 1.4$, $\alpha_3^{BEV} = 0.5$ $\alpha_4^{BEV} = 2$, $\alpha_5^{BEV} = 2.8$, $\alpha_1^{GV} = 1$ $\alpha_2^{GV} = \alpha_3^{GV} = \alpha_4^{GV} = 0$, $\alpha_5^{GV} = 2.8$, $\alpha_6^{GV} = 0.5$.

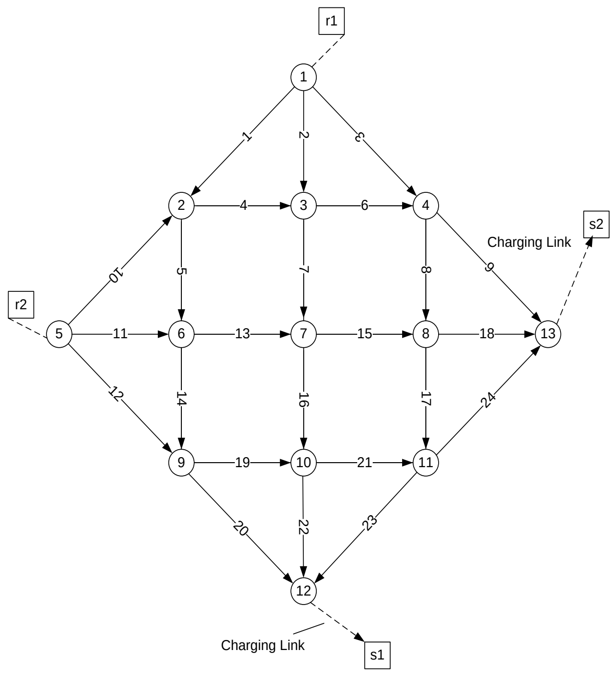


Figure 5 Test Network

Link ID	free flow time	length	Capacity	Link ID	free flow time	length	Capacity
1	0.25	25	800	13	0.3	15	500

2	0.2	10	700	14	0.3	20	500
3	0.25	25	800	15	0.2	15	600
4	0.2	20	700	16	0.2	5	600
5	0.2	20	500	17	0.3	20	500
6	0.2	20	600	18	0.1	20	650
7	0.3	10	500	19	0.3	20	500
8	0.2	20	500	20	0.1	25	700
9	0.2	25	650	21	0.2	20	600
10	0.2	25	800	22	0.2	15	650
11	0.2	20	800	23	0.1	15	700
12	0.2	13	800	24	0.2	15	650

Table 1. Network Parameters

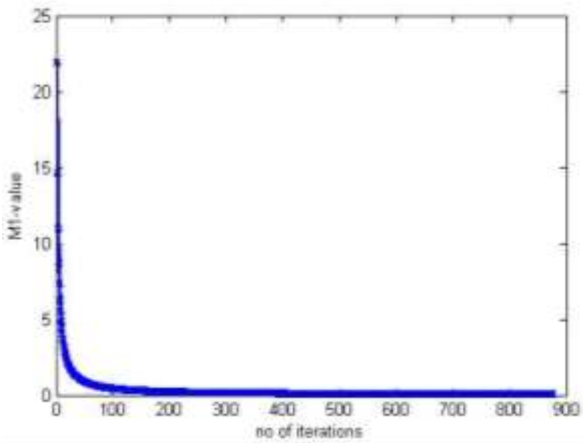
The GVs drivers do not encounter the electricity-charging cost or the charging time penalty cost or the charging waiting/congestion cost. Instead GVs drivers encounter operating cost. The per unit distance operating cost for GV drivers is set as of $g = 0.2$. The operating cost for GV drivers is $g\sum_a d_a \delta_{a,k}^{rs,GV}$. The weighting parameter this operating cost is represented using α_6^{GV} for GV drivers The electricity-charging cost parameters for BEVs are $e_h = 0.05$, $e_s = 0.15$, $\sigma = 0.03$.

Convergence check

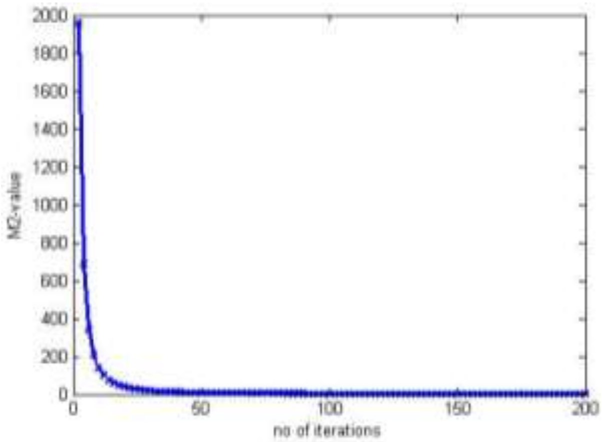
The solution quality is checked on the tested network. The convergence criterion is defined by the difference between solution results and equilibrium conditions results of two-level choices respectively (Eq. 41-42). The convergence pattern of the solution is presented in Figure 6. It can be seen that the algorithm achieve good convergence patterns for both level of choices. The logit-based equilibrium for all joint choice is satisfied.

$$M_1 = \frac{\sum_{rs}\sum_{m}\sum_{t}\sum_{k} \left| f_k^{rs,m}(t) - q^{rs,m}(t)\frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'}\exp(-\theta C_{k'}^{rs,m}(t))} \right|}{\sum_{rs}\sum_{m}\sum_{t}\sum_{k} f_k^{rs,m}(t)} \tag{41}$$

$$M_2 = \frac{\sum_{rs}\sum_{m}\sum_{t} \left| q^{rs,m}(t) - q^{rs,b,j}\frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)}\exp(-\beta L_t^{rs,m})} \right|}{\sum_{rs}\sum_{m}\sum_{t} q^{rs,m}(t)} \tag{42}$$



(a) for the stochastic user equilibrium.



(b) for the stochastic user equilibrium

Figure 6 Convergence pattern

The results

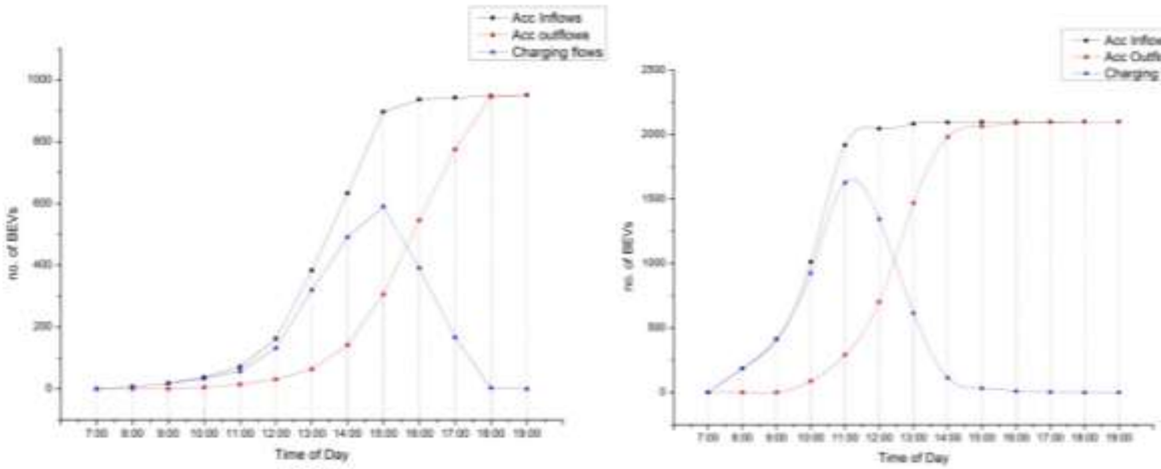
The results of the combined demand of departure time and duration of stay, as well as the behavior of accumulative arrivals, departures and charging curves at the charging facility verifies that the proposed model can indeed generate a network equilibrium solution of the joint choices.

Table 2 is the results for the demand matrix with different departure time and duration of stay for BEVs. It shows that there is no demand at any time slices for staying at destination 1 hour or less for BEV drivers. The reason is, even the shortest length route between (5,13) is too long that all drivers need to recharge their vehicle at destination longer than 1 hour. The demands for l =1h, l =2h, and l =3h between O-D pair (1,12) for BEVs are 872.3496 , 1039.567, and 1088.0826 respectively. The demands for l =1h, l =2h, and l =3h between O-D pair (5,13) for BEVs are 0, 1027.9349, and 1072.065 respectively. This indicates that BEVs drivers tend to stay longer at the destination. The reason for this might be one or more of the following facts: (1) the BEVs drivers needs to wait until the necessary electricity-recharging finished; (2) the BEVs drivers needs to wait for an unoccupied charging infrastructure, where the congestion in charging stations exists. The results also suggest that most of the drivers try to reach their destination within the time window, and thus reduce their schedule delay costs.

Duration of stay			l =1hour		l =2 hours		l =3 hours	
Departure Time			(1,12)	(5,13)	(1,12)	(5,13)	(1,12)	(5,13)
6:00	~	7:00	21.68	0.00	22.75	176.07	23.09	181.89
7:00	~	8:00	38.09	0.00	39.96	275.51	40.56	284.05
8:00	~	9:00	69.63	0.00	72.76	294.10	73.77	309.37
9:00	~	10:00	121.40	0.00	126.91	193.76	128.69	204.71
10:00	~	11:00	129.57	0.00	144.85	63.21	149.57	65.76
11:00	~	12:00	145.58	0.00	184.83	18.08	196.01	18.81
12:00	~	13:00	139.61	0.00	184.71	5.16	197.85	5.36
13:00	~	14:00	125.71	0.00	173.98	1.46	187.87	1.52
14:00	~	15:00	62.03	0.00	68.71	0.41	70.79	0.43
15:00	~	16:00	15.52	0.00	16.46	0.12	16.77	0.12
16:00	~	17:00	2.90	0.00	3.07	0.03	3.13	0.03
17:00	~	18:00	0.54	0.00	0.57	0.01	0.00	0.00
18:00	~	19:00	0.10	0.00	0.00	0.00	0.00	0.00

Table 2 Departure rate for BEVs

The following Figure 7 presents the total number arrivals, departures and accumulation of BEVs that recharging at charging locations at different time of the day. The accumulation for charging is the vertical distance between the cumulative arrival and cumulative departure curves. The capacity of the charging location at destination node 12 is 2400 and is 1680 at destination node 13. The maximum recharging flow at destination 12 is 590.3221 and is 1627.611 at destination 13. This number is related link length and the utilized paths' length between each O-D pair. Thus, although the BEV trip demand between O-D (1,12) is higher than that of O-D (5,13), the total number of recharging flows between (1,12) is much less than that between (5.13).The charging facility started to be occupied at around 12:00pm and the peak occupied time is at around 3:00pm at destination 12; while the peak occupation at charging facility at destination 13 is between 9:00am~1:00pm. The behavior of the arrivals, departures and charging is consistent with our expectations in terms of time of the day and duration of stay.



(a) charging facility 12. (b) charging facility 13

Figure 7 charging facility arrival, departure, and accumulation

CONCLUSION

New opportunities exist for studying the traveling behaviors of BEV drivers and for investigating the impacts from introducing BEVs on to the road. Tractable modeling techniques are critical for developing realistic and effective travel behaviors models. The model presented in this paper accounts for BEV drivers' behavior in the time-dependent, multi-class, combined travel choices network equilibrium. Special consideration given to BEVs drivers includes the electricity-charging cost and extra-charging time. The problem is formulated as a variational inequality model where the joint choices are proved to follow the NL structure. The formulation was solved using a optimization-based heuristic technique where the two-level travel choices are solved iteratively. Numerical experiments have been conducted on an example network. As the numerical results demonstrate, the proposed model formulation and solution approach can capture the feasible duration of stay choices for BEVs, the electricity-charging cost of BEV drivers at destination, and the recharging behavior of the BEV drivers at destination. With the developed model, further work may focus on the sensitivity analysis of the flow patterns with different vehicle ranges and different BEV penetration rate.

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