Robust Tolling Schemes for High-Occupancy/Toll (HOT) Facilities Under Variable Demand

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Abstract

High-Occupancy/Toll (HOT) lanes have become an increasingly prevalent form of congestion management pricing in the U.S. over the past few decades. The success of a HOT facility is dependent on the pricing scheme implemented, which determines the utilization of the HOT lane, and the corresponding congestion relief on the parallel general-purpose (GP) lanes. An additional complexity in determining HOT tolls arises from the inevitable variability in travel demand, which is inherent to transport networks. A successful tolling scheme, whether fixed or time-varying, must therefore be robust to changes in travel demand. In this paper we examine various tolling schemes for HOT facilities in efforts to identify robust pricing policies. The expected performance and corresponding variability of the facility is evaluated under each pricing scheme for different demand profiles.

The focus of this paper is on non-correlated demand uncertainties (i.e. the number of arrivals during a given time interval is independent of the number of arrivals in the preceding and following time intervals), which we model by considering the number of arrivals in each minute as independent random variable with known distribution and time-of-day dependent mean. The performance model for a given demand realization is deterministic.

The results show that a fixed toll can achieve about two thirds of the benefit of an ideal HOT system. The performance of a pre-scheduled toll system is between the fixed toll and the ideal system: closer to the ideal when the coefficient of variation is below 40%, and closer to the fixed toll otherwise. A relatively simple real-time system, with density-based linear adjustment to the pre-scheduled toll, has practically equivalent performance to the ideal system.

Keywords: high-occupancy/toll lanes; tolling; managed lanes; value of time; robust pricing; stochastic demand

1. INTRODUCTION

High-occupancy/toll (HOT) lanes represent one of many traffic demand management strategies available to address peak period congestion. HOT lanes evolved from high-occupancy vehicle (HOV) lanes which were introduced in the 1970's to encourage ride sharing and reduce vehicle demand. However, the designated HOV lane was often underutilized, resulting in increased congestion on the general-purpose (GP) lanes. In attempts to utilize the remaining capacity of HOV lanes, HOT facilities emerged, which allowed lower occupancy vehicles (LOV) to use the managed lane (ML) for a fee. Since introduced in the 1990's, HOT lanes have become an increasingly prevalent form of congestion management in the U.S.[1]

The level of service on a HOT lane can be controlled by regulating the volume of entering LOVs. The volume of LOVs that will choose to pay and take the HOT lane is dependent on the toll value, the user's value of time (VOT), value of reliability and the travel time savings gained from taking the HOT lane relative to the GP lane *(1)*. The pricing scheme implemented is therefore a determining factor in the success of a HOT facility.

HOT tolling strategies can be fixed throughout the day, pre-scheduled by time of day, or dynamically determined in real time in view of prevailing traffic conditions. Expected travel demand is rarely constant throughout a peak period. Furthermore, travel demand is inherently uncertain, and therefore it is unlikely that the realized demand will be the same as the expected demand profile. Moving from fixed tolls, to pre-scheduled, to dynamic tolls, provides system managers additional flexibility for adaptations of the toll to the actual demand. The importance of this advantage may vary by scenario.

On the other hand, additional management flexibility has a price: it may complicate toll collection; it may require additional infrastructure investment (sensors, signs); it reduces

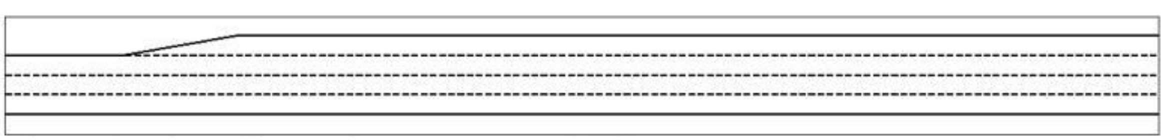
[1] To the best of our knowledge, outside the U.S. there is one HOT facility, in Israel.

transparency of the toll scheme; it may complicate the driver choice process; in turn, there may be influences on the effectiveness of the management scheme; and it may increase the risk of counter-productive toll changes due to algorithmic issues or measurement errors (equipment related or statistical).

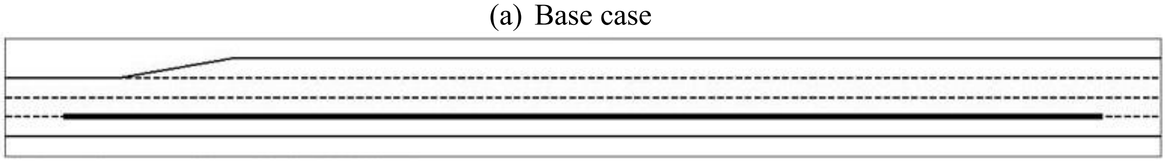
Therefore, a methodology is needed for evaluation of fixed, pre-scheduled and dynamic pricing schemes, in consideration of demand uncertainty. Each option should be robustly designed as much as possible. The purpose of this work is to perform such evaluation in a relatively simplified context, in which insights and key behaviors are more easily identified.

Demand uncertainty may have contributions from several sources. Our focus in this paper is on non-correlated day-to-day demand uncertainties that can be modeled by considering the number of arrivals in each minute as an independent random variable with known distribution and time-of-day dependent mean.

A specific facility configuration is used throughout the paper, as a basis for numerical evaluation, as well as for concrete demonstration of the discussed concepts. The configuration consists of a freeway corridor with parallel managed and GP lanes, with a downstream bottleneck for each lane group (corresponding, for instance, to a lane drop on the GP lanes or a reduction in capacity due to roadway geometry at the merge). We assume that the managed lane is priced in such a way that no queue forms at the diverge point.



(a) Base case



(b) Bottleneck with Managed Lane

Figure 1 **Case study facility. (a) base case - all lanes are for general purpose (GP); and (b) one lane is managed.**

The remainder of this paper is organized as follows: Section 2 introduces the relevant literature, section 3 describes the traffic simulation and lane choice model implemented, section 4 defines the evaluated tolling schemes, section 5 presents the numerical analysis and section 6 concludes the paper.

2. BACKGROUND

Previous literature offers background on several aspects relevant for this paper, including (1) the prevalence of HOT facilities and their classification; (2) methods used for determining toll values; (3) models used for HOT corridor scenario evaluation, particularly the component in the evaluation model used for the representation of HOT lane choice by toll-paying drivers; and finally (4) the role of both day-to-day and long term demand uncertainty on facility performance. This section discusses past research on each topic in turn.

Multiple region-specific case studies have been conducted to investigate the impact of implementing HOT lanes and other types of managed lanes *(2-14)*. In addition, current HOT facilities have been evaluated in attempts to identify user preferences and improve future performance *(15-18)*.

HOT facilities can be classified by the tolling strategy used as: pre-scheduled by time of day, or determined in real time *(19)*. In a relatively early study Dahlgren *(20)* evaluated HOT lanes by assuming "… that as long as there is congestion on the main lanes, the toll can be set so that the HOT lane is fully utilized but not congested." We shall refer to this option as *full-utilization* tolls. In *(21)* the authors established that this strategy is optimal for minimizing average passenger time, under the assumption of deterministic demand and an idealized single-bottleneck model. However, in practice, these assumptions may not hold. For example Lou, Yin and Laval *(22)* claim that "since the moving-bottleneck effect always exists, HOT lanes may never be fully utilized." Nevertheless, the full-utilization strategy may be an appropriate idealized reference point for comparison with other alternatives.

It is challenging to set toll values in practice, particularly in real time. Strategies which have been applied include pre-defined lookup tables based on density *(23)*, scenario-based robust optimization *(24)*, simulation *(25-27)*, or reactive, self-learning approaches *(22, 28)*. These studies suggest that there is considerable value to real-time adjustments to toll values, in terms of increasing average speed, decreasing average roadway occupancy, and producing higher revenue. Under additional simplifying assumptions, optimal toll values can be analytically obtained *(21, 29-33)*.

Models for HOT corridor scenario evaluation have two main components: traffic flow and HOT lane choice. As mentioned above, several different approaches have been used for traffic flow representation, including microscopic simulation *(25)* and mesoscopic simulation *(26)*. A key dynamic aspect relevant for HOT corridor evaluation is queue accumulation, which can also be represented by analytic models, such as a point queue, single-bottleneck model *(34)*.

HOT lane choice by individual drivers is represented in most of the above mentioned studies by a logit model *(22)*. The ratio of cost and time parameters in the logit model can be interpreted as a "representative" VOT value. Variation in VOT across the population plays an important role. A simple approach to deal with VOT variations is by discrete user classes. Alternatively, the choice model can be based on continuous VOT distribution *(35)*.

In a previous publication *(21)* we demonstrated the advantages of a VOT-based choice model for HOT lane evaluation, relative to the Logit model. We used the VOT-based model to compare the performance of the full utilization pricing scheme with a fixed pricing scheme, under a scenario with deterministic demand. In the chosen scenario, the base case (all lanes are GP) and HOV strategies lead to similar performance in terms of average person travel time (APTT), 14 min and 15.1 min respectively. The full utilization strategy reduced

APTT to 9 min (35% less than the base case). The best fixed toll provided approximately two thirds of this benefit (10.7 min APTT, 24% less than the base case).

This paper extends *(21)* by introducing uncertainty into the demand profile. The successful implementation of HOT lane facilities depends on the ability of both fixed and time varying tolls to accommodate the inevitable stochasticity associated with travel demand.

Realistically, it is difficult to accurately predict the demand profile of any transport system. Previous work has illustrated that excluding long term demand uncertainty from the planning process can result in underestimation of system performance *(36)*, and lead to sub-optimal network design decisions *(37)*. Long term demand uncertainty plays a role in the success of tolled transportation systems as well *(38, 39)*. Optimal tolling schemes which account for long term demand uncertainty were proposed by *(40-43)*. Furthermore, Gardner *et al.* *(44)* developed a framework to compare the benefits of real-time travel information against the benefits of responsive pricing under both demand and supply uncertainty.

Demand uncertainty on the short term (day-to-day) scale has been introduced into network modeling in several ways. Asakara & Kashiwadani *(45)* used simulation to study the impact of demand variations on travel reliability. Bell et al. *(46)* used equilibrium sensitivity analysis techniques to develop a more computationally-efficient approach, and Clark & Watling *(47)* develop an analytical formulation for estimating the distribution of total travel time under day-to-day demand uncertainty. Shao et al. *(48)* developed a projection-based heuristic for a stochastic user equilibrium variant of this problem, while Nakayama *(49)* provides a fixed-point formulation.

In this paper we seek robust pricing schemes for a HOT lane facility subject to stochastic demand during a peak travel period. The expected performance and corresponding variability of the facility is evaluated under several pricing schemes for different demand profiles.

3. MODEL

This section describes the model used for the experiments in this paper. The traffic flow model and lane choice model are similar to that described in Gardner et al. *(21)*. These are briefly reviewed in the following subsections for the sake of completeness. The main difference from the previous study is the stochastic nature of demand.

Demand uncertainty may have different sources. Long-term uncertainty may be the result of changes in land-use patterns and demography, among other reasons. We assume a context in which the managing agency has the data sources and the capabilities to update the tolling scheme on a monthly or quarterly basis, and therefore long-term uncertainties are not a concern.

Another source for demand variability is the changes in traffic throughout the day, in a more-or-less repeatable daily pattern. In this respect we assume that the recurrent pattern of daily demand profile can be observed, and thus treated as a deterministic contribution.

Some of the fluctuations in traffic can be attributed to *events*. These include on road accidents and incidents, as well as off roads demand generating gatherings. Downstream on-road events influence mainly the supply of available capacity, but upstream on-road events may influence the observed travel demand at the entrance to the managed lane. The main feature of events is that their influence on travel demand exhibits temporal auto-correlations. As a result, a generalized representation of events, and the joint distribution of their attributes (start time, duration, and intensity), could be fairly difficult to model. We recognize that the ability of HOT lane systems to deal with events is important. Whether event treatment should be automatic by a real-time controller, or alternatively by intervention of a human-in-the-loop procedure, is not obvious at all. Either way, addressing correlated day-to-day variations is beyond the scope of the present paper.

The last type of demand uncertainty is the non-correlated day-to-day variations. In other words, the causes for differences between today and tomorrow in the count of arriving vehicles at 8:16, which will not affect the counts at 8:17. This type of demand uncertainty is the focus of the present paper. Therefore, our stochastic demand model assumes that the number of vehicles of class v (LOV, HOV, Transit) arriving at the GP/HOT diverge point during time interval t is a normally distributed random variable d_v^t with mean μ_v^t and standard deviation σ_v^t, respectively.

Based on these demand realizations, a toll policy will be chosen in one of several ways, and performance metrics calculated. Section 3.1 presents the traffic flow and lane choice model used. Section 3.2 then presents four toll models which will be compared using the performance metrics presented in Section 3.3. The stochastic nature of the demand implies that these tolls and performance metrics are themselves random variables, and a Monte Carlo sampling approach will be used for evaluation.

3.1 The Deterministic Performance Model

Given a specific demand realization d_v^t, the remaining evaluation process is deterministic, as described in this section. Both the GP and HOT lane groups are spatially homogeneous, with bottlenecks at the downstream end of capacity $\bar{q}_{GP}$ and $\bar{q}_{HOT}$. The free-flow time on the two lane groups are denoted τ_{GP}^0 and τ_{HOT}^0, both assumed to be integers in the unit system chosen. We assume there are no onramps or offramps in the region of interest. We discretize time into $T + 1$ intervals of equal length, and index these intervals with $t \in (6\ T)$, where the index t refers to the time at the start of the t-th interval. Let V be the set of vehicle classes (for instance, single-occupant vehicles, HOVs, and transit) and $\bar{V} \subseteq V$ the set of vehicle classes which must pay the toll. The median value of time for vehicles of class v is ζ_v.

