Evaluation of a Strategic Road Pricing Scheme Accounting for Day-to-Day and Long-Term Demand Uncertainty

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Transport network pricing schemes are an integral traffic management strategy that can be implemented to reduce congestion, among other network impacts. However, the problem of determining tolls becomes much more complex when multiple sources of demand uncertainty are considered. This paper proposes a novel tolling model based on a particular variant of strategic user equilibrium in which users base their route choice decisions on a known demand distribution. The study showed that by using an average daily demand, a marginal social cost–based tolling approach could induce near optimal conditions in a strategic network. However, uncertainty was associated with the long-term future planning demand; inaccurate forecasts of future demand could result in poor realized tolling scheme performance. Therefore, this paper also proposes a method to test the robustness of a tolling scheme, which is the reliability of the link tolls under a range of future demand scenario realizations. Results demonstrated that evaluations of strategic tolling schemes differed when both the short-term and the long-term uncertainty in demand were accounted for, and furthermore suggested that future research into the integration of multiple sources of uncertainty into the pricing scheme evaluation is merited.

Transport network road pricing is a topic of great interest to researchers and practitioners alike. Pricing is one of the primary management tools available to road operators to improve network performance for the benefit of users in the system. In addition, a well-planned tolling scheme will not only help relieve congestion, but can also produce revenue that will support infrastructure health and contribute to a stronger, more reliable network.

However, the problem of road pricing becomes more complex when the inherent uncertainty in origin–destination (O-D) trip demand is considered. While traditional deterministic models such as the marginal social cost (MSC) approach can be easily solved, the models may overestimate performance when factors such as the future planning demand vary from the forecast value. In addition, a deterministic model does not capture the effect of day-to-day demand volatility on user route choice behavior.

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This paper explores a first-best tolling framework when the impact of short-term day-to-day demand uncertainty on user behavior is included by implementing a variant of a strategic user equilibrium–based assignment model, referred to as StrUE (*1*). Under StrUE, users determine route choice according to the expected minimum cost path for a known distribution of the day-to-day demand. The strategic model output is a set of fixed link flow proportions that defines link flow patterns. Then, on any given day, the actual link flow volumes will be a function of the strategic fixed proportions and the realized demand. Therefore, a particular demand realization will result in nonequilibrium link flows, representing the volatile network behavior observed in reality. This study uses a marginal social cost–based approach to propose a tolling methodology that attempts to induce strategic system optimal (StrSO) behavior from users in a strategic equilibrium with tolls (StrT) model.

The long-term uncertainty in planning demand also plays an important role; if the future planning demand scenario varies from the forecast, the performance of a tolling scheme may be overestimated. A robust pricing scheme will consistently estimate system performance for a range of possible future demand realizations. Therefore, this paper proposes a procedure to evaluate the robustness of a tolling scheme, in which possible future demand scenario realizations are sampled from a future planning demand distribution. The methodology introduced in this study isolates the effect of day-to-day demand uncertainty in the short term from the effect of the long-term planning demand uncertainty and presents a method to clearly compare the effects of accounting for each source on tolling scheme evaluation. Thus, this paper demonstrates the importance of including both sources of uncertainty when the system performance of a tolling scheme is evaluated.

LITERATURE REVIEW

Marginal social cost pricing based on Pigouvian taxes has a rich history in the literature (*2*). This method aims to set tolls in such a way that a collective system optimal behavior is induced, rather than having drivers choose routes unilaterally to minimize their own travel time (selfish behavior) (*3, 4*). The tolling framework addressed in this paper is classified as first best, which means that it is possible to toll every link in the network to achieve some objective. While maximizing social welfare by relieving congestion may be a common goal for public planning agencies, many other objectives have also been explored, among those aims that may represent the interests of private tolling agencies, such as maximizing revenue, minimizing tolling locations, and minimizing the maximum toll collected (*5, 6*).

Second-best tolling scenarios, in which not all links in the network are available to be tolled because of political or social restrictions, have also been well explored in the literature (*7, 8*). However, to introduce the impact of the StrT model, only schemes in which all links in the network are priced are considered in this work.

While the pioneering works on pricing road networks assumed travel demand and other network characteristics (such as link capacity) to be fixed values, the impact of uncertainties on transport models has become another popular topic in the literature. This topic is particularly important for tolling scenarios because optimal prices that are calculated for an unrealized level of demand could have an unpredictable impact on network conditions, a fact that is further discussed by Lemp and Kockelman (*9*). It is commonly agreed that the main sources of uncertainty in a transport network result from the demand (*10, 11*), supply (*12*), and behavioral choices of travelers (*13*). Boyles et al. examined first-best pricing while accounting for uncertainty in road capacity and further looked at the impact of supplying users with information about the state of the network (*14*). This study highlights the difference between tolling schemes that respond to network conditions and tolls that are intended to address recurring, predictable congestion. Each of these sources could affect optimal toll design in different ways. Researchers begin by analyzing different sources in isolation, but more complicated models such as the approach by Gardner et al., who account for uncertainty in both demand and supply, may offer more realistic insights into the road network (*15*).

A number of studies have approached the issue of travel demand uncertainty and its impact on tolling. Gardner et al. examine the impact of long-term demand uncertainty—such as that resulting from changes in land use, technology, and petrol prices—on robust tolling schemes and evaluate a number of approaches to solve this problem (*16*). The authors show that MSC tolls that are calculated by using an expected demand can result in suboptimal system performance, especially when the actual system performance differs significantly from what was forecast. Gardner et al. further explore a number of solution methods for solving a similar problem and find that using an inflated demand scenario gave the most consistently robust results (*17*). Li et al. propose a bilevel mathematical programming formulation to solve for first-best tolls aimed at increasing network reliability, in which users' choices are determined with a multinomial logit model (*18*). Sumalee and Xu also examine the impact of stochastic demand by treating network demand and link flows as random variables (*19*). Their work addresses uncertainty in user behavior by considering how different risk attitudes from users might affect pricing results, which is additionally a method of incorporating users' value of travel time reliability. Li et al. extend this model to find the optimal tolls with the objective of minimizing emissions (*20*).

The work introduced here differs from previous contributions in its novel behavioral model to capture users' strategic decisions. Strategic traffic assignment was introduced by Dixit et al. and found equilibrium flows on the basis of expected path costs; their work is detailed in the next section (*1*). This model results in link volumes that will vary from day to day, thus accounting for short-term demand uncertainty that users face making day-to-day route choice decisions. Waller et al. propose a linear formulation for a dynamic version of the strategic problem that finds optimal route flows across a discrete set of possible demand scenarios (*21*).

This study extends the strategic assignment model to a StrT first-best pricing application. Previous work has examined the impact of short-term demand uncertainty or long-term demand uncertainty on first-best tolling in isolation, but rarely in combination. This study proposes a flexible framework to fill that gap.

PRICING MODEL DESCRIPTION

StrUE MSC Theoretical Framework

The strategic route choice assignment model accounts for the day-to-day volatility in demand by assuming that users know the day-to-day demand distribution and make their choices strategically on the basis of that knowledge. Travelers then follow a route choice decision based on expected cost regardless of manifested travel demand, but the number of users traveling in each demand actualization will change. The result of this approach is a fixed proportion of flow that will travel on each link; the actual link flow will then vary according to realizations from the day-to-day travel demand distribution. Equations 1 through 4 show the mathematical formulation of the StrUE model that minimizes the strategic user equilibrium objective function $z(p, T)$ (*1*). Table 1 contains an explanation of the notation and variable definitions.

$$\text{minimize}\, z(p,T) = \int_0^\infty \sum_{a\in A} \int_0^{p_a} t_a(p,T)\, g(T)\, dp\, dT \tag{1}$$

subject to

$$\sum_{k\in K} p_k^{rs} = q_{rs} \qquad \forall r, s \tag{2}$$

$$f_k^{rs} \geq 0 \qquad \forall r, s \tag{3}$$

$$p_a = \sum_{r\in R}\sum_{s\in S}\sum_{k\in K} f_k^{rs}\delta_{a,k}^{rs} \qquad \forall a \tag{4}$$

To ensure uniqueness of the link flows for each O-D, path flow proportion is assumed to be equal under all demand scenarios. Therefore, each path will be altered proportionally when the total

TABLE 1 Notation for General Strategic Assignment Approach

Variable or Quantity	Definition
$a \in A$	Index for link a in set of all network links A
$r \in R$, $s \in S$	Index for origin r in set of all origins R and destination s in set of all destinations S, respectively
p_a	Proportion of total travel demand on link a
f_k^{rs}	Proportion of total travel demand on path k connecting origin r and destination s
T	Random variable representing total number of trips for all O-D pairs
$g(T)$	Probability distribution for day-to-day travel demand, representing number of trips T
M_k	kth analytical moment of demand distribution $g(T)$
$t_a\,(pT)$	Travel cost function on link a
$\tau_a \in \Phi$	Tau (τ) value on link a contained within set of tau (τ) values Φ
$\delta_{a,k}^{rs}$	Indicator equal to 1 if link a is contained on path k connecting origin r and destination s, and 0 otherwise

O-D demand varies. The system performance measures in the strategic approach can be found through either analytical derivations or simulation-based sampling methods and will be detailed in the next section.

The purpose of an MSC-based pricing scheme is to ensure that the traffic patterns that result from individual decision makers seeking to maximize their own utility from a myopic perspective can be improved to social optimal through the implementation of tolls. The problem of setting optimal tolls in the strategic assignment scenario becomes significantly more complex than in the deterministic case. The reason is in part the result of the way each model handles the individual traveler. The first-order output of the deterministic user equilibrium model is link flows, representing the number of individuals on each link. Traditional pricing schemes target the individual vehicle on a link by pricing the individual impact on system travel time. Furthermore, realistic applications of traditional tolling are also constrained by the individual, because in a real setting users must be charged a certain amount on a road each day.

However, the first-order output from the strategic approach is proportions on each link; the link flows are an extension of this proportion and change according to the realization of the day-to-day travel demand. Thus, a pure MSC strategic pricing approach would target the proportion of flow on a link by pricing the proportional impact on system travel time; however, system travel time is a product of random variable T and will be changing with each realization of the demand. It follows that the actual toll price on each link would also be changing with the realization of the total trips T. Therefore, to set an MSC pricing scheme that would result in perfect StrSO flow patterns, the network operator would need to have perfect knowledge of all demand realizations; obviously, that is unrealistic.

However, with a slight modification in approach, StrT can be derived to fit the more realistic data constraints of the problem. Therefore, the approach is based on the concept of an average daily demand (AD) total system travel time [AD(TSTT)]. In this method, the day-to-day demand realization is still a changing random variable T, but an average daily total travel time, defined as the proportion on a link multiplied by the first moment (i.e., the mean) of the demand distribution, is targeted. In the strategic case the tolls are set so that the system travel time for an average daily demand is minimized. In the strategic case the tolls are set such that the AD(TSTT) is minimized, as shown in Equation 5.

$$\begin{aligned}\mathrm{AD(TSTT)} &= \sum_a \int_0^\infty (p_a M_1)\, t_a(pT)\, g(T)\, dT \\ &= M_1 \sum_a \int_0^\infty p_a t_a(pT)\, g(T)\, dT \end{aligned} \tag{5}$$

Equation 5 shows that because the first moment of the demand distribution (M_1) is constant and a property of the system demand, minimizing AD(TSTT) is equivalent to minimizing the expected total system travel time $E(\mathrm{TSTT}) = \sum_a \int_0^\infty p_a t_a(pT) g(T) dT$. To derive the tolls that should be implemented on each link so as to minimize AD(TSTT), one must consider the integration by parts of the following term:

$$\begin{aligned}&\sum_a \int_0^\infty \int_0^{p_a} p \frac{dt_a(pT)}{dp} dp\; g(T)\, dT \\ &= \sum_a \int_0^\infty \left(p_a t_a(p_a T) - \int_0^{p_a} t_a(pT)\, dp \right) g(T)\, dT \end{aligned} \tag{6}$$

$$\begin{aligned}&\sum_a \int_0^\infty \int_0^{p_a} \left[\left(t_a(pT) + p \frac{dt_a(pT)}{dp} \right) dp \right] g(T)\, dT \\ &= \sum_a \int_0^\infty p_a t_a(p_a T)\, g(T)\, dT \end{aligned} \tag{7}$$

Minimizing the first part of the left-hand side of Equation 7 represents the StrUE objective, and minimizing the right-hand side of Equation 7 represents minimizing AD(TSTT). Therefore the marginal toll (toll_a) that needs to be applied on each link would be

$$\mathrm{toll}_a = \int_0^\infty \int_0^{p_a^*} p \frac{dt_a(pT)}{dp} dp\; g(T)\, dT \tag{8}$$

where p_a^* is the AD(TSTT) flow pattern.

StrT Application

This section describes the specific notation, equations, and assumptions made for the application of the StrT model and the MSC approach in this work. Table 2 contains a detailed summary of the notation and definitions of variables introduced in this section.

In regard to the day-to-day travel demand, this approach assumes a lognormal (LN) distribution with random variable $T \sim LN(\mu_s, \sigma_s)$, and it assumes that the O-D demand follows fixed, specified proportions. Travelers make their route choices according to knowledge of the distribution and the resulting expected travel costs. In addition, this work uses a modified version of the well-known Bureau of Public Roads function to make the formulation presented in the previous subsection tractable:

$$t_a(p_a T) = t_{f_a} \left(1 + \alpha \left(\frac{p_a T}{c_a} \right)^\beta \right) \tag{9}$$

To derive the link toll values in this work, where the α and β parameters are the same for all links in the network, and $\bar{p}$ is the optimal link proportion patterns resulting from the StrAD, the tolls can be calculated in Equation 10, where $\bar{p}_a$ is the StrSO link proportion for link a.

$$\tau_a = t_f \alpha \beta M_\beta \left(\frac{\bar{p}_a}{c_a} \right)^\beta \tag{10}$$

The four assignment problems necessary in this approach (StrUE, StrSO, StrAD, and StrT) result in three possible system performance measures. The exact value of a system performance measure will differ depending on the assignment problem. The three system performance measures are expected total system travel time E, average demand total system travel time AD, and standard deviation (SD) of total system travel time. In addition, the StrT problem includes tolling and outputs expected revenue R. While this combination results in 14 possible system performance measures, not all of these combinations are necessary to evaluate the pricing model performance. This work focuses on E and SD.

Each of these performance measures can be analytically derived with the theoretical framework described in the previous subsection and the assumptions concerning the demand distribution. The

TABLE 2 Notation for Specific Application of StrUE Approach

Variable or Quantity	Definition
T	Day-to-day random variable for demand following a lognormal distribution, $T \sim LN(\mu_s, \sigma_s)$; assume a fixed proportion of demand for all O-D pairs.
$E_s(T)$	Expected total number of trips, where $E(T) = e^{\mu_s + \theta_s^2/2}$
$\text{var}_s(T)$	Variance of total number of trips T, where $\text{var}(T) = \left(e^{\theta_s^2} - 1\right)e^{2\mu_s + \theta_s^2}$
CV_S	Coefficient of variation of day-to-day travel demand distribution equal to ratio of mean to standard deviation: $\frac{E(T)}{\sqrt{\text{var}(T)}}$
$g(E_s, \text{CV}_S)$	Convenient notation of lognormal strategic day-to-day demand distribution with expected value of demand E_S and standard deviation of demand $\text{CV}_S * E_S$; assume that parameters μ_s and σ_s are found as above.
$\boldsymbol{p}$	Set of link flow proportions for all $a \in A$ output by a strategic assignment model
c_a	Capacity on link a (vehicles per hour)
$t_{f,a}$	Free-flow travel time on link a (min)
α, β	Geometric link parameters for Bureau of Public Roads cost function equal to 0.15 and 4, respectively
VOTT	Value of travel time for network users; for simplicity, assumed to be $10/min
TSTT	Abbreviation for total system travel time
n	Sample realized demand values where n: $T \sim LN(\mu_s, \theta_s)$
N	Total number of demand samples
E	A system performance measure representing expected value of TSTT (min)
AD	A system performance measure representing expected value average demand system travel time based on average daily demand (min)
SD	A system performance measure representing standard deviation of TSTT (min)
R	A system performance measure representing expected revenue from a pricing scheme Φ ($)
$\Diamond(\cdot)$	Symbol meaning that value "$\cdot$" is analytically derived, e.g., $\Diamond E$ is the analytical TSTT
$\odot(\cdot)$	Symbol meaning that value "$\cdot$" was obtained through simulation testing, e.g., $\odot E$ is the average TSTT from n demand samples
$\Delta(\cdot,\cdot)$	Percentage of difference between two system performance measures; e.g., $\Delta(\Diamond E_{\text{StrUE}}, \odot E_{\text{StrUE}})$ is difference between analytical and simulated E values resulting from StrUE (%)
M_1	First moment
M_β	M_k, where β is a parameter
μ	Mean
σ	Standard deviation

symbol $\Diamond$ indicates that a measure was calculated from the analytical equation. ($\Diamond$ SD is not included in this study.) The three analytical performance measures can be found as

$$\Diamond E = \sum_{a \in A}\left(t_f\left(p_a M_1 + \alpha \frac{p_a^{\beta+1}}{c^\beta} M_{\beta+1}\right)\right) \tag{11}$$

$$\Diamond \text{AD} = \sum_{a \in A}\left(t_f M_1 p_a\left(1 + \alpha \frac{p_a^{\beta+1}}{c^\beta} M_\beta\right)\right) \tag{12}$$

$$\Diamond R = \sum_{a \in A} p_a M_1 \tau_a \tag{13}$$

In addition, system performance measures can be found through simulation testing, in which random numbers are generated from the strategic demand distribution to represent demand realizations. Dixit et al. show that analytical and simulation results converge (*1*). Through empirical testing it was observed that a high number of demand samples N were necessary for the analytical and simulation results to reliably converge. This finding is a reflection of the complex behavior of the StrT assignment problem. To find a balance between computation and convergence reliability, a value of N = 50,000 (unless specified otherwise) is assumed for the rest of this work.

The method for finding the strategic marginal social cost–based tolls, called Procedure A, follows. The symbol $\odot$ before a value means that the value was obtained through simulation testing. CV_S represents the coefficient of variation of day-to-day travel demand distribution.

Begin Procedure A

Step 1. Given demand distribution $g(E_s, \text{CV}_S)$, solve for $\boldsymbol{p}_{\textbf{StrUE}}$ and $\Diamond E_{\text{StrUE}}$.
Step 2. Use $\boldsymbol{p}_{\textbf{StrUE}}$ and the simulation subprocedure to obtain $\odot E_{\text{StrUE}}$ and $\odot \text{SD}_{\text{StrUE}}$.
Step 3. Given demand distribution $g(E_s, \text{CV}_S)$, solve for StrAD assignment pattern to obtain $\bar{\boldsymbol{p}}_{\textbf{StrAD}}$.
Step 4. Calculate network tolls Φ by using $\bar{\boldsymbol{p}}_{\textbf{StrAD}}$ and Equation 6.
Step 5. Set network tolls equal to Φ.
Step 6. Given demand distribution $g(E_s, \text{CV}_S)$, solve for $\boldsymbol{p}_{\textbf{StrT}}$ and $\Diamond E_{\text{StrT}}$.
Step 7. Use $\boldsymbol{p}_{\textbf{StrT}}$ and the simulation subprocedure to obtain $\odot E_{\text{StrT}}$ and $\odot \text{SD}_{\text{StrT}}$.
Step 8. Calculate Δ, measures of effectiveness.

End Procedure A

Begin simulation subprocedure

Step 1. With strategic proportions $\boldsymbol{p}$ for specified assignment problem,

Step 2. For each of N simulation realizations

a. Sample a demand realization n from $g(E_s, \mathrm{CV}_S)$,

b. Set $T = n$, and

c. Calculate TSTT (or ADSTT or R) by using T and $\boldsymbol{p}$.

Step 3. Find $\odot E$ and $\odot$SD as mean and standard deviation of the set of N(TSTT) results.

End simulation subprocedure

The simulation subprocedure can be easily adapted to StrUE, StrSO, StrAD, and StrT by using the correct strategic proportions in Step 2c. Further, all sampling results (e.g., $\odot E$, $\odot$SD) can be calculated as running averages to prevent unnecessary memory storage and computation time.

DEMONSTRATION

This demonstration focuses on clarifying the MSC StrT approach and studying the impact of the strategic day-to-day demand uncertainty on system performance. The demonstration network is similar to the well-known Braess's paradox network, in which the addition of a link between Nodes 2 and 3 causes an increase in TSTT resulting from the difference in equilibrium versus system optimal behavior. This network was chosen to capture the interaction between strategic user behavior and the presence of tolls. Figure 1 shows the demonstration network, network parameters, and demand. The initial demand lognormal distribution in this problem has parameters $E_s(T) = 20$ and $\mathrm{CV}_S = 0.2$.

The results from the analytical method compared with the simulation method converge closely, in part as a result of the high number of demand samples. Table 3 shows the analytical and simulation results for E and AD resulting from the StrUE and the StrT assignment problems. While the values of AD and E are not the same, solving the StrAD and the StrSO assignment problems will result in identical proportions.

This demonstration illustrates the impact on system performance of the variation in the day-to-day demand, quantified as the CV_S of the strategic demand distribution. To capture this effect, Procedure A described in the previous subsection was implemented by using the same $E_S(T)$ but varying $0 \leq \mathrm{CV}_S \leq 0.6$ in increments of 0.05. Figure 2 displays the results of $\odot E$ and $\odot$SD from the untolled assignment StrUE and the assignment including tolls StrT from the varying CV_S experiment. As reflected in Table 3, the analytical and simulation methods converge in this problem; therefore, only simulation results are studied in Figure 2.

TABLE 3 Convergence Results for *E* and AD for StrUE and StrT Assignment Patterns

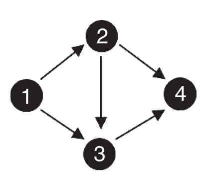
Variable	$\lozenge(\cdot)$	$\odot(\cdot)$	$\Delta(\lozenge, \odot)$ (%)
E_{StrUE} (min)	1,526	1,531	0.31
$\mathrm{AD}_{\mathrm{StrUE}}$ (min)	1,394	1,397	0.20
$\Delta(E_{\mathrm{StrUE}}, \mathrm{AD}_{\mathrm{StrUE}})$ (%)	8.7	8.8	na
E_{StrT} (min)	1,066.87	1,066	0.0
$\mathrm{AD}_{\mathrm{StrT}}$ (min)	1,026.51	1,026.11	0.00
$\Delta(E_{\mathrm{StrT}}, \mathrm{AD}_{\mathrm{StrT}})$ (%)	3.78	3.74	na

NOTE: na = not applicable.

Figure 2 displays the results from StrUE and StrT in two ways: Figure 2*a* shows the absolute results while Figure 2*b* shows the relative results. The horizontal axis in Figure 2, *a* and *b,* shows the CV_S of the strategic demand distribution. The vertical axis of Figure 2*a* shows the value of $\odot E$ and $\odot$SD in minutes. The vertical axis of Figure 2*b* shows the percentage difference between the StrUE results and the StrT results, $\Delta(\odot\mathrm{StrUE}, \odot\mathrm{StrT})$, for both E and SD. Δ is a reflection of system performance improvement that resulted from the implementation of tolls.

To facilitate visual comprehension, the results relating to $\odot E$ are in blue, and the results relating to $\odot$SD are in red. Therefore, visually speaking, for any value of CV_S, the difference between the two red lines in Figure 2*a* is equal to the red bar for that value of CV_S in Figure 2*b,* and the same for the blue lines. For the case of $\mathrm{CV}_S = 0.05$, Figure 2*a* shows $\odot E_{\mathrm{StrUE}} = 1{,}397$ min and $\odot E_{\mathrm{StrT}} = 978$ min. The difference between these two values is about 30%, which is the Δ value shown by the blue bar for $\mathrm{CV}_S = 0.05$ in Figure 2*b*. The 30% represents the reduction in expected TSTT resulting from the tolling scheme, which also reduced $\odot$SD by 65%.

Figure 2 illustrates the relationship between variation in day-to-day demand and network tolling behavior. When $0.05 \leq \mathrm{CV}_S \leq 0.3$, the addition of tolls consistently reduced $\odot E$ and $\odot$SD in the network in a nonlinear fashion. However, when $0.4 \leq \mathrm{CV}_S$, this relationship dismantles, and the $\odot$STD for both StrUE and StrT becomes much greater than $\odot E$. In addition, Figure 2*b* indicates that the relative differences between the tolled and untolled networks are smaller for higher CV_S values. Figure 2 is not scaled to include these values because an SD that is so much greater than the E value seems unrealistic. While, of course, observations are network specific, results indicate that the strategic pricing model may be best applied in networks in which the $\mathrm{CV}_S < 0.4$.



From link	*To link*	c_a	$t_{f,a}$
1	2	50	12
1	3	50	36
2	3	50	1.2
2	4	50	36
3	4	50	12

$T \sim g\,(20, 0.2)$

FIGURE 1 Demonstration network and network parameters.

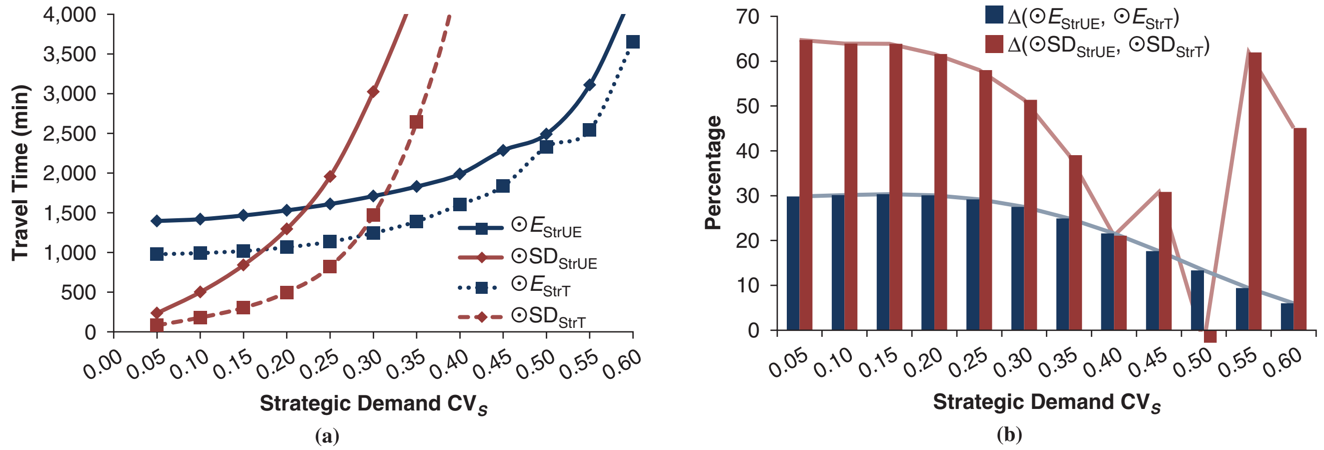


FIGURE 2 Simulation results on demonstration network: (*a*) absolute results and (*b*) relative results.

EVALUATION OF LONG-TERM DEMAND UNCERTAINTY

While the strategic pricing approach accounts for the short-term uncertainty in demand users face when making route choice decisions, planners must still be concerned about the uncertainty in the long-term future planning demand. In the deterministic approach, the interpretation of this concept lies in the exact value of demand that is used to make planning decisions. In cases accounting for long-term uncertainty, the future realization of the travel demand may be different from the predicted planning value. Gardner et al. show that not accounting for possible variation in realized planning demand may result in overestimation of toll performance (*16*).

An analogous situation exists with the strategic approach. However, in the strategic approach the long-term planning uncertainty concerns a future demand scenario. In each demand scenario, planners know that travelers will react strategically by using their knowledge of $g(E_S, \mathrm{CV}_S)$ to make route choices, but the planner does not know the exact value of E_S that will be realized. To have a reliable estimation of the performance of a pricing scheme, the network operator needs to test the impact of the long-term uncertainty associated with a strategic planning demand scenario. A robust pricing scheme will give reliable evaluations when the realized strategic demand scenario differs from the forecast strategic planning demand scenario.

This section describes the necessary assumptions and the method to test the robustness of a pricing scheme that is applied to evaluate the impact of long-term demand uncertainty on the StrT model. Table 4 introduces the additional notation concerning long-term demand uncertainty.

The system performance measures are similar to the approach without long-term demand uncertainty. However, because of the added sampling method, mean and standard deviation results for all strategic system performance measures can be found. This work places emphasis on results obtained through the simulation approach: $M(\odot E)$ is the simulation-based expected TSTT including the impact of long-term planning demand scenario uncertainty, and $\mathrm{SD}(\odot E)$ is the long-term standard deviation of the simulation-based expected total travel time. The mean value of $\odot\mathrm{SD}$ is a robust reflection of variation in the strategic demand scenario TSTT, while $\mathrm{SD}(\odot\mathrm{SD})$ reflects the variation of the variation in future demand scenarios.

Finally, long-term measures of effectiveness are necessary. This study focuses on the change in $\odot E$ and $\odot\mathrm{SD}$ between the strategic tolling scenario, StrT, and the do-nothing StrUE scenario, in which the long-term strategic demand is evaluated without tolls. The difference in travel time is denoted $\Delta(M(\odot E_{\mathrm{StrUE}}), M(\odot E_{\mathrm{StrT}}))$; the reduction in future system variation in travel time is denoted $\Delta(\mathrm{SD}(\odot E_{\mathrm{StrUE}}), \mathrm{SD}(\odot E_{\mathrm{StrT}}))$.

The method for testing the robustness of a set of strategic marginal social cost–based tolls, called Procedure B, is as follows:

Begin Procedure B

Step 1. Set network tolls Φ for $g(E_S, \mathrm{CV}_S)$ by using Procedure A, Steps 3 through 5.

Step 2. For each of Q strategic planning demand scenarios,

a. Sample a demand scenario realization ω from planning distribution $\Omega(\mu_\Omega, \theta_\Omega)$;

b. Set $E_S' = \omega$;

c. Given $g'(E_S', \mathrm{CV}_S)$, solve for $\boldsymbol{p}_{\mathrm{StrT}}'$ and $\Diamond E_{\mathrm{StrT}}'$; and

d. Follow the simulation subprocedure by using $\boldsymbol{p}_{\mathrm{StrT}}'$ and demand distribution $g'(E_S', \mathrm{CV}_S)$.

TABLE 4 Notation for Long-Term Demand Uncertainty

Variable or Quantity	Definition
ω	Possible long-term (future) demand scenario realization ω
$\Omega(\mu_\Omega, \sigma_\Omega)$	Distribution of long-term (future) planning demand scenarios $\omega \sim N(\mu_\Omega, \sigma_\Omega)$
CV_Ω	Coefficient of variation of long-term planning demand scenario distribution to the ratio of mean to SD: $\frac{\mu_\Omega}{\sigma_\Omega}$
Q	Number of long-term demand scenario samples where $Q{:}\omega \sim N(\mu_D, \sigma_D)$
$M(\cdot)$	Mean of a quantity obtained from set of Q planning demand samples; i.e., $M(\Diamond E)$ is long-term expected analytical total system travel time
$\mathrm{SD}(\cdot)$	Standard deviation of a quantity obtained through set of Q samples; i.e., $\mathrm{SD}(\odot\mathrm{SD})$ is SD of a set of SDs of each demand scenario obtained through simulation

Step 3. Find $M(\lozenge E)$, $\mathrm{SD}(\lozenge E)$, $M(\odot E)$, $\mathrm{SD}(\odot E)$, $M(\odot \mathrm{SD})$, and $\mathrm{SD}(\odot \mathrm{SD})$ by using the set of results from each demand scenario realization.

Step 4. Repeat Procedure B with $\Phi = 0$ to obtain M(StrUE) and SD(StrUE) performance measures, and then calculate Δ effectiveness measures.

End Procedure B

This procedure reflects a robust evaluation that accounts for the long-term uncertainty in demand. This procedure can be easily adapted to evaluate the impact of long-term uncertainty in StrUE by setting network tolls $\Phi = 0$, or in StrSO by solving for the appropriate assignment pattern in Step 2c. In addition, this procedure will sample from two distributions (both Ω and g), so it is critical that adequate Q and N values be chosen to minimize sampling bias.

DEMONSTRATION OF EVALUATION OF LONG-TERM DEMAND SCENARIO UNCERTAINTY

The demonstration network from a previous subsection is revisited to provide clarification between the impact of the day-to-day uncertainty resulting from the strategic approach and the impact of long-term uncertainty in the strategic planning demand scenario.

The network parameters in Figure 1 remain the same, with the exception of E_S, which is no longer a known value. The future planning demand scenario in this demonstration has a mean of $\mu_\Omega = 20$ and $\mathrm{CV}_\Omega = 0.2$; therefore, demand realization $\omega \sim N(20,4)$, and for this demonstration, $Q = 1{,}000$. Procedure B was then implemented to obtain an evaluation of tolling scheme Φ that reflected the impact of long-term demand uncertainty.

Similar to Figure 2, Figure 3 shows the results from the varying CV_S experiment; however, now the impact of planning demand scenario uncertainty is accounted for. Again, CV_S was varied from $0 \le \mathrm{CV}_S \le 0.6$ in increments of 0.05. CV_S is not affected by the uncertainty in the planning demand scenario. For each possible CV_S value, Procedure B was implemented to obtain $M(\odot E)$ and $M(\odot \mathrm{SD})$ in the StrUE and StrT models. The horizontal axis of Figure 3 shows each possible CV_S value. The vertical axis of Figure 3*a* shows the values of travel time resulting from the long-term planning demand scenario sampling, while the vertical axis of Figure 3*b* shows the percentage reduction in $M(\odot E)$ and $M(\odot \mathrm{SD})$ resulting from the presence of tolls.

For the case of $\mathrm{CV}_S = 0.05$, Figure 3*a* shows $M(\odot E_{\mathrm{StrUE}}) = 1{,}411$ min and $M(\odot E_{\mathrm{StrT}}) = 1{,}048$ min. The difference between these two values is about 25%, which is the Δ value shown by the blue bar for $\mathrm{CV}_S = 0.05$ in Figure 3*b*. Again for the case of $\mathrm{CV}_S = 0.05$, a robust evaluation of the StrT model results in a 25% reduction in travel time and 54% reduction in standard deviation of travel time, as opposed to 30% and 65%, respectively, for the results without considering long-term uncertainty.

While Figure 3 shows behavior similar to the results in Figure 2 (showing the same experiment but without the added consideration of long-term uncertainty), the behaviors are not the same. The implication is that a network operator should not rely on a pricing scheme without evaluating its robustness by using a method such as Procedure B, lest system performance measures be overestimated. In addition, the unrealistic behavior observed when $0.4 \le \mathrm{CV}_S$ in Figure 2 is less prominent in Figure 3.

MEDIUM NETWORK DEMONSTRATION: SIOUX FALLS, SOUTH DAKOTA, AND ANAHEIM, CALIFORNIA

Results from Evaluation of Long-Term Performance

This section implements Procedure A and Procedure B on networks of Sioux Falls, South Dakota, and Anaheim, California, to demonstrate results and illustrate the scalability of the proposed method. Those networks are both well-known transportation network modeling test networks, the data for which were obtained from Bar-Gera (*22*). The Sioux Falls network consists of 24 nodes, 76 links, and 24 zones, while the Anaheim network consists of 416 nodes, 914 links, and 38 zones. All link parameters are as specified in the known data, with the additional strategic demand parameter of $g(T{:}360{,}600;\ \mathrm{CV}_S)$ and future planning scenario parameter of $\Omega(\omega{:}360{,}600;\ 0.2)$ for Sioux Falls, and $g(T{:}106{,}176;\ \mathrm{CV}_S)$ and $\Omega(\omega{:}106{,}176;\ 0.2)$ for Anaheim. For these models, $N = 50{,}000$ and $Q = 1{,}000$.

The experiment varying $0 \le \mathrm{CV}_S \le 0.6$ in increments of 0.05 described in previous sections was repeated for the case in which

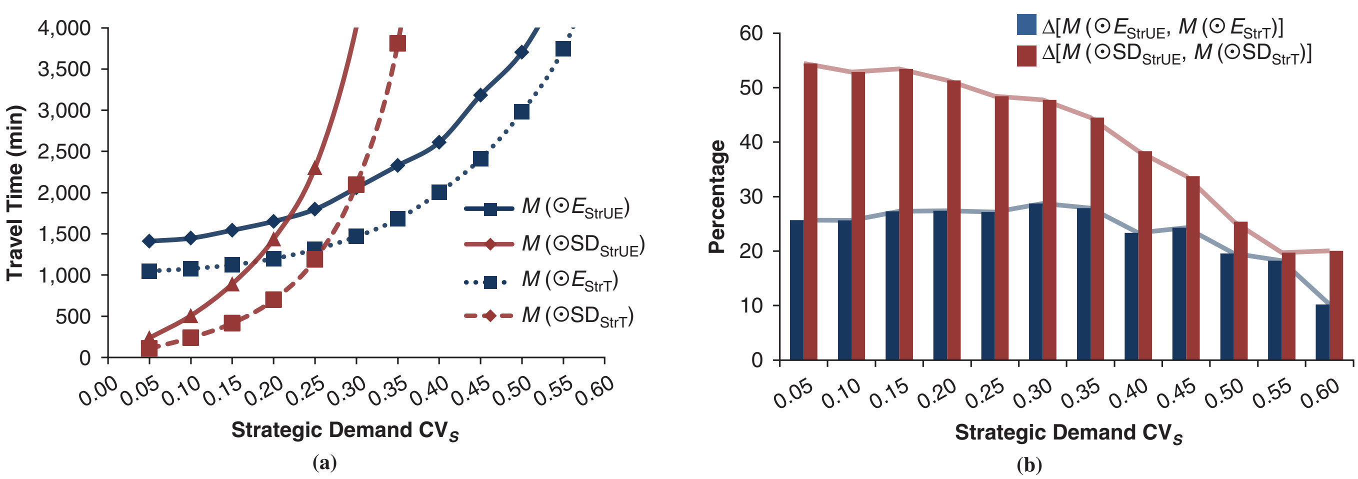
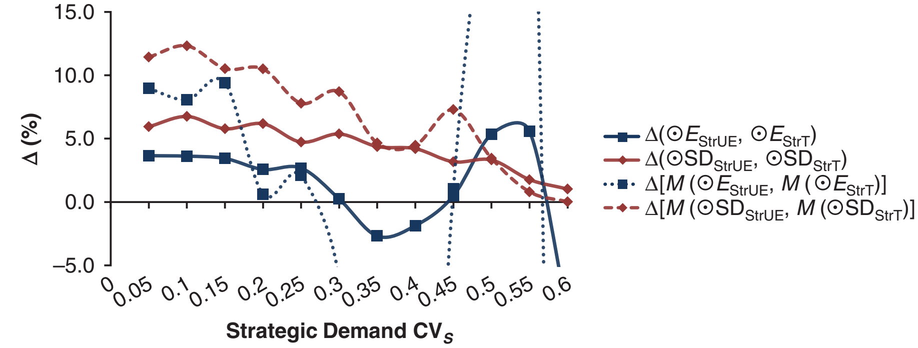
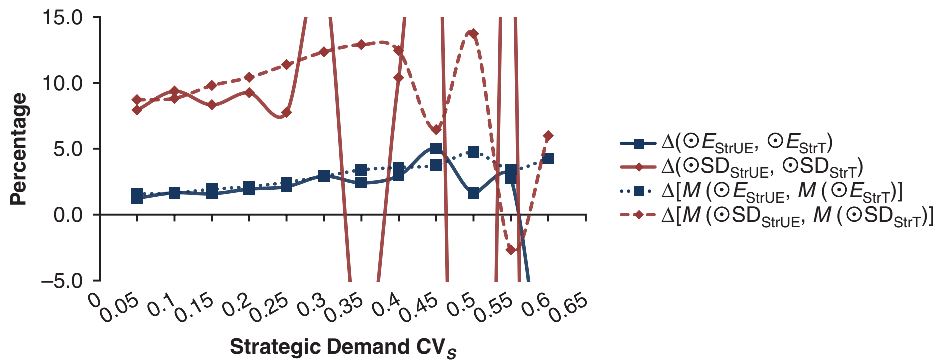


FIGURE 3 Results for evaluating impact of planning demand scenario uncertainty on demonstration network.



(a)



(b)

FIGURE 4 Results comparing approaches including and not including long-term planning demand scenario uncertainty for (*a*) Sioux Falls network and (*b*) Anaheim network.

long-term planning scenario uncertainty was not included, which yields performance measures $\Delta(\odot E)$ and $\Delta(\odot SD)$ reflecting the reduction in system travel time resulting from the addition of the tolls. The same experiment varying CV_S was then repeated for Procedure B to illustrate the different values for effectiveness that might be obtained when the robustness of tolls is included in the evaluation.

Figure 4 shows the results of this experiment for networks in Sioux Falls and Anaheim. The horizontal axis in both of these figures shows the varying CV_S in increments of 0.05. The vertical axis in both figures then represents the Δ values. Once again, the blue lines represent $\odot E$ and $M(\odot E)$, and the red lines represent $\odot SD$ and $M(\odot SD)$.

These two figures suggest a number of observations about the behavior of the StrT model. In both networks, when $0 \leq CV_S \leq 0.25$, not accounting for planning demand scenario uncertainty seems to underestimate system effectiveness. However, at larger values of CV_S, the StrT model seems to dismantle and the results vary wildly. This effect may result from sampling bias, but initial empirical observation indicates that the system performance can vary widely and model convergence is a complicated issue. Nonetheless, this outcome clearly shows that ignoring future planning scenario uncertainty can result in incorrect predictions of tolling scheme performance and supports the need for further research.

Results from Average Demand–Based Tolls

Figure 4 shows the StrT model performance considering long-term demand uncertainty; however, it is also important to consider the case in which tolls are determined on the basis of an average demand (i.e., short-term demand uncertainty is not included in the toll-setting process). The same experiment from the previous subsection was performed for the set of tolls determined on the basis of deterministic conditions, assuming that $CV_S = 0$, representing the average demand. On the Sioux Falls and Anaheim networks, results for $M(\odot E)$ and $M(\odot SD)$ were similar for the cases in which tolls were determined on the basis of average demand versus strategic demand. However, the results for $SD(\odot E)$, a measure of system volatility, differed substantially. In the Anaheim network, for the case of $CV_S = 0.25$, average demand tolls resulted in $\Delta SD(\odot E_{StrUE}, \odot E_{StrT}) = 75\%$ and strategic demand tolls resulted in $\Delta SD(\odot E_{StrUE}, \odot E_{StrT}) = 62\%$, while Sioux Falls showed a similar pattern. These results illustrate that neglecting short-term demand uncertainty may result in an overestimation of toll performance with regard to system robustness.

CONCLUSION

This work introduced a strategic marginal social cost–based pricing methodology. The strategic tolling model (StrT) approach accounts for the influence of day-to-day demand volatility on user route choice behavior and sets tolls such that users are priced for the marginal impact of their myopic route choice on system travel time.

However, network operators must be aware of the additional uncertainty in the long-term planning demand scenario—that is, a future strategic demand scenario realization in which the expected value of total trips E_S differs from the forecast value. A procedure to evaluate the robustness of a strategic pricing scheme was proposed. Initial results show that if both sources of uncertainty are not included in an evaluation of a strategic pricing approach, the performance of a tolling scheme could be underestimated or overestimated, and how the system will behave is not intuitive.

This work contains an introduction to a strategic pricing approach and has juxtaposed two sources of demand uncertainty to clearly differentiate between them. There are a number of research directions that emerge from the comparison. In particular, the use of two sampling distributions may result in unknown convergence behavior that requires further investigation. In addition, the use of Bayesian statistical inference to describe the prior probability distribution of the strategic day-to-day travel demand may present an interesting avenue of research. Finally, the strategic pricing approach to the next-best pricing problem has been left for future research.

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