A SINGLE DEPOT CONCRETE DELIVERY WITH TIME WINDOWS MODEL USING INTEGER AND ASSIGNMENT VARIABLES

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ABSTRACT

We address the logistics and planning problem of delivering Ready Mixed Concrete (RMC) to a set of demand customers from a single depot while respecting time-windows induced restrictions. The RMC Dispatching Problem (RMCDP) is closely related to the Vehicle Routing Problem (VRP) with the difference that demand nodes in the RMCDP may be visited more than once by a truck. This class of routing and scheduling problems can be represented using Mixed-Integer Programming (MIP), however the joint decisions involved often lead to hard-to-solve formulations, due to either the number of variables required or the number of constraints imposed in the models. In this work, we propose a novel formulation for the single-depot RMCDP that attempts to integrate time windows restrictions by assigning vehicles to feasible delivery slots and uses integer variables to represent repeated trips between depot and customer nodes. We show that the proposed formulation is able to provide promising computational results on a realistic dataset representative of an active RMCDP in the region of Adelaide, Australia.

Keywords: Concrete Delivery, Integer Programming, Dispatching

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1. Introduction

In this paper, we address the problem of delivering Ready Mixed Concrete (RMC) to a set of demand customers. This logistics and planning problem arise in many real-world applications where a large amount of RMC needs to be delivered to several construction sites while respecting some scheduling and haul time constraints. The underlying routing problem within RMC delivery is closely related to the Vehicle Routing Problem (VRP), with the difference that in the RMC Dispatching Problem (RMCDP) a customer may be visited more than once by the same truck to be entirely serviced. The RMCDP can be represented mathematically using Mixed Integer Programming (MIP) and therefore this class of routing problems requires dedicated models and solution methods. In this work, we examine a novel formulation for the single-depot variant of the problem where time windows induced restrictions are taken into consideration to represent the potential scheduling constraints arising therein.

In the Single-depot RMCDP it is desirable to find the best allocation of delivery trucks to customers so that transportation costs are minimized and a maximum number of customers are fully supplied. In this paper, we propose a novel MIP formulation that uses integer – instead of binary – variables to represent the routing decisions and binary variables to assign delivery trucks to feasible service slots. This model provides an alternative approach to represented repeated trips between concrete depots and customer sites, which are often required to meet the customer demand in RMC. We implement the proposed model using a realistic dataset representative of an active RMCDP in the region of Adelaide, Australia and report promising results.

The paper is organized as follows: Section 2 summarizes the existing literature on the RMC delivery problems and related works; Section 3 presents the mathematical framework of and the formulation of the proposed MIP to represent the S-RMCDP; Section 4 details the implementation of the proposed formulation and the results obtained and Section 5 concludes this research.

2. Literature Review

The section is devoted to reviewing the literature by focusing on RMC dispatching problem RMCDP. Within the last decade a number of growing publications have been focused on introducing mathematical formulation for single and multi-depot RMC dispatching problem. Single-depot RMCDPs aim to represent small to medium sized delivery problems which only have an active batch plant and an assumed homogeneous fleet. The multi-depot variant seeks to represent the case where multiple batch plants (depots) are available to load fresh concrete into delivery trