Network Flows of Plug-In Electric Vehicles: Impacts of Electricity-Charging Price

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ABSTRACT

This paper is intended to model and evaluate the impacts of electricity-charging prices on work-related commuting network flows of electric vehicles. A stochastic user-equilibrium network flow problem is formulated and analyzed for commuters who drive battery electric vehicles and incorporate charging costs into routing decision makings. Given the spatial difference of electricity-charging prices, solving the problem requires tracking all individual paths. As such, a path-based solution algorithm based on the disaggregated simplicial decomposition scheme, with some modifications in the path generation phase and an added k-shortest path search procedure for eliminating the solution inaccuracy issue, is proposed for problem solutions. We implemented the modeling and solution methods for evaluating the network performance changes caused by varying electricity-charging prices in multiple network-level and link-level evaluation matrices. The evaluation results show that the traffic network of battery electric vehicles incurs a higher vehicle miles traveled (VMT) value and a lower vehicle hours traveled (VHT) value compared to the network of gasoline vehicles, and the VMT value decreases and the VHT value increases with the increase of either the origin-based or destination-based electricity-charging price.

INTRODUCTION

Petroleum has long been the largest single energy source in many countries. In the U.S., 19.2 million barrels of petroleum are consumed per day in 2010, and the transportation sector dominates the petroleum consumption [1]. Among all transportation modes, road transportation alone consumes 85 percent of the total energy used by the transportation sector and it has been almost the sole transportation mode responsible for additional energy demands [2]. Road transportation's overdependence on petroleum has created numerous negative impacts on our society's environment and economic development. Despite this, in the foreseeable future, the caused climate change will very likely become more obvious and petroleum prices are anticipated to continuously climb, which greatly threaten in the long term the environmental and economic sustainability of our society. In view of these serious global issues, an increasingly growing interest has arisen all over the world of taking emergent actions to reduce the petroleum dependence and diversify the energy consumption structure.

Plug-in electric vehicles (PEVs), especially battery electric vehicles (BEVs), have received tremendous attention in recent energy policy discussions and are believed as one of the most promising and implementable strategies for achieving the petroleum consumption reduction. The U.S. federal government highlighted electricity as a strategic energy replacement to gasoline for the transportation sector in the future [3] and anticipated that PEVs can capture a significant market share (5% to 15%) in the next 15 to 20 years [4]. The national academies also predicted that there will be 13 to 40 million PEVs out of a total of 300 million vehicles running on the U.S. roads in 2030 [5].

While a massive adoption of BEVs implies tremendous environmental and economic sustainability benefits, it must be recognized that driving BEVs may be subject to some technological and infrastructure limits at the initial stage of the market. These limits inevitably affect the purchase and usage willingness of prospective BEV consumers. An average BEV travels a shorter distance before recharging than the distance an average gasoline vehicle (GV) can travel before refueling, which raises the range anxiety concern in the current and potential BEV consumer population. Unlike gasoline stations, the charging infrastructures for electric vehicles, even if available, nowadays are only located at parking places such as on-street parking slots, parking lots and home garages. These parking places are typically the origins and destinations of vehicle trips. A technical reason for such a charging infrastructure distribution is that the charging time for electric vehicles is usually up to a number of hours under the current battery technologies. For example, a midsize BEV with a 20 kWh battery pack may require 6 to 8 hours for a full charge with a level 2 charger (providing the 240 VAC charging), and up to 20 hours with a level 1 charger (providing the 120 VAC charging) [6]. It is less likely that a BEV driver stops at a charging station in the midst of a trip for vehicle charging only; instead, parking times (at origins or destinations) seem to be the only feasible time-of-day periods for charging vehicles if a significant amount of electricity needs to be injected. This leads to the so-called *park-and-charge* concept.

On the other hand, with the recently proposed battery-swapping or battery-switching service, quickly replacing BEV batteries at swapping stations seems to be an attractive time-saving solution that can eliminate the above park-and-charge requirement. However, developing automated battery-swapping equipment for a variety of vehicle and battery models and standardizing the entire operational process pose a set of complicated technical and institutional issues and may require a long-term public-private partnership involving multiple stakeholders, including vehicle manufacturers, electricity providers, legislation institutions, and vehicle owners. Because of such an accommodation complexity, it may take a long period to commercialize and consolidate a generic battery-swapping service in the market, and the market might be finally segmented by a number of competitive but mutually incompatible battery-swapping services. In view of the unclear vision of the competitiveness evolution between different battery technologies and electricity supply modes, it is very likely that the future BEV market is supported by a mix of all competitive electricity provision modes, including both the battery-charging and battery-swapping services among others.

Nevertheless, the battery-charging service provides the lowest price and most convenient accessibility and has gained the widest acceptance among all existing electricity provision services in the emerging market. With the advancement of emerging battery technologies (e.g., lithium-air batteries), the electricity-charging time is expected to be continuously reduced in the future. Given the argument that the electricity for most BEVs nowadays and in the near future is provided by home-based and workplace-based charging equipment, at least for those BEVs primarily used for commuting trips, our modeling effort in this paper is focused on the electricity-charging activities of individual drivers in a work-related commuting network. More specifically, we are concerned about how electricity-charging performance—evaluated in terms of charging cost—impacts BEV drivers' travel behaviors and network flows. Our initial focus here is given to the route choice behavior, though the impact on other travel choices could be included in a similar way by more complex models. We believe that charging cost is one of the most important efficiency measures used by individual BEV drivers to evaluate their electricity-charging experiences. As such, this paper finally comes up with a network flow model for BEV commuters who explicitly incorporate charging price into their routing decision makings and a numerical analysis on how the electricity-charging price influences the individual route choice behavior on the network level.

The remaining part of this paper is structured as follows. The next section reviews the literature of related modeling techniques and solution methods for the research tasks we mentioned above. Section 3 discusses the specific problem settings and assumptions pertaining to modeling commuting networks of BEV drivers and presents an equilibrium network flow model that takes the electricity-charging cost into account for individual route choices. Section 4 presents a solution algorithm for the model. Section 5 discusses the experimental results and behavioral insights we gained from applying the model and solution methods for a numerical analysis. Finally, Section 6 concludes the paper.

RELEVANT LITERATURE

According to Wardrop's first principle [7], the user equilibrium (UE) conditions in a traffic network are reached when no driver can improve his or her travel cost by unilaterally changing routes. The stochastic user equilibrium (SUE) principle is considered as a more realistic generalization of the deterministic UE, in that the presumption is relaxed that every driver in the network has an accurate perception on travel costs, by introducing randomness into drivers' travel cost perceptions. The definition of SUE can be described as that no driver can improve his or her perceived travel cost by unilaterally changing routes [8]. In this paper, we adopt SUE as the behavioral basis and modeling framework for characterizing BEV flows in congested commuting networks. The preference of SUE to UE is not only because SUE is based on a more behaviorally sound modeling paradigm (i.e., random utility theory) but also because any SUE solution contains a unique path flow pattern. As we will see later on, the path solution uniqueness is required for evaluating individual travel times and charging costs.

The existing literature shows that the SUE problem and its variants can be, in general, modeled by three types of mathematical techniques: (i) optimization problems [8-10]; (ii) variational inequalities [11, 12]; and (iii) fixed-point problems [13, 14]. Among all these problem formulations, Fisk's logit-based SUE model presents a convex optimization problem with linear constraints, which favors a number of efficient solution algorithms for its solutions [9]. Our model formulations developed in this text can be regarded as its extensions with alternative travel impedance structures. It is noted that Fisk's model is formulated in the path flow variable, but have been solved by link-based stochastic network loading approaches most of the time [15, 16]. In our case, however, a path-based solution algorithm is preferable. Because the charging cost for BEVs is a function of the path length traveled, the path flow pattern is required for deriving the solution.

The path-based SUE problem can also be solved by path enumeration and column generation techniques [17]. The explicit path enumeration technique assigns the probability to the pre-selected paths, which is called the selective-explicit enumeration approach [18]. From a realistic perspective, typical drivers do not consider or perceive all routes in a traffic network. Following such behavioral considerations, a number of researchers assumed that only those paths satisfying some efficiency or impedance rules can become feasible paths to be considered or perceived, which results in a restricted path set in the solution process [14, 15, 18, 19]. Damberg [20] extended the disaggregated simplicial decomposition (DSD) algorithm proposed by [21] for solving the SUE problem, which appears to be the first work of implementing an algorithm that explicitly provides route flows for the SUE problem. The algorithm is an iterative procedure consisting two main phases: a restricted master problem phase in which the equilibrium of path flows are solved within a restricted path set and a (column generation) subproblem phase in which new routes are generated and added into the restricted path set.

In our network equilibrium problem discussed below, feasible paths are generated by solving a constrained shortest path (CSP) problem. A number of different solution algorithms have been developed for the CSP problem, which can be divided into three categories: (i) k-shortest path algorithms, (ii) dynamic programming algorithms, and (iii) Lagrangian relaxation algorithms. Dynamic programming algorithms based on labeling schemes have been believed as a very efficient solution strategy in the existing literature, suggested by [22-25]. Recently, new techniques based on Lagrangian relaxation were proposed, such as the Lagrangian relaxation plus enumeration (LRE) algorithm [26]. An efficient method for enumerating the near-shortest path (NSPs) suggested by [27] was used in the LRE algorithmic framework, which is typically more efficient than the k-shortest path algorithms in some particular cases. A comparison of the LRE algorithm with the label-setting algorithm of [25] was conducted in [27], and it suggested that it is more computational efficient for solving the single-resource-constrained shortest path problem. Therefore, we adopt the LRE algorithm by [26] with some necessary modifications for solving the CSP in the DSD framework.

PROBLEM FORMULATION

We consider a traffic network $G = (N, A)$, where N is a finite set of nodes and A is a finite set of directed arcs. Let R be the set of origins and S be the set of destinations, and (r, s) represent an origin-destination (O-D) pair, where $r \in R$ and $s \in S$. K_{rs} is the set of routes between O-D pair (r, s) and $\widehat{K}_{rs}$ is a restricted subset of routes between O-D pair (r, s) generated by column generation. Other parameters and variables are listed as follows.

Parameters

d_a	Length/distance of link a
d_k^{rs}	Length/distance of route k connecting O-D pair (r, s)
$\delta_{a,k}^{rs}$	Link-path inclusion indicator for link a and route k connecting O-D pair (r, s)
e_h	Time-equivalent charging cost per unit distance for electricity charging at origin h
e_s	Time-equivalent charging cost per unit distance for electricity charging at destination s
q^{rs}	Demand rates between O-D pair (r, s)
θ	Dispersion parameter of the logit model, which indicates the variance of the perceived travel costs of BEV drivers

Variables

t_a	Travel time functions of link a
e_k^{rs}	Operating cost for BEV drivers using route k between O-D pair (r, s)
f_k^{rs}	Flow rate on path k between O-D pair (r, s), where the path flow pattern is denoted by $\mathbf{f} = [f_k^{rs}]$
x_a	Flow rate on link a, where the link flow pattern is denoted by $\mathbf{x} = [x_a]$
U_k^{rs}	Perceived utility of route k between O-D pair (r, s)
V_k^{rs}	Actual utility of route k between O-D pair (r, s)
ε_k^{rs}	Perception error of route k between O-D pair (r, s)
C_k^{rs}	Travel cost of route k between O-D pair (r, s)
p_k^{rs}	Probability of choosing route k between O-D pair (r, s)

The O-D demands for the entire analysis period, the duration distributions of drivers staying at destinations, electricity-charging prices at destinations (e.g., workplaces, schools, or shopping malls) and at origins (i.e., homes) are assumed to be known a priori. For modeling static traffic networks, the analysis period is typically referred to as a specific steady-state time-of-day period, such as morning peak hours, afternoon peak hours, or midday off-peak hours. The analysis period of interest in this study is the morning peak period and all travel demands are assumed to be home-to-workplace commuting trips. As we will discuss throughout the text, different time-of-day periods may imply different electricity-charging behaviors of BEV drivers.

We specify the route choice behavior in the random utility maximization framework, by which each driver chooses a route that maximizes his/her perceived random travel utility. The path flow of each route k between an O-D pair (r, s) is determined by:

$$f_k^{rs} = q^{rs} p_k^{rs} = q^{rs} P\left(U_k^{rs} = \max_{k'} U_{k'}^{rs}\right) \tag{1}$$

The SUE network flow pattern is obtained by applying both the random utility theory and equilibrium principle to congested networks. The perceived travel utility on route k between an O-D pair is defined as $U_k = V_k + \varepsilon_k$, where V_k is the actual travel utility associated with a given flow pattern and ε_k is a random component. The systematic utility function for choosing route k between O-D pair (r, s) is defined by $V_k^{rs} = -\theta C_k^{rs}$. Depending on the probability distribution chosen for this random term, different models can be resulted. In particular, we employ a logit-based SUE model developed by [9], where the random term ε_k is an independent and identically distributed Gumble variable. The path flow pattern implied by the equilibrium solution of Fisk's model is:

$$f_k^{rs*} = \frac{\exp\{-\theta C_k^{rs}(\mathbf{f}^*)\}}{\sum_{k'} \exp\{-\theta C_{k'}^{rs}(\mathbf{f}^*)\}} \quad \forall k, r, s \tag{2}$$

where $\mathbf{f}^* = \left[f_k^{rs*}\right]$ is the equilibrium path flow vector over the network.

Modeling assumptions

Assumption 1 (Equivalence of trip lengths). For a home-based work tour or trip chain, which includes a home-workplace trip and a workplace-home trip, we assume that a driver will choose a route for his/her workplace-home trip parallel to the route for his/her home-workplace trip. The distances of the home-workplace trip and workplace-home trip are assumed to be equal. ■

This assumption is quite strict and may not be fully consistent with the equilibrium behavior. However, it greatly simplifies our modeling complexity that we can use separate trip-based models instead of a tour-based model to describe the whole work tour or trip chain.

Assumption 2 (Existence of feasible paths). We assume that for any O-D pair with a positive demand rate, there are a number of feasible routes, the length of which is less than or equal to the effective distance limit of BEVs. ■

The effective distance limit is defined as the maximum driving distance that a BEV can travel after a full charge under ideal driving conditions minus a "safety" or "buffer" distance. The consideration of the safety distance is necessary since extra electricity energy will more or less consumed under varying, stochastic driving conditions. In this text, we say that a path between an O-D pair is feasible if its length is no farther than the effective distance limit. Moreover, given this assumption, it is readily known that the threshold distance for recharging requirement on a one-way trip is the half of the effective distance limit.

Assumption 3 (Spatial price difference). We assume that the electricity-charging price at origins (e.g., homes) is typically lower than destinations (e.g., workplaces), since the home-based electricity price is for residents and the workplace-based electricity price is for the business or commercial purpose. ■

In view of the spatial price difference, whenever possible, all BEV drivers prefer to charge their vehicles at home garages as much as possible. As a result, all BEV drivers tend to fully charge their vehicles at origins, and do not charge their vehicles or charge their vehicles at destinations as little as the amount that is sufficient for their return trips from destinations to origins. If a full charge at origins is not sufficient to support the entire round tour, BEV drivers will have to charge their vehicles at destinations. Given that the electricity-charging price at origins is lower, however, they will not charge more electricity energy than what is needed for their return trips. Under this assumption, it is readily known that the "whether-to-charge" behavior of an arbitrary BEV commuter will depend on the vehicle's battery capacity and electricity consumption (or driving distance) along the home-workplace trip.

Assumption 4 (Homogeneity of drivers and vehicles). For modeling simplicity, we assume that all drivers in the network have the same value of time and all vehicles have the same effective distance limit. ■

The homogeneity assumption can be readily relaxed by specifying a multi-class version of the models developed in this text or incorporating a continuous distribution of relevant parameters into the model.

Electricity-charging cost function

What travel impedance items should be included in the disutility function of a traffic assignment model and how to specify/evaluate these items have been a debated topic for decades and resulted in a large number of different traffic assignment problems. Without loss of generality, we employ a generic travel disutility function in this paper to describe individual travel cost: *travel time* + *operating cost*. Compared to a GV driver, a BEV driver may experience a different travel time and operating cost, even if both the drivers travel on the same route. This travel cost composition difference calls for alternative models and solution methods to account for BEV network flows. This is the major motivation that we launch this research.

Prior to the model development, it is important to understand the difference of travel cost composition of BEVs from that of GVs. The discussion is concentrated on the operating cost. If we ignore the vehicle depreciation cost, the operating cost for a GV is mainly from its gas consumption; similarly, the operating cost for a BEV is from its electricity consumption. The gas and electricity cost difference causes the most visible advantage that motivates individuals to choose BEVs rather than GVs.

Gas stations are accessible almost everywhere and the gas prices at different gas stations in a daily commuting region are typically the same or very close. In contrast, the construction of public electricity-charging stations, however, is now on its infancy stage; even in those regions that have been designated to provide electricity-charging business, the distribution of stations is still very sparse. As we discussed earlier, most available charging stations at present are located only at origins and destinations of trips, including, for example, homes, offices, schools, shopping centers, and so on. On the other hand, the electricity prices at a home garage and at a public charging station may be considerably different. For the lower electricity price and the charging time requirement, most BEV drivers may have to or prefer to charge their vehicles at home, preferably during evenings. Thus, unlike GV drivers, BEV drivers will consider where to charge their vehicles in addition to which route to take to minimize the operating costs of their whole tours.

Moreover, although both gasoline consumption and electricity consumption can be roughly estimated in terms of driving distance, the total operating cost of GVs in a network may be estimated by simply evaluating link flows, while the network wide operating cost of BEVs must be estimated by tracing individual paths. The underlying reason for the latter phenomenon is from the fact of different electricity prices between origins and destinations, which leads to the operating cost of an arbitrary BEV is possibly a result of mixing different electricity-charging costs, depending on how far the trip is and when and where the BEV is charged.

These prominent features of operating costs can be further quantified by the following function. We construct a charging cost function for BEV drivers based on the problem settings and assumptions stated in the previous section. For a BEV driver choosing path k between O-D pair (r,s), if $D/2 < d_k^{rs} \leq D$, where $d_k^{rs} = \sum_a d_a \delta_{a,k}^{rs}$, the BEV needs to be recharged with at least an equivalent amount of electricity to $(2d_k^{rs} - D)$ (in terms of distance) for powering its return trip, i.e., the destination-origin trip. For modeling convenience, the electricity-charging cost is represented by its time-equivalent travel cost here. If a BEV driver can complete the round tour (i.e., the origin-destination trip and destination-origin trip) without recharging at the destination, the total electricity-charging cost spent on the tour will be $2e_h d_k^{rs}$; if the driver needs to recharge the vehicle at the destination, the additional electricity-charging cost incurred is $e_s(2d_k^{rs} - D)$ and the total electricity-charging cost for the tour is $e_h D + e_s(2d_k^{rs} - D)$.

From Assumption 1, we can simply derive that the electricity-charging cost for the origin-destination trip is half of the above total electricity-charging cost derived, as shown in Eqn. 3. The charging cost function can be further illustrated by Fig. 1, where the piecewise solid line represents the electricity-charging cost, which is a function of the distance traveled.

$$e_k^{rs} = \begin{cases} e_h d_k^{rs} & \text{if } d_k^{rs} \leq \dfrac{D}{2} \\ \dfrac{1}{2}[e_h D + e_s(2d_k^{rs} - D)] & \text{if } \dfrac{D}{2} < d_k^{rs} \leq D \end{cases} \qquad \forall k, r, s \tag{3}$$

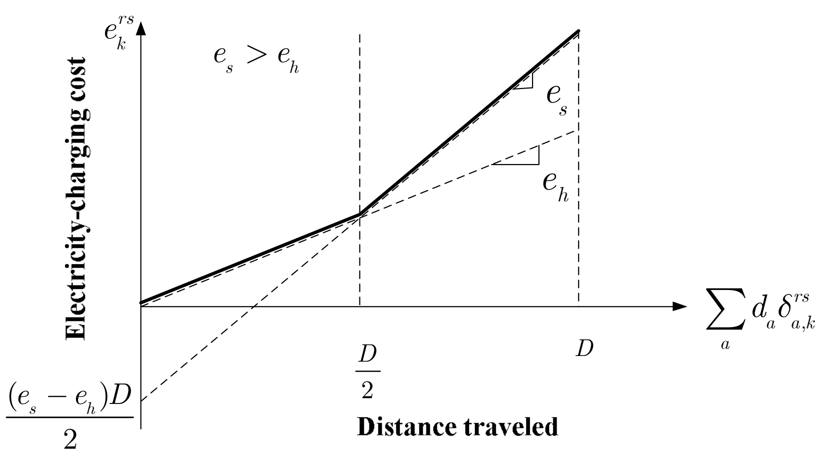


FIGURE 1 The charging cost function

In the following, we consider a network equilibrium problem case that incorporates the charging cost into route choices.

Problem formulation

Due to the electricity-charging cost function, the route choice behavior of BEV drivers is evidently different from GV drivers. To accommodate these alternative impedances pertaining to BEV drivers, we propose a network equilibrium model that takes the electricity-charging cost into account. The model is established on the behavioral basis of minimization of the sum of travel time and operating cost. It is assumed that the latter cost term is fully attributed by charging cost. The model implicitly uses the origin-destination-origin tour as the basic travel analysis unit. Following Assumption 1, however, we only need to pay attention to the network for either origin-destination trips or destination-origin trips and accordingly form trip-based network equilibrium models.

Moreover, we claim that BEV drivers suffer from random perceptions on travel times but know travel distances accurately. Travel times are flow-dependent and affected by many disturbing factors, such as traffic conditions, accidents, weather conditions, light conditions, and so on. Different travelers may experience different travel times in a time-varying traffic network and in different days. The distance of a route among all drivers, however, is fixed and hence can be learned reasonably well on a long-term basis.

The model is formulated as the following optimization problem:

$$\min_{\mathbf{f}} z = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw + \sum_{rs}\sum_{k} f_k^{rs}\max\left(e_s\sum_{a} d_a\delta_{a,k}^{rs} - \frac{e_s - e_h}{2}D, e_h\sum_{a} d_a\delta_{a,k}^{rs}\right) \quad (4)$$

subject to

$$\sum_{k} f_k^{rs} = q^{rs}, \quad \forall r,s \quad (5)$$

$$f_k^{rs} \geq 0, \quad \forall k,r,s \quad (6)$$

where

$$x_a = \sum_{rs}\sum_{k} f_k^{rs}\delta_{a,k}^{rs}, \quad \forall a \quad (7)$$

The first two terms in the objective function z are identical to the well-known Fisk model [9]. The last term in the objective function is the total charging cost for all BEV drivers in the network. Constraints (5)-(6) are the demand reservation constraint and the flow non-negative constraint, and constraint (7) is a definitional constraint specifying the relationship of link and path flows. The above model is strictly convex. Fisk [9] showed that the first two terms of the objective functions (4) are convex. The last term in the objective function (4) is a linear function of path flows. In overall, the objective function (4) is a strictly convex function.

We then show that the optimality conditions of the proposed formulations are equivalent to their implied logit-based route choice behaviors. We construct the Lagrangian problem as follows,

$$\begin{aligned} L(\mathbf{f}, \boldsymbol{\lambda}) = \frac{1}{\theta} \sum_{rs} \sum_{k} f_k^{rs} (\ln f_k^{rs}) + \sum_{a} \int_0^{x_a} t_a(w) dw \\ + \sum_{rs} \sum_{k} f_k^{rs} \cdot \max\left(e_s \sum_{a} d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_{a} d_a \delta_{a,k}^{rs} \right) \\ + \lambda^{rs} \left(\sum_{k} f_k^{rs} - q^{rs} \right) \end{aligned} \tag{8}$$

where λ^{rs} is the Lagrangian multiplier corresponding to Eqn. (5). Equating the partial derivative of the Lagrangian problem with respective to each path flow variable f_k^{rs} to zero gives the conditions of the stationary point. The derivative analysis result ensures that each path flow variable is positive, because $\partial L_A(f, \lambda)/\partial f_k^{rs} = \infty$ when $f_k^{rs} = 0$. Therefore, a solution will only be valid if all components of the stationary point of the feasible region are strictly positive. The derivative of L with respective to f_k^{rs} is,

$$\begin{aligned} \frac{\partial L(\mathbf{f}, \boldsymbol{\lambda})}{\partial f_k^{rs}} = \frac{1}{\theta} (\ln f_k^{rs} + 1) + \sum_{a} t_a(x_a) \delta_{a,k}^{rs} + \max\left(e_s \sum_{a} d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_{a} d_a \delta_{a,k}^{rs} \right) \\ + \lambda^{rs} \qquad \forall k, r, s \end{aligned} \tag{9}$$

By setting $\partial L(f, \lambda)/\partial f_k^{rs} = 0$, we get

$$\begin{aligned} f_k^{rs*} = \exp\left\{ -\theta \left[\sum_{a} t_a(x_a^*) \delta_{a,k}^{rs} + \max\left(e_s \sum_{a} d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_{a} d_a \delta_{a,k}^{rs} \right) + \lambda^{rs*} \right] \right. \\ \left. - 1 \right\} \qquad \forall k, r, s \end{aligned} \tag{10}$$

Incorporating the O-D demand conservation constraint, i.e., Eqn. (5), we obtain

$$\begin{aligned} p_k^{rs*} &= \frac{f_k^{rs*}}{q^{rs}} \\ &= \frac{\exp\left\{ -\theta \left[\sum_a t_a(x_a^*) \delta_{a,k}^{rs} + \max\left(e_s \sum_a d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k}^{rs} \right) \right] \right\}}{\sum_{k'} \exp\left\{ -\theta \left[\sum_a t_a(x_a^*) \delta_{a,k'}^{rs} + \max\left(e_s \sum_a d_a \delta_{a,k'}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k'}^{rs} \right) \right] \right\}} \qquad \forall k, r, s \end{aligned} \tag{11}$$

or

$$f_k^{rs*} = q^{rs} \cdot \frac{\exp\left(-\theta c_k^{rs*}\right)}{\sum_{k'} \exp\left(-\theta c_{k'}^{rs*}\right)} \qquad \forall k, r, s \tag{12}$$

where the route travel cost c_k^{rs} is the sum of the travel time and charging cost

$$c_k^{rs} = \sum_a t_a(x_a)\delta_{a,k}^{rs} + \max\left(e_s \sum_a d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k}^{rs}\right) \qquad \forall k, r, s \tag{13}$$

The solution uniqueness can be readily seen from the fact that both the objective function and constraint set shown in Eqn. (4)-(7) are convex with respect to path flow f_k^{rs}.

SOLUTION ALGORITHM

We have shown that the charging cost of an individual BEV driver is a function of the length of his/her chosen route (see Eqn. (3)). Note that the function is not purely linear, but a piecewise linear function of the length. This implies that evaluating the total charging cost over the network requires tracking all individual paths. For the same reason, a solution algorithm that can derive path flow solutions is required for the above SUE problem.

Among all possible algorithmic choices, the disaggregated simplicial decomposition (DSD) method developed by Larsson and Patriksson [21] has been shown to be an efficient solution algorithm for the UE problem. The DSD algorithm is based on a disaggregated representation of the feasible solutions for convex problems over the Cartesian product sets. The basic algorithmic logic of this algorithm is to iterate between a restricted master problem (equilibrium) phase and a subproblem (path generation) phase until convergence. Damberg et. al. [20] extended the DSD algorithm to the logit-based SUE problem. In the restricted master problem phase of Damberg et al.'s implementation, a path flow pattern is obtained over the subset of routes $\widehat{K}_{rs}$ between O-D pair (r, s), by directly applying the logit probability function. In the subproblem phase, new routes are generated by a shortest path algorithm and added into the restricted master set.

We adopt Damberg et al.'s DSD algorithm with some necessary modifications in the path generation phase to solve our alternative SUE problems for BEV network flows. In particular, we need to solve two constrained shortest path (CSP) problems to find the to-be-added routes. This algorithmic requirement is discussed below.

Constrained Shortest Path Search

Let x_{uv} be a binary variable, where x_{uv} equals to 1 if the link $(u, v) = a$ is on a given feasible path and equals to 0 if the link $(u, v) = a$ is not on the given feasible path. Then the two CSP subproblems for the problem are:

Subproblem 1

$$\min_{x_{uv}} z_1 = \sum_{(u,v)\in A} t_{uv}x_{uv} + e_s \sum_{(u,v)\in A} d_{uv}x_{uv} = \sum_{(u,v)\in A} (t_{uv} + e_s d_{uv})x_{uv} \tag{14}$$

subject to

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{15}$$

$$\frac{D}{2} < \sum_{(u,v)\in A} d_{uv}x_{uv} \le D \tag{16}$$

$$x_{uv} = \{0,1\}, \ \forall (u,v) \in A \tag{17}$$

Subproblem 2

$$\min_{x_{uv}} z_2 = \sum_{(u,v)\in A} t_{uv}x_{uv} + e_h \sum_{(u,v)\in A} d_{uv}x_{uv} = \sum_{(u,v)\in A} (t_{uv} + e_h d_{uv})x_{uv} \tag{18}$$

subject to Eqn. (15), Eqn. (17), and

$$\sum_{(u,v)\in A} d_{uv}x_{uv} \le \frac{D}{2} \tag{19}$$

After the two CSP subproblems are solved, their optimal objective values, z_1^* and z_2^*, are compared: If $z_1^* < z_2^*$, the resulting shortest path from Subproblem 1 will be chosen and added into the restricted master problem set; otherwise, the chosen and added path is the shortest path generated by Subproblem 2. Note that Assumption 2 ensures the feasibility of at least one of the subproblems, since the feasible sets of Subproblem 1 and Subproblem 2 are exclusive but supplementary in the region of $0 < \sum_{(u,v)\in A} d_{uv}x_{uv} \le D$. Furthermore, if an additional constraint $\sum_{(u,v)\in A} d_{uv}x_{uv} \le D/2$ is added into Subproblem 1, i.e., $\sum_{(u,v)\in A} d_{uv}x_{uv} \le D$ instead of $D/2 < \sum_{(u,v)\in A} d_{uv}x_{uv} \le D$ is included in the feasible set of Subproblem 1, z_2^* is always less than z_1^*, because of $e_h < e_s$ (see Assumption 1). Following this result, the constraint of $D/2 < \sum_{(u,v)\in A} d_{uv}x_{uv} \le D$ can be replaced by $\sum_{(u,v)\in A} d_{uv} \cdot x_{uv} \le D$ in Subproblem 1.

We suggest all the above CSP problems to be solved by the Lagrangian Relaxation and Enumeration (LRE) method developed by Carlyle et al. [26]. The algorithmic details of this method are omitted here due to the space limit.

Algorithm synthesis

An adaptive DSD algorithm for solving the problem is proposed below.

Step 0 (Initialization). Find an initial subset of routes $\widehat{K}_{rs}$ for each O-D pair (r,s). Compute and initialize the route flows vector $\mathbf{f}^0$. Set $i = 0$.
Step 1 (Restricted master problem phase). Set $j = 0$. Let $\mathbf{f}^j = \mathbf{f}^i$. Repeat the following until the stop criteria is met:

Compute at the jth iteration the path costs of all routes , $C_k^{rs}(\mathbf{f}^j)$, in the subset $\widehat{K}_{rs}$.

Compute the auxiliary route flows vector h^j according to the formulation.

$$h^j = q^{rs}\frac{\exp\{-\theta C_k^{rs}(\mathbf{f}^j)\}}{\sum_{k'\in\widehat{K_{rs}}}\exp(-\theta C_{k'}^{rs}(\mathbf{f}^j))}$$

Stop if $|\mathbf{f}^j - \mathbf{h}^j| < \epsilon$. Otherwise, find the step size t_j, and let the new point be $\mathbf{f}^{j+1} = \mathbf{f}^j + t_j(\mathbf{h}^j - \mathbf{f}^j)$. Set $j = j + 1$. Output $\mathbf{f}^j$.

Step 2 (Subproblem phase). Using the LRE to solve the two CSP problems, i.e., subproblem 1 and subproblem 2, get objective value and binary vector as $(z_1^*, \mathbf{x_1^*})$ and $(z_2^*, \mathbf{x_2^*})$, respectively.

If both subproblem 1 and subproblem 2 are feasible, and $z_1^* \le z_2^*$, output x_1^*; if $z_1^* > z_2^*$, output $\mathbf{x}_2^*$.

If subproblem 2 is infeasible, then output $\mathbf{x}_1^*$.

Let $\mathbf{f}^{i+1} = \mathbf{f}^j$. Set $i = i + 1$. Go to Step 1.

Note that at each iteration of executing the above DCD algorithm, we generate and add the constrained shortest path of each O-D pair into the restricted path set. The iteration terminates when there is no new path to be added to the master problem or the solution converges to the pre-specified level. The former "no-new-path" termination condition will possibly lead to inaccurate solutions if the termination occurs earlier than expected, as discussed by [20]. To overcome this possibly arising solution inaccuracy problem, we propose two supplementary or replaceable approaches below.

The first approach is to find the constrained k-shortest path instead of the constrained shortest path in the column generation phase and add these k-shortest paths into the restricted route set. Obviously, the accuracy of solutions depends on the value of k: the larger the k value is, the higher possibility there is at least one new path (to the restricted path set) in the generated k-shortest paths. Some k-shortest path algorithm finds k-shortest paths in an ascending order from 1 to k, i.e., it first finds the 1st-shortest path, then 2nd-shortest path, …, and finally kth-shortest path. If such an ascending k-shortest path algorithm is employed here, we do not need to specify a fixed k value a priori, but to generate k-shortest paths as many as needed. More specifically, the algorithm is executed in a way of checking whether or not the newly generated path has been included in the existing restricted path set when a new path is generated. Whenever a path that has not been included in the restricted path set is generated by the k-shortest path algorithm, we stop the algorithm and add this path into the restricted path set; otherwise, we continue the k-shortest path search process to find the next shortest path. The search process at each iteration is finished until a path that has not been included in the existing path set is found.

The second approach is to solve the SUE problems by an alternative (probably link-based) algorithm that does not suffer the aforementioned solution inaccuracy issue. Then we derive the path flow pattern from the obtained link flow pattern by applying a k-shortest path algorithm and the logit probability function. As long as k is sufficiently large, the resulting k-path flow pattern will be sufficiently close to the link flow pattern. Moreover, we can implement the k-shortest path algorithm in an iterative way of generating a new path and checking the closeness of the resulting path flow pattern to the link flow pattern until the expected solution accuracy is reached.

NUMERICAL ANALYSIS

On the basis of an implementation of the modeling and solution methods developed above, we present a numerical example of evaluating the impacts on the aggregated routing behavior and network flow pattern of BEVs from the electricity-charging price. The main purpose of this evaluation is twofold: (i) to highlight the difference of routing behavior and flow pattern between BEV and GV networks; (ii) to quantify how electricity-charging prices e_h and e_s influence the individual behavior and network performance. The Sioux Falls network (see Fig. 3), which has 24 nodes, 76 links and 528 O-D pairs, is used as the numerical example. According to Assumption 2, the electricity in a full charge should be enough for a BEV to finish at least the origin-destination trip. The effective vehicle range we set is 60 distance units (e.g., miles), which satisfies Assumption 2. In this particular numerical test, we set time-equivalent electricity-charging prices $e_h = 0.005$hr/mile and $e_s = 0.01$hr/mile, and time-equivalent gasoline-charging price $g = 0.02$hr/mile, which are equivalent to \$0.045/mile, \$0.09/mile, and \$0.18/mile, respectively, given the value of time \$9/hour. These electricity and gasoline prices are estimated based on the transportation economics data in [29, 30].

Changes of network flows

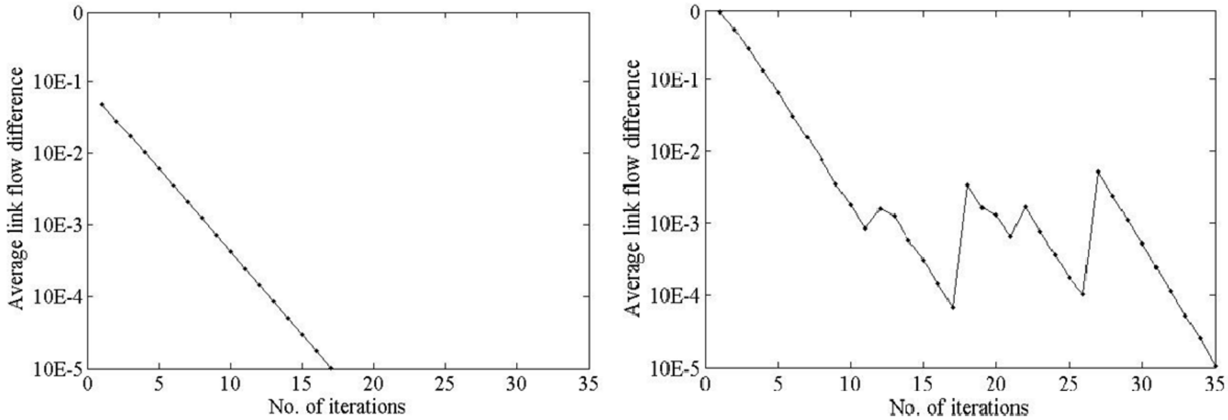
We first discuss the difference of the BEV and GV network flow patterns in the same traffic network with the same O-D demands and the same supply and demand parameters (i.e., the link performance function parameters $\alpha = 0.15$ and $\beta = 4$ and the logit dispersion parameter = 1). The solution difference is only caused by the different travel cost compositions. The travel impedance includes the travel time and charging cost. In contrast, the GV network flow pattern is produced by solving the following SUE problem,

$$\min_{\mathbf{f}} z_{gv} = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}\,(\ln f_k^{rs}) + \sum_{a}\int_0^{x_0} t_a(\omega)d\omega + \sum_{a} g\cdot d_a\cdot x_a \qquad (20)$$

subject to (5)-(7)

in which the travel impedance includes the travel time and fueling cost. In this SUE formulation for GV flows, g represents the travel time-equivalent fueling cost per unit distance for GVs. For comparison fairness, the SUE problem for GV flows is also solved by the DSD algorithm.

Given that the convergence criterion is set as $\epsilon = 10^{-5}$, the solution convergence curves from the BEV and GV solution processes are given in Fig. 2. While both the convergence curves show approximately a linear convergence rate, the convergence process for the BEV network costs significantly more iterations to reach the same solution precision level, without mentioning that each iteration of the BEV solution process is typically much longer than that of the GV solution process. The former involves solving the constrained shortest path problem while the latter requires only solving the basic shortest path problem. Meanwhile, the resulting network flow patterns of BEVs and GVs on the link level are shown in Table 1. From this table, we see that the link flows between the two driver populations are significantly different on a large number of links, especially links 13, 19, 22, and 24, among others. With the gasoline price set as 4 times the home-based electricity price and 2 times the destination-based electricity price, it is apparent that under the equilibrium conditions BEV drivers tend to use longer routes than GV drivers if these longer routes result in lower total travel costs. As a result, the BEV network flow pattern delivers a higher vehicle miles traveled (VMT) value and lower vehicle hours traveled (VHT) value compared to the GV network flow pattern: 2,134,887.2 miles (BEV) versus 2,105,536.2 miles (GV), and 1,709,663.9 hours (BEV) versus 1,916,974.2 hours (GV).



(a) The SUE Problem in the GV Network (b) The SUE Problem in the BEV network

FIGURE 2. Solution Convergence Performance of the SUE-GV and SUE-BEV Problems

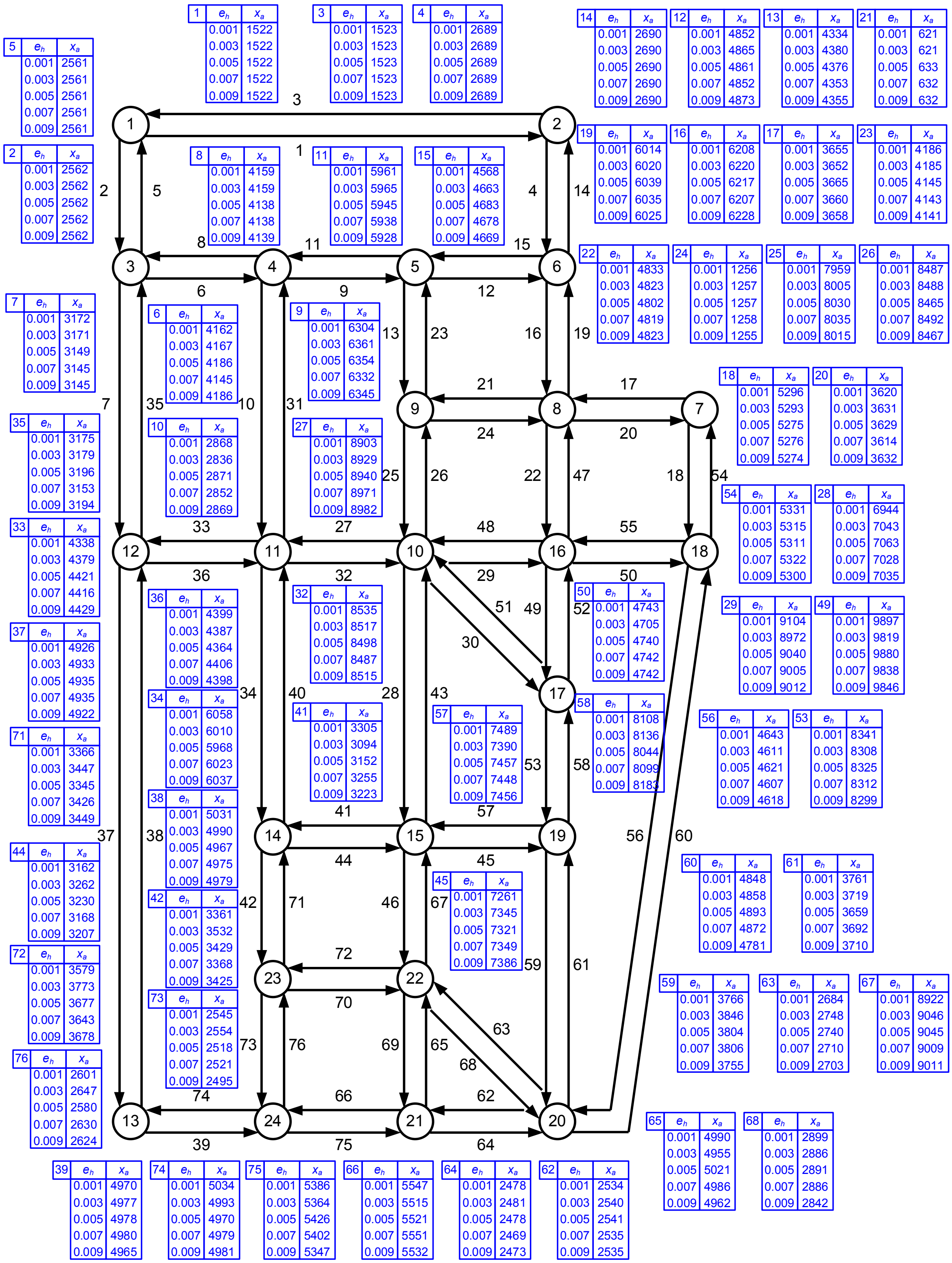
TABLE 1 Link Flow Patterns of the GV and BEV Networks

Link	GV	BEV	Link	GV	BEV	Link	GV	BEV	Link	GV	BEV
1	1583	1522	20	3435	3629	39	4730	4978	58	8473	8044
2	2501	2562	21	417	633	40	6201	6027	59	2474	3804
3	1583	1523	22	6048	4802	41	3532	3152	60	5023	4893
4	2750	2689	23	3278	4145	42	3554	3429	61	2474	3659
5	2501	2561	24	417	1257	43	6073	7063	62	1954	2541
6	4249	4186	25	7386	8030	44	3532	3230	63	2300	2740
7	2843	3149	26	7428	8465	45	8138	7321	64	1923	2478
8	4259	4138	27	8346	8940	46	9235	9145	65	5508	5021
9	6057	6354	28	6031	7063	47	6048	3965	66	5545	5521
10	2808	2871	29	10577	9040	48	10618	9933	67	9235	9045
11	6067	5945	30	339	465	49	10148	9880	68	2300	2891
12	4559	4861	31	2850	3274	50	5646	4740	69	5508	5051
13	3278	4376	32	8263	8498	51	339	406	70	3681	3596
14	2750	2690	33	4491	4421	52	10148	9657	71	3554	3409
15	4569	4683	34	6201	5968	53	8473	8325	72	3681	3677
16	7013	6217	35	2833	3196	54	5409	5311	73	2377	2518
17	3444	3665	36	4491	4364	55	5687	5018	74	4720	4970
18	5399	5275	37	5177	4935	56	5013	4621	75	5513	5426
19	7023	6039	38	5208	4967	57	8138	7457	76	2377	2580

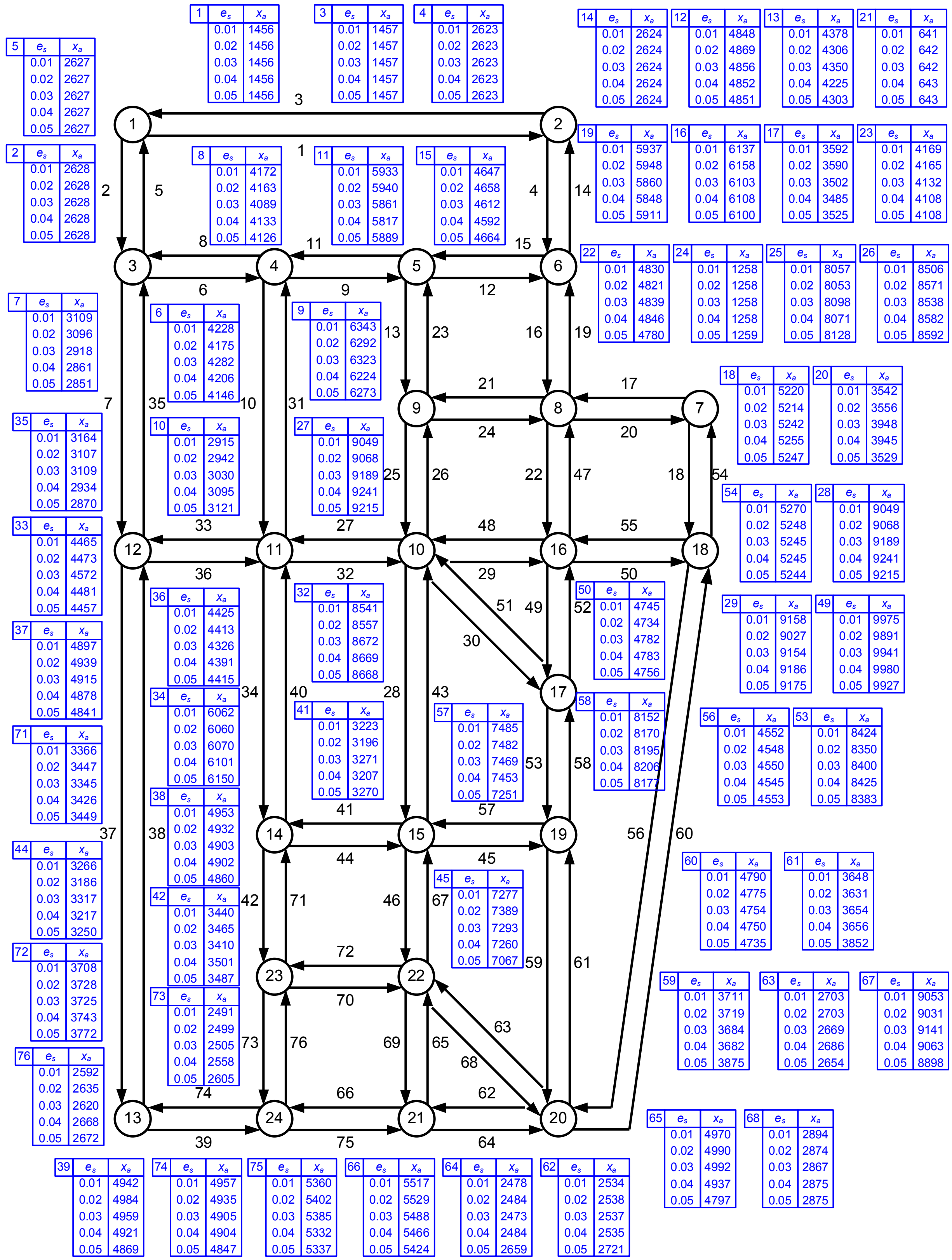
Impacts of the electricity-charging price

The impacts of the electricity-charging price on network flows are evaluated under different values of the home-based price, e_h, and workplace-based price, e_s. For this purpose, we consider two scenarios: one is to change the workplace-based price from 0.01hr/mi to 0.05hr/mi while keeping the home-based price fixed at 0.005hr/mi, and another is to change the home-based price from 0.001hr/mi to 0.009hr/mi while keeping the workplace-based price fixed at 0.01hr/mi. Certainly, these two scenarios cannot provide a complete evaluation for the impacts of the electricity-charging prices on network flows. Our purpose here is illustrative rather than comprehensive; we simply intend to provide an initial assessment on the magnitude of network flow changes given a reasonable range of electricity-charging price changes.

The network flow changes in the above two evaluation scenarios are presented in Figure 3. First, we notice that most of network links experience relatively small changes in the given spectrum of either the home-based charging price e_h or the workplace-based charging price e_s. For instance, some links, such as link 1, link 3, link 62 and link 64 have almost invisible changes following the variation of e_h from 0.01hr/mi to 0.05 hr/mi. In contrast, some other link flows encounter a more significant change, for example, the flow on link 72 is changed by at most 3% when e_h varies and flow on link 35 is changed by at most 9% when e_s varies. As an overview, we counted the number of links that experience different flow variations as a result of the full-scale change of e_h (from 0.001hr/mi to 0.009hr/mi) and the full-scale change of e_s (from 0.01hr/mi to 0.05hr/mi), as shown in Figure 4.

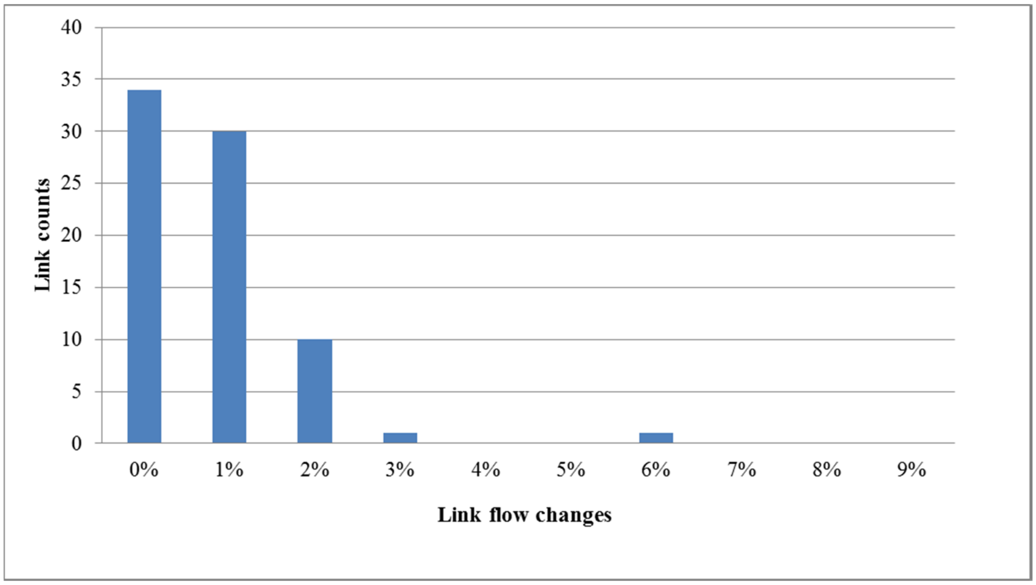


(a) Link flow changes with varying e_h (with a fixed value of e_s at 0.01hr/mi)

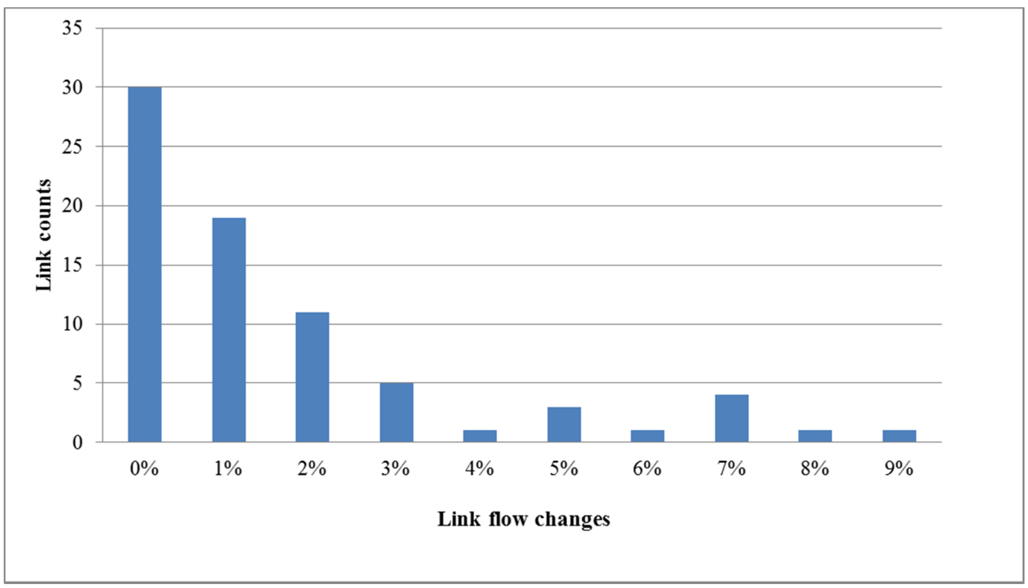


(b) Link flow changes with varying e_s (with a fixed value of e_h at 0.005hr/mi)

FIGURE 3. Link flows with Varying Charging Price



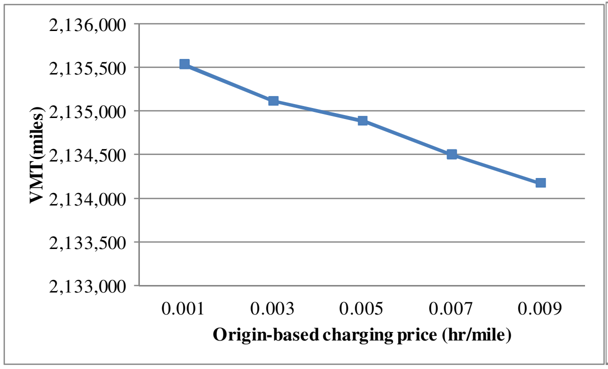
(a) Given Fixed e_s, Link Flow Changes with e_h=0.001hr/mi and e_h=0.009hr/mi

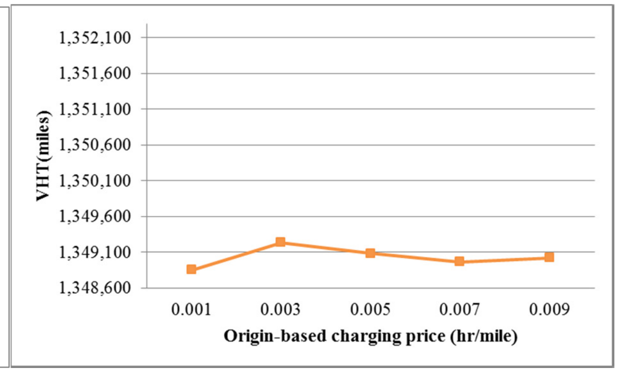


(b) Given Fixed e_h, Link Flow Changes with e_s=0.01hr/mi and e_s=0.05hr/mi

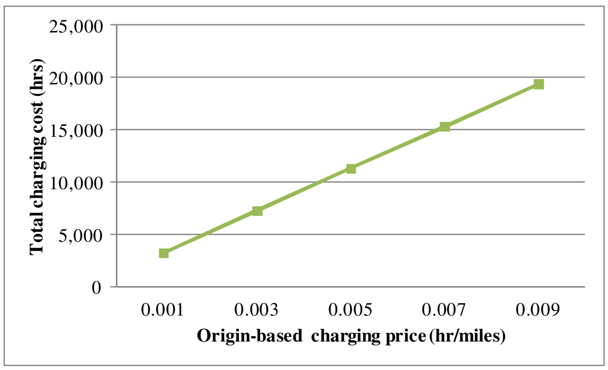
FIGURE 4. Counts in Each Range of Link Flow Changes

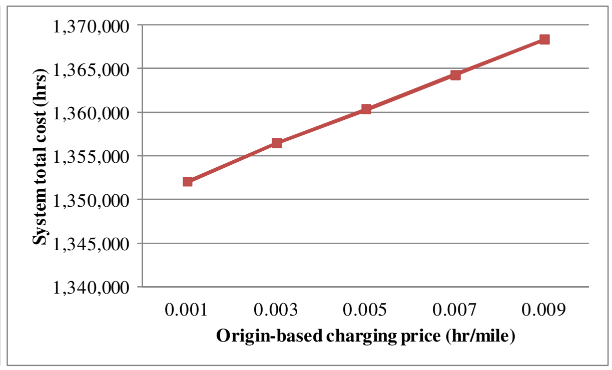
Finally, we evaluate the impacts of electricity-charging prices on the network level. Figure 4 depicts the vehicle miles traveled (VMT), vehicle hours traveled (VHT), total travel costs, and total charging costs, with different values of e_s and e_h. Among all these network-level matrices, the VMT value decreases with the increase of both the home-based and workplace-based electricity-charging prices, while the VMT value decreases with both the electricity-charging prices. Such a network variation is a result of travelers' route choice changes in response to the varying operating cost.



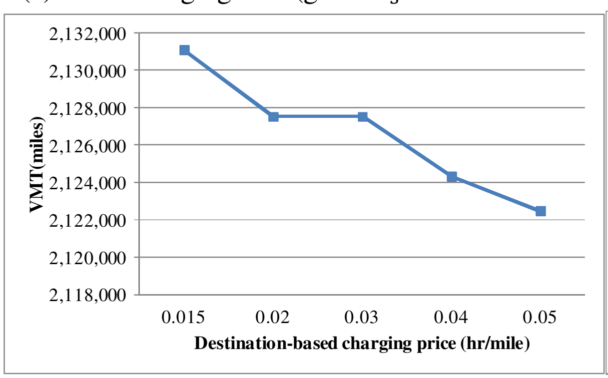


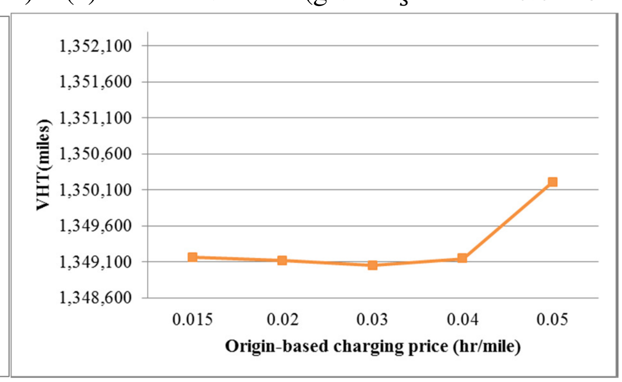
(a) VMT (given e_s fixed at 0.01hr/mi) (b) VHT (given e_s fixed at 0.01hr/mi)



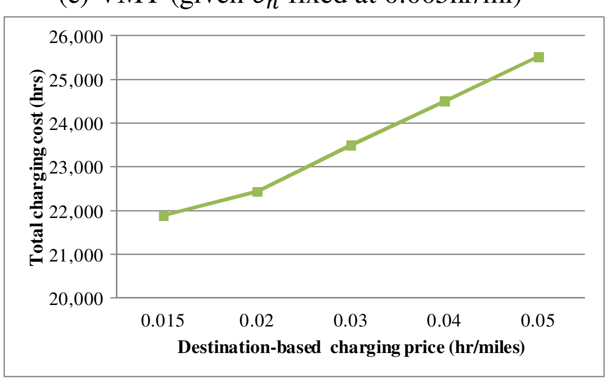


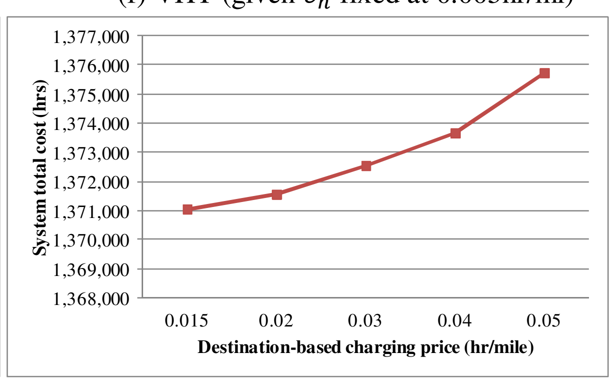
(c) Total charging cost (given e_s fixed at 0.01hr/mi) (d) Total travel cost (given e_s fixed at 0.01hr/mi)





(e) VMT (given e_h fixed at 0.005hr/mi) (f) VHT (given e_h fixed at 0.005hr/mi)





(g) Total operating cost (given e_h fixed at 0.005hr/mi) (h) Total travel cost (given e_h fixed at 0.005hr/mi)

FIGURE 4. System performance with different electricity-charging prices

CONCLUSIONS

The purpose of this paper is twofold: (i) developing the modeling and solution methods for characterizing BEV equilibrium network flows; (ii) evaluating the impacts of electricity-charging prices on network flows. For the former purpose, the possible model could be in various forms. Our focus is given to a special logit-based SUE model, which is written as an optimization problem that contains in its objective function a special piecewise linear function of the distance-based electricity-charging cost. The particular reasons for this modeling choice are due to its concise structure and (path flow) solution uniqueness. The piecewise cost setting is the modeling result of different electricity-charging prices at origins and destinations. For the requirement of evaluating the total charging cost, we adopted a solution algorithm that can derive path flow solutions—the DSD algorithm developed by Damberg et al. (1996)—and proposed two supplementary or replaceable algorithmic schemes that can be used to eliminate the solution inaccuracy issue within the original DSD algorithm.

For the second purpose, we implemented the modeling and solution methods for evaluating a benchmark network—the Sioux Falls network—with different electricity-charging price scenarios. The evaluation results show that the BEV network flow pattern is significantly different from the GV network flow pattern in the same network with the same O-D travel demands due to the difference of the electricity-charging and gasoline-fueling prices and the BEV network flow pattern is considerably affected by the difference of the origin-based (or home-based) and destination-based (or workplace-based) electricity-charging prices. In particular, on the network level, the BEV network incurs a higher VMT value and a lower VHT value compared to the GV network, and the VMT value decreases and the VHT value increases with the increase of either the origin-based or destination-based electricity-charging price.

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