The proportion of vehicles of class v choosing the HOT lane over time is denoted p_v^t, which is determined by the toll on the HOT lane (denoted c^t), the travel time savings relative to the GP lane (denoted $\Delta\tau^t$), and the distribution of VOT for this user class, represented by the cumulative distribution function F_v. Travel time calculation is described later in this section. Assuming $\in \bar{V}$, then we have the relation

$$p_v^t = 1 - F_v\left(\frac{c^t}{\Delta\tau^t}\right) \tag{1}$$

with the conventions that $p_v^t = 0$ if $\Delta\tau^t = 0$ and c^t is nonzero, and that $p_v^t = \frac{\bar{q}^{HOT}}{\bar{q}^{HOT} + \bar{q}^{GP}}$ if $\Delta\tau^t = c^t = 0$. (That is, no class v vehicles will use the HOT lane if there is a toll but no time savings; and if there is neither toll nor time savings the two lanes are equivalent, and vehicles will split proportional to downstream capacity.) For vehicle classes which need not pay the toll $v \notin \bar{V}$, lanes are chosen in an "all-or-nothing" manner: $p_v^t = 0$ if $\Delta\tau^t < 0$, $p_v^t = 1$ if $\Delta\tau^t > 0$, and $p_v^t = \frac{\bar{q}^{HOT}}{\bar{q}^{HOT} + \bar{q}^{GP}}$ if $\Delta\tau^t = 0$. In this paper, we assume that the VOT distribution follows a modified Burr distribution, that is,

$$F\left(\frac{c_v^t}{\Delta\tau_v^t};\zeta_v,\gamma\right) = 1 - \frac{1}{\left[1 + \left(\frac{c}{\zeta_v \cdot \Delta\tau}\right)^{\gamma}\right]} \tag{2}$$

where γ is a shape parameter and ζ_v is the median value of time for class v. This distribution has been used to model household income *(50)*.

To describe the evolution of congestion and vehicle flows, we use upstream and downstream cumulative counts: $N_l^{\uparrow}(t)$ and $N_l^{\downarrow}(t)$ are the arrival and departure curves, respectively representing the total number of vehicles that have passed the upstream and downstream end of lane group $l \in \{\text{GP, HOT}\}$ at the start of time interval t. The upstream count equations are:

$$N_{GP}^{\uparrow}(t) = N_{GP}^{\uparrow}(t-1) + \sum_{v} d_v^t \left(1 - p_v^t\right) \tag{3}$$

$$N_{HOT}^{\uparrow}(t) = N_{HOT}^{\uparrow}(t-1) + \sum_{v} d_v^t p_v^t \tag{4}$$

for $t > 0$ with $N(0) = 0$ for all lane groups. The bottleneck constrains the exiting flow in any time interval to be no greater than $\bar{q}_l$. Thus

$$N_l^{\downarrow}(t) = \min\left\{N_l^{\downarrow}(t-1) + \bar{q}_l, N_l^{\uparrow}\left(t - \tau_l^0\right)\right\} \tag{5}$$

for $l \in \{\text{GP, HOT}\}$. To reflect system conditions at times not corresponding to one of the discretization points t, linear interpolation is used:

$$N(t) = (t - \lfloor t \rfloor) N(\lfloor t \rfloor) + (\lceil t \rceil - t) N(\lceil t \rceil) \qquad \text{if } t \in (0, T) \text{ is not an integer} \tag{6}$$

The same formula applies to both upstream and downstream counts of all lane groups. Using this continuous formula, an inverse function can be meaningfully defined:

$$T(n) = \min\{t : N(t) \geq n\} \tag{7}$$

representing the time at which the n-th vehicle passes the point where N is measured (omitting subscripts and superscripts for brevity). Thus, the travel time for a vehicle entering lane group l at time t is

$$\tau_l(t) = T_l^{\downarrow}\left(N_l^{\uparrow}(t)\right) - T_l^{\uparrow}\left(N_l^{\uparrow}(t)\right) \tag{8}$$

3.2 Toll Algorithms

In this paper, we compare four toll-setting methods. Three represent the main groups of tolling schemes discussed above, and the fourth is an idealized method used as a reference for evaluation. The four tolling schemes are (1) fixed tolls (constant across time); (2) pre-scheduled full-utilization tolls based on the mean demand values (FU-M); (3) real-time density-modified full-utilization toll (FU-DM), in which tolls are set in ignorance of the current demand value, but with knowledge of the HOT lane density; and (4) perfect information full-utilization tolls (FU-PI), where the demand realization is known to the operator before tolls are set.

In the group of pre-scheduled tolls there is room for additional improvement, by taking the variance into consideration. Yet we believe that the proposed FU-M toll option is a reasonable representative of the potential of this group. In the group of real-time tolls, there are many possible options, some of which were mentioned in section 2. Often, a calibration process is needed per scenario, and it is not always clear how to automate the calibration. Since we evaluate many scenarios, we preferred a representative that can be easily calibrated automatically, as described in section 5.1. The choice of density as a key parameter for real-time operation is in-line with the typical approaches described in the literature.

To find the FU-M toll, corresponding to full utilization of the HOT lane, we solve the following equation for *c*:

$$\sum_{v\in\bar{V}} \mu_v^t p_v^t(\Delta\tau, c) + \sum_{v\in V\setminus\bar{V}} \mu_v^t = \min\left\{\bar{q}_{HOT}, \sum_v \mu_v^t\right\} \qquad (9)$$

In this way, the inflow to the HOT lane, represented by the left-hand side of (9), is the highest possible value for the given demand level without exceeding bottleneck capacity, represented by the right-hand side, based on expected demand.

The FU-DM toll uses the FU-M toll as a basis; let $c_{FU-M}(t)$ represent this toll (the solution to equation (9)). This toll is modified based on the number of vehicles in the HOT lane. If higher than the "target" full-utilization value $\bar{q}_{HOT}\tau_0^{HOT}$, the HOT lane has too many vehicles, and the toll will be increased; if the number of vehicles on the HOT lane is lower than this target value, additional vehicles can be allowed on without causing a queue and the toll will be decreased. Mathematically, the FU-DM toll $c_{FU-DM}(t)$ is given by equation (10):

$$c_{FU-DM}(t) = c_{FU-M}(t) + \alpha(N_{HOT}^{\uparrow}(t) - N_{HOT}^{\downarrow}(t) - \bar{q}_{HOT}\min\{t, \tau_0^{HOT}\}) \qquad (10)$$

where α is a parameter which must be calibrated.

Finally, the FU-PI toll is calculated as in equation (11):

$$\sum_{v \in \bar{V}} d_v^t p_v^t(\Delta\tau, c) + \sum_{v \in V \setminus \bar{V}} d_v^t = \min\left\{\bar{q}_{HOT}, \sum_v d_v^t\right\} \tag{11}$$

Note that the only difference between the FU-M and FU-PI toll is that the former is calculated using the expected demand (from equation 9), while the latter is calculated using the realized demand value (from equation 11).

3.3 Performance Metrics

In order to evaluate the facility performance under the proposed pricing schemes, six performance metrics are computed and compared for each of the tolling methods, as well as the Base and HOV cases. Let t_{GP}^t and τ_{HOT}^t define the travel times of a vehicle entering the network at time *t*. Each of these quantities in addition to the average vehicle travel time (*AVTT*), and the average passenger travel time (*APTT*) are endogenous. The APTT and AVTT can be calculated as shown below:

$$AVTT = \frac{1}{\bar{d}} \sum_t \sum_v d_v^t (p_v^t \tau_{HOT}^t + (1 - p_v^t)\tau_{GP}^t) \tag{12}$$

$$APTT = \frac{1}{\bar{o}} \sum_t \sum_v o_v d_v^t (p_v^t \tau_{HOT}^t + (1 - p_v^t)\tau_{GP}^t \tag{13}$$

where $\bar{d}$ is the total number of traveling vehicles and $\bar{o}$ is the total number of traveling occupants. The (random) revenue is given by:

$$R = \sum_t \sum_{v \subseteq \bar{V}} d_v^t p_v^t c^t. \tag{14}$$

The focus of this work is incorporating the role of demand uncertainty; therefore we compute the following six performance measures to evaluate the HOT facility:

1. Expected APTT (E[*APTT*])
2. Standard deviation of APTT (S[*APTT*])
3. Expected AVTT (E [*AVTT*])
4. Standard deviation of AVTT (S [*AVTT*])
5. Expected revenue (E [*R*])
6. Standard deviation of revenue (S [*R*])

Each of these performance metrics is calculated based on a random sample of demand scenarios. The measures will help illustrate the impact of demand uncertainty on the HOT lane facility under the proposed lane choice model and pricing schemes.

4. IMPLEMENTATION

The system described above is implemented in a simulation program written in C, based on K scenarios generated using Monte Carlo sampling. This simulation performs the following steps:

1. Initialize sample set: set $k = 0$.
2. Initialize traffic flow model: set $t = 0$, travel times on GP and HOT lanes to free flow.
3. Sample demand: Generate demand sample, d_v^t, from known distribution
4. Calculate toll c (either constant or based on one of the full-utilization formulas (9 - 11))
5. Calculate lane choice probability p_v^t using the lane choice mappings described by (1)
6. Propagate flow, updating upstream counts using (3) and (4), and updating downstream counts using (5).
7. Update statistics (lane group travel time, revenue, and other metrics)
8. If $t < T$, increment t and return to step 3. Otherwise, go to step 9.
9. If $k < K$, increment k and return to step 2. Otherwise, terminate.

Note that our model recalculates the toll at every time step. Thus, the time step should be adjusted based on the temporal resolution of input data, the desired response rate, and any regulatory constraints on how frequently the toll can be changed. In our numerical analyses the time step is set to one minute.

5. NUMERICAL ANALYSIS

The results presented in this section are based on the facility depicted in Figure 1. The HOT lane has a capacity of 1800 vph and each GP lane has a capacity of 2100 vph. The length is

10 km and the free flow speed is 100 km/h. The expected demand profile (provided in Table 1) is chosen such that the LOV demand exceeds the GP lane capacity and a queue is formed at the bottleneck. The VOT distribution for single-occupant vehicles is the Burr distribution described in Section 3 with parameter γ =2, and the median VOT is assumed to be 15$/hour.

TABLE 1 Peak Period Travel demand Profile

	Average Occ.	**7:00-8:00**	**8:00-9:00**	**9:00-10:00**
LOV	1.2	6300	5100	3900
HOV	4	600	600	600
Transit	40	300	300	300
Total		**7200**	**6000**	**4800**

The focus of this work is the impact of demand uncertainty on the performance of the HOT facility under various tolling schemes, therefore the demand is assumed to be a Normal random variable, and a range of uncertainty levels are considered. The level of uncertainty is described by the coefficient of variation, CV=σ/μ. In addition to analyzing the HOT lane facility under various pricing scenarios, results are also provided for the base case where all lanes are GP, and the HOV case where only HOV and transit users use the managed lane.

5.1 Parameter Calibration

A calibration process is needed for the fixed and FU-DM tolling schemes to determine the optimal fixed toll value and DM parameter, respectively, under given demand scenarios. In addition to identifying the optimal toll for a range of demand distributions, we also explore the robustness of the two tolling methods (fixed and FU-DM) to the chosen parameter by evaluating the performance in terms of E[APTT] for a selection of pre-specified fixed tolls and DM parameters. In this section we present analysis which reveals the optimal fixed toll (see Figure 2) and sensitivity to pre-specified fixed toll values (see Figure 3) for a range of uncertainty levels. We also present the optimal DM parameter (see Figure 4) and sensitivity to pre-defined DM parameters (see Figure 5) for a range of uncertainty levels. .

5.1.1 Fixed toll calibration and sensitivity

Figure 2 illustrates the facility performance in terms of minimizing E[APTT] under a range of fixed toll values between \$0 and \$40 (using resolution of 0.25\$) for five different Normal demand distributions. For each distribution the expected demand is that shown in Table 1, and the CV ranges from 0% to 80%, in increments of 20%.

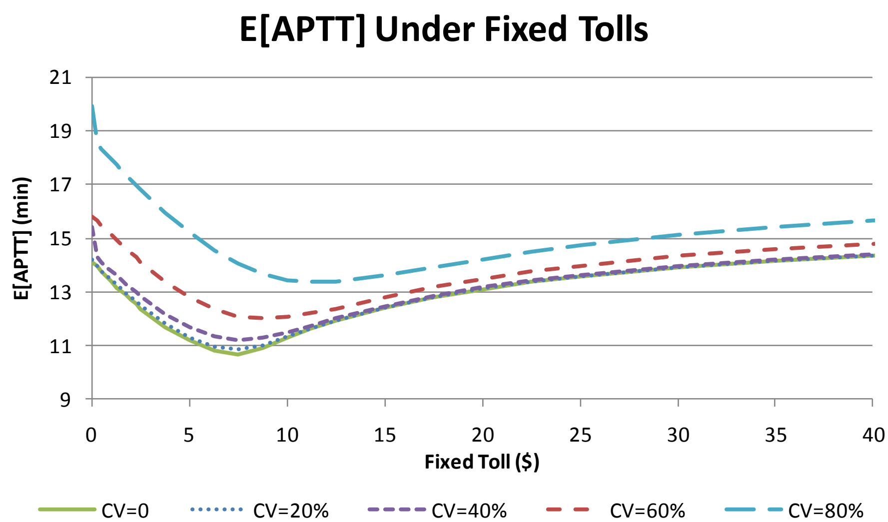


Figure 2 **Facility Performance for a range of fixed tolls under pre-specified demand distributions**

Based on Figure 2, the fixed tolls behave similarly across distributions. A toll=\$0 results in the highest E[APTT], followed by a general decrease in E[APTT] until an optimal toll is reached, and then a steady increase in E[APTT] as the toll is increased above the optimal value. However, the optimal fixed toll value in terms of minimizing E[APTT] varies depending on the level of uncertainty, with higher uncertainty corresponding to higher optimal tolls. The E[APTT] corresponding to the optimal toll also increases with the uncertainty level. For the deterministic demand and CVs below 50% the optimal toll is the same, and equal to \$7.50, however the corresponding E[APTT] increases from 10.67 min at the deterministic case, to 11.56 min for a CV of 50%. When the CV is 60% the optimal fixed toll increases to \$8.75 with an E[APTT] of 12 min, and for a CV of 80% the optimal toll is \$11.25, and the E[APTT] is 13.4 min.

As revealed in Figure 2, the optimal toll value is dependent on the realized demand distribution. However, the future demand profile cannot be known with certainty at the time the tolls are set, therefore the chosen toll value should be robust to fluctuations in demand. To evaluate the robustness of the fixed tolling scheme to future demand variability, the optimal toll values from Figure 2 are evaluated under a range of demand scenarios. This analysis is presented in Figure 3.

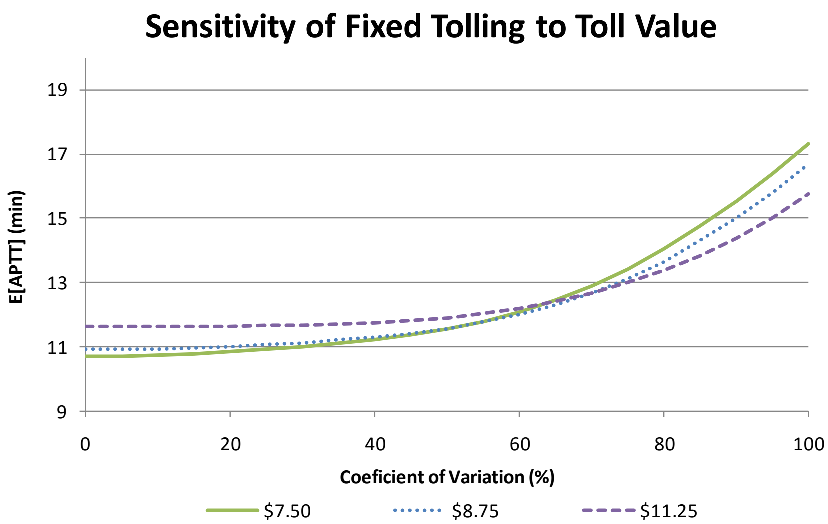


Figure 3 **Facility Performance for pre-specified fixed tolls as the level of uncertainty increases**

Figure 3 suggests fixed tolls are relatively robust to variation in demand. As expected, the lower toll of \$7.50, which is optimal for lower CVs, performs better than the higher toll of \$11.25, until the CV exceeds 60%. Above 60% CV the higher toll outperforms the lower toll, with a maximum difference in E[APTT] of 90 sec.

5.1.2 Real time toll calibration and sensitivity

Figure 4 illustrates the facility performance in terms of minimizing E[APTT] under a range of DM parameters for the real-time FU-DM tolling scheme. Results are shown for the

DM parameter evaluated between 0 and 0.5, and the same set of Normal demand distributions evaluated in Figure 2.

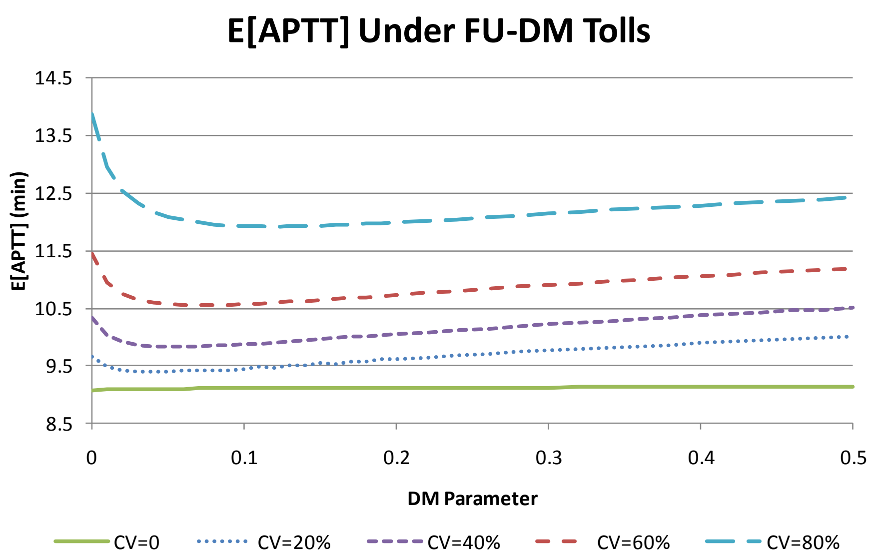


Figure 4 **Facility Performance under FU-DM tolls for range of DM parameters and demand distributions**

Based on Figure 4 the E[APTT] performance relative to the DM parameter is similar across demand distributions. A DM parameter value = 0 represents the FU-M tolling method. For all levels of uncertainty, as the parameter increases from zero, the E[APTT] decreases until reaching an optimal parameter value which varies between (0,0.12) depending on the level of uncertainty, then steadily increases as the parameter is increased beyond its optimal value. The optimal DM parameter increases as the CV increases, as does the corresponding E[APTT]; the E[APTT] corresponding to the optimal parameter varies from 9.08 min for a deterministic demand to 11.92 for 80%. When the uncertainty is high (CV=80%), the pre-scheduled option (α=0) performs worse than FU-DM tolling under all other parameter values evaluated. However, when the uncertainty is modest (CV=20%), the performance of the pre-scheduled option (α=0) is nearly optimal, and noticeably better than the over-reacting option with α=0.5. Therefore, it cannot be taken for granted that a real-time system will perform better than a pre-scheduled one.

A similar sensitivity analysis is conducted for the DM parameter as was done for the fixed tolls to evaluate the robustness of the FU-DM tolling scheme to future variability in demand. Figure 5 illustrates the optimal DM parameter values from Figure 4 evaluated under a range of demand scenarios.

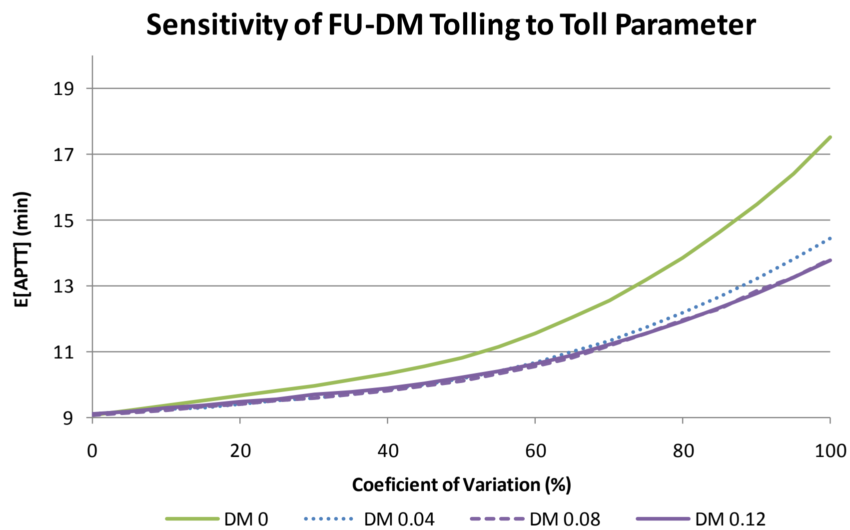


Figure 5 **Facility Performance under FU-DM tolls with specified parameter for increasing level of uncertainty**

From Figure 5 the FU-DM tolls outperform the FU-M tolls under all scenarios evaluated, though for very low levels of uncertainty (e.g. CV < 20%) the difference in E[APTT] is almost indistinguishable. Additionally, the FU-DM tolls are relatively robust to the parameter value implemented, especially for a CV below 80%. When evaluated for a CV=100% the maximum difference in E[APTT] between FU-DM tolls with a parameter of 0.04 (which is optimal for a CV=40%) and a parameter of 0.12 (which is optimal for a CV=80%) is only 42 seconds. The E[APTT] for both cases is 14.5 and 13.8 minutes, respectively. In contrast at a CV=100% the FU-M tolls result in a E[APTT] that is 17.5 minutes.

5.2 Comparison of various tolling schemes under demand uncertainty

The performance of each tolling scheme under varying levels of uncertainty for a normal demand distribution is illustrated in Figures 6. For the analysis presented the fixed toll and the DM parameters are optimized for each level of uncertainty.

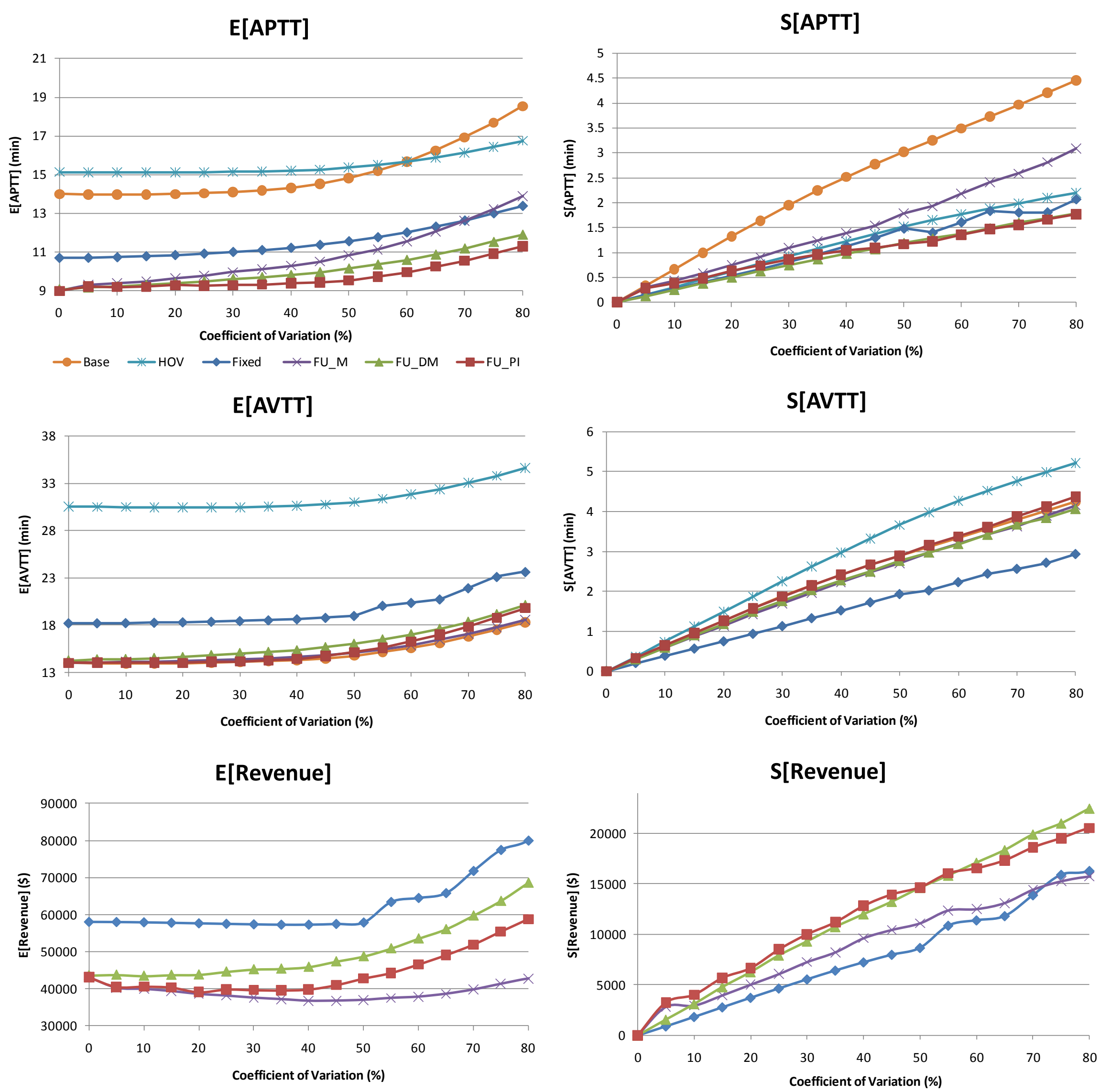


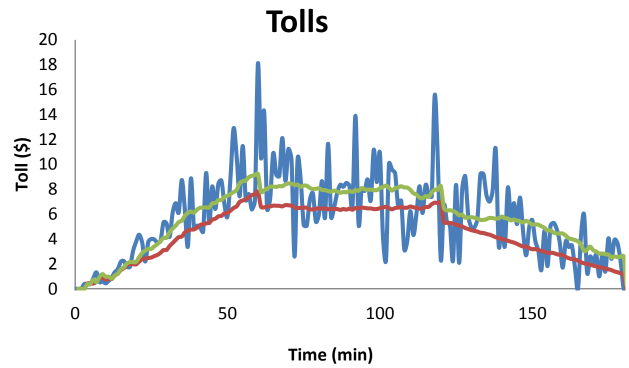
Figure 6 **Facility Performance for Base, HOV, and tolling schemes Fixed, FU-M, FU-DM, and FU-PI tolls under varying levels of uncertainty for a normal demand distribution**

From Figure 6 it is apparent that the expected performance in terms of APTT remains nearly constant unless demand uncertainty is very high (CV > 50%). The performance of the real-time option (FU-DM) is practically equivalent to the reference idealized option (FU-PI), with a maximum difference in APTT of 42 seconds. The improvement of both options (FU-DM & FU-PI) relative to the HOV case is fairly constant around 6 min, regardless of variability. The base case is more sensitive to uncertainty than the HOV case, with slightly better performance at lower levels of uncertainty, and slightly worse expected performance when the uncertainty is high.

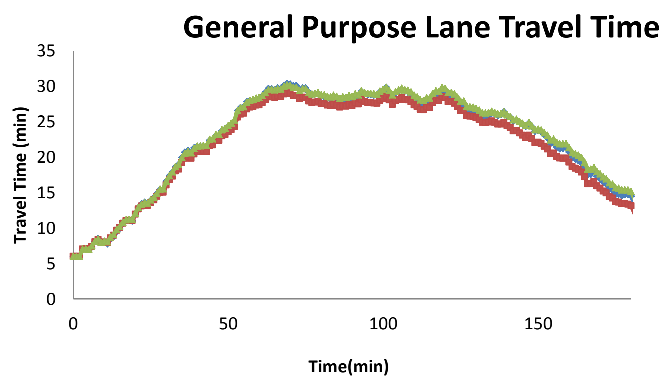
According to these results, a fixed toll can generally achieve about two-thirds of the potential benefit in terms of expected APTT, at all levels of uncertainty, saving approximately four minutes relative to the HOV case, compared to six minutes saved by the FU-PI tolls, a finding that was relatively stable across different levels of uncertainty. Under deterministic demand, pre-scheduled FU-M tolls are identical to the ideal FU-PI tolls (by definition), and as the uncertainty increases they gradually transition to the level of performance of the fixed tolls. The medium point where the pre-scheduled FU-M tolls are in the middle between the fixed toll and the ideal FU-PI tolls, appear to be in this case when the CV is around 40%. This result suggests that with lower levels of variability the performance of pre-scheduled tolls may provide most of the potential benefit of the HOT system, which helps quantify the additional value of variable-tolling schemes relative to fixed tolls. Interestingly, at very high levels of uncertainty, setting tolls based on the mean demand alone (FU-M) actually leads to worse system performance than using a fixed toll.

In terms of system reliability, i.e. S[APTT], the base case performs very poorly, but the other options exhibit relatively similar results until the CV reaches 40-50%. At higher CV values the ranking is similar to the ranking in terms of E[APTT], except for the surprisingly good performance of the HOV option. It seems that when variability is high, protecting the level of service for transit and HOVs can be quite beneficial.

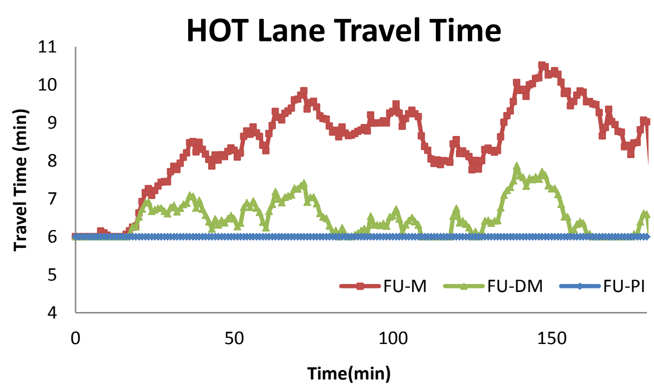
For the three full-utilization tolling schemes, the tolls vary dynamically over the course of the peak travel period, and are calculated as described in section 3.2. Figure 7 shows the evolution of tolls over time for a normal demand distribution with a CV=30%. For the analysis presented the DM parameter is 0.05, which is optimal for the chosen CV.



(a)



(b)



(c)

Figure 7 **A single simulation evolution during the peak period with Normal demand distribution (CV=30%) for three tolling schemes:FU-PI, FU-M and FU-DM. a) tolls; b) general-purpose lane travel times; and c) HOT lane travel times.**

It can be seen that the pre-scheduled FU-M tolls are fairly well behaved, the real-time FU-DM exhibit slightly more frequent changes, while following the demand on a minute-by-minute basis leads to very substantial volatility. It is not at all clear whether ~10$ and ~three-fold changes in the toll from one minute to the next can be acceptable in real-world scenarios.

The GP travel time under all tolling schemes is almost indistinguishable, with a maximum difference on 1.7 minutes. In contrast the HOT travel time is substantially higher under FU-M tolls compared with FU-DM tolls, with a maximum difference of 3.5 minutes. Additionally, both FU-M and FU-DM tolls result in relatively unstable HOT travel times, although they behave similarly. For the FU-PI tolls the HOT lane travel time is stable and minimized at 6 minutes, as would be expected. HOT travel time under the FU-DM tolls can be up to 8 minutes, but is able to achieve a travel time of 6 minutes at various points in time. The HOT FU-M toll are often 2-3 minutes higher than under FU-DM tolls, and increase to 10.5 minutes at one point.

6. CONCLUSIONS

In this paper we study HOT tolling schemes under demand uncertainty. We focus on non-correlated day-to-day demand uncertainties, which we model by considering the number of arrivals in each minute as independent random variable with known distribution and time-of-day dependent mean. The performance model for a given demand realization is deterministic, thus allowing identification of ideal tolls, as a reference for the performance of other schemes.

In the studied scenario, the base case (without managed lanes) and the HOV option have similar expected performance. The results show that a fixed toll can achieve about two thirds of the benefit of an ideal HOT system. The performance of a pre-scheduled toll system is between the fixed toll and the ideal system: closer to the ideal when the coefficient of variation is below 40%, and closer to the fixed toll otherwise. A relatively simple real-time system, with density-based linear adjustment to the pre-scheduled toll, has practically equivalent performance to the ideal system.

In conclusion, this study proposed and evaluated various tolling methods on a simple facility with stochastic demand, a single downstream bottleneck, and a known VOT distribution. Expanding the analysis to a more complex facility, such as including on-ramps and off-ramps, incorporating travel time reliability, adding operational considerations such as traffic monitoring, exploring the sensitivity of the model to errors in input data, and empirically estimating VOT distributions among HOT-lane drivers are all planned extensions of this research.

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REFERENCES

1. P. Devarasetty, Burris, M. and Shaw, W. The Value of Travel Time and Reliability - Evidence from a Stated Preference Survey and Actual Usage. *Transportation Research Part A* Vol. 46 No. 8, 2012, pp. 1227 - 1240.
2. M. W. Burris, Geiselbrecht, T.C., Goodin, G.D. and MacGregor, M. E., Predetermining Performance-Based Measures for Managed Lanes, in Proceedings of the 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011, No. 1482.
3. M. W. Burris, Ungemah, D. H., Mahlawat, M. and Pannu, M. S. Investigating the Impact of Tolls on High-Occupancy-Vehicle Lanes Using Managed Lanes. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2099, 2009, pp. 113–122.
4. H. Bar-Gera, The Fast Lane to Tel-Aviv: High-Occupancy Toll Project with a Pareto Package, in Proceedings of the 91st Annual Meeting of the Transportation Research Board, Washington, D.C., 2012, No. 0712.
5. X. Cao, Xu, Z. and Huang, A., Safety Benefits of Converting HOV Lanes to HOT Lanes: Case Study of I-394 MNPass in Proceedings of the 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011, No. 0361.
6. E. Parkany. Can HOT lanes encourage carpooling? A case study of carpooling behavior on the 91 express lanes. *Transportation Research Record: Journal of the Transportation Research Board*, No. 1682, 2007 pp. 46-54.
7. Y. Chung and Choi, K. Dynamic Toll Concept to Assess Feasibility of High-Occupancy Vehicle Lane on Kyungbu Freeway, South Korea. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2178, 2010 pp. 162–169.
8. M. Fosgerau. How a fast lane may replace a congestion toll. *Transportation Research Part B* Vol. 45, No. 6, 2011, pp. 845-851.
9. P. Gordon, Moore II, J. E., Richardson, H. W. and Williamson, C., *The Economic Effects of Highway Widening: Tolled Lanes vs. General-Purpose Lanes – Using an Integrated Impact Model*, Report FHWA/CA/OR-2005/12, 2004.
10. K. R. Buckeye. Performance Evaluation of I-394 MnPASS Express Lanes in Minnesota *Journal Transportation Research Record: Journal of the Transportation Research Board* No. 2278, 2012, pp. 153-162.
11. I. C. Barnes, Frick, K.T., Deakin, E. *et al.* Impact of Peak and Off-Peak Tolls on Traffic in San Francisco-Oakland Bay Bridge Corridor in California. *Journal Transportation Research Record: Journal of the Transportation Research Board* No. 2297, 2012, pp. 73-79.
12. X. Liu, Schroeder, B., Thomson, T. *et al.* Analysis of Operational Interactions Between Freeway Managed Lanes and Parallel General Purpose Lanes. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2262, 2011, pp. 62-73.
13. X. Liu, Zhang, G., Lao, Y., and Wang, Y., Modeling Traffic Flow Dynamics on Managed-Lane Facility: Approach Based on Cell Transmission Model. *Transportation Research Record: Journal of the Transportation Research Board* No. 2278. 2012, pp. 163-170.
14. B. Kuhn, Goodin, G., Ballard, A., Brewer, M., Brydia, R., Carson, J., Chrysler, S., Collier, T., Eisele, W., Fenno, D., Fitzpatrick, K., Ullman, B., Ullman, G., and

Venglar, S., *Findings from Texas: Five Years of Research on Managed Lanes,* Report Number: FHWA/TX-06/0-4160-25, 2005.

15. A. Borrell-Rovira and Supernak J., Peak-Period Utilization of I-15 Corridor in San Diego: Long Term Impact of the Fastrak Program, in Proceedings of the 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011, No. 3751.
16 R. Goel, Burris, M.W., HOT-Lane Policies and Their Implications, *Transportation*, 2011, DOI 10.1007/s11116-011-9382-5.
17. Southern California Association of Governments (SCAG). Regional south-California high-occupancy vehicle lane system performance study. Final summary Report. 2004. Available http://www.scag.ca.gov/publications/pdf/2005/HOVStudy_Final1104.pdf
18. M. Burris, S. Nelson, P. Kelly *et al.* Willingness to Pay for High-Occupancy Toll Lanes Empirical Analysis from I-15 and I-394 *Journal Transportation Research Record: Journal of the Transportation Research Board,* No. 2297, 2012, pp. 47-55.
19. C. Chung and Recker, W., State-of-the-Art Assessment of Toll Rates for High-Occupancy and Toll Lanes, in Proceedings of the 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011, No. 0496.
20. J. Dahlgren. High-occupancy/toll lanes: where should they be implemented? . *Transportation Research Part A* Vol. 36, No. 2002, pp. 239-255.
21. L. M. Gardner, Boyles, S. and Bar-Gera, H., Development and comparison of choice models and tolling schemes for high-occupancy/toll (HOT) facilities. *Transportation Research Part B* Vol. 55, No. 2013, pp. 142–153.
22. Y. Lou, Yin, Y. and Laval, J. Optimal dynamic pricing strategies for high-occupancy/toll lanes. *Transportation Research Part C* Vol. 19, No. 1, 2011, pp. 64-74.
23. R. Halvorson, Nookala, M. and Buckeye, K.R. High occupancy toll lane innovations: I-394 MnPASS. *Public Works Management & Policy* Vol. 10, No. 1, 2006, pp. 242-255.
24. D. Michalaka, Lou, Y. and Yin, Y., Proactive and Robust Dynamic Pricing Strategies for High-Occupancy-Toll (HOT) Lanes, in Proceedings of the 90th Annual Meeting of the Transportation Research Board Washington, D.C., 2011, No. 2617.
25. Morgul, Ender Faruk and K. Ozbay, Simulation-Based Evaluation of a Feedback Based Dynamic Congestion Pricing Strategy on Alternate Facilities, in Proceedings of the 90th Annual Meeting of the Transportation Research Board, Washington, D.C., 2011, No. 3535.
26. K. Ozbay, Morgul, E.F., Ukkusuri, S., Iyer, S. and Yushimito, W.F., Mesoscopic Simulation Evaluation of Dynamic Congestion Pricing Strategies for New York City Crossings, in Proceedings of the 90th Annual Meeting of the Transportation Research Board, , Washington, D.C., 2011, No. 3612.
27. L. Yang, Saigal, R. and Zhou, H., Distance-Based Dynamic Pricing Strategy for Managed Toll Lanes. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2283, 2012, pp. 90–99.
28. Y. Yin and Y. Lou., Dynamic tolling strategies for managed lanes. *ASCE Journal of Transportation Engineering* Vol. 135, No. 2, 2009, pp. 45-52.
29. R. Arnott, de Palma, A. and Lindsey, R. “Recent developments in the bottleneck model.” In K.J. Button and E.T. verhoef, eds. *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*, Cheltenham: Edward Elgar, 1998.
30. X. Chu. Endogenous trip scheduling: the Henderson approach reformulated and compared with the Vickrey approach. *Journal of Urban Economics,* Vol. 37, No. 3, 1995, pp. 324-343.

31. L. N. Liu and McDonald, J.F., Economic efficiency of second-best congestion pricing schemes in urban highway systems. *Transportation Research Part B* Vol. 33, No. 3, 1999, pp. 157-188.
32. H. Yang, and Huang, H.J. Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. *Transportation Research Part B* Vol. 31, No. 6, 1997, pp. 425-440.
33. M. Kuwahara, A theoretical analysis on dynamic marginal cost pricing, *Transportation Research Part A* Vol 41 No. 7, 2007, pp. 627-643.
34. W. S. Vickrey. Congestion theory and transport investment. *American Economic Review* Vol. 59 No. 2, 1963, pp. 251-60.
35. R. Dial. A model and algorithm for multicriteria route-mode choice. *Transpn Res.* Vol. 138, 1979, pp. 31 i-316.
36. S. Waller, Schofer, JL, and Ziliaskopoulos, AK, Evaluation with traffic assignment under demand uncertainty. *Transportation Research Record: Journal of the Transportation Research Board*, No. 1771, 2001, pp. 69–75.
37. J. Duthie, Unnikrishnan, A., and Waller, S. T., Influence of demand uncertainty and correlations on traffic predictions and decisions. *Computer-Aided Civil and Infrastructure Engineering*, Vol 26, No 1, 2011, pp. 16-29.
38. E. Malini, Kumar, A., Kansal, M., Minocha, V., Kumar, S., Lam, W., and Tam, M., Risk Analysis of Traffic and Revenue Forecasts for Road Investment Projects, *J. Infrastruct. Syst.*, Vol. 5, No. 3, 1999, pp. 112–114.
39. L. Gardner, Unnikrishnan, A, and Waller, ST Robust pricing of transportation networks under uncertain demand. *Transportation Research Record*, No. 2085, 2008, pp. 21–30.
40. A. Chen and Subprasom, K. Analysis of regulation and policy of private toll roads in a build-operate-transfer scheme under demand uncertainty. *Transportation Research Part A,* Vol. 41, 2007, pp. 537–558.
41. L. M. Gardner, Unnikrishnan, A., and Waller, S. T. Solution methods for robust pricing of transportation networks under uncertain demand. *Transportation Research, Part C,* Vol. 18, 2010,pp. 656-667.
42. T. Nagae, and Akamatsu, T. Dynamic revenue management of a toll road project under transportation demand uncertainty *Networks and Spatial Economics,* Vol. 6 2006, pp. 345–357.
43. H. Li, Bliemer, M.C.J., and Bovy, P.H.L. Optimal toll design from reliability perspective. Proceedings of the Sixth Triennial Symposium on Transportation Analysis (TRISTAN VI), Phuket, Thailand., 2007.
44. L. Gardner, Boyles, S. and Waller, S.T., Quantifying the Benefit of Responsive Pricing and Travel Information in the Stochastic Congestion Pricing Problem. *Transport. Res. Part A,* Vol. 45, No. 3, 2011, pp. 204-218.
45. Y. Asakura, Road network reliability caused by daily fluctuations of traffic flow. Proceedings of the 19th PTRC Summer Annual meeting, Brighton, 1991, pp. 73–84.
46. M.G.H. Bell, Cassir, C., Iida, Y. & Lam, W.H.K. A sensitivity based approach to network reliability assessment. Proceedings of the 14th International Symposium on Transportation and Traffic Theory', Jerusalem, 1999, pp. 283–300.
47. S. Clark and Watling, D. Modelling network travel time reliability under stochastic demand. *Transportation Research B* Vol. 39, No. 2, 2005, pp. 119–140.
48. H. Shao, Lam, W.H.K. & Lam, M.L., A reliability based stochastic traffic assignment model for network with multiple user classes under uncer-tainty in demand. *Networks and Spatial Economics* Vol. 6, 2005, pp. 169–172.

49. S. Nakayama. Stochastic network equilibrium model under uncertain demand. Proceedings of the 6th Triennnial Conference on Transportation Analysis', Phuket, Thailand., 2007.
50 C. Kleiber, and Kotz, S., Statistical Size Distributions in Economics and Actuarial Sciences. Hoboken, NJ: John Wiley and Sons. 2003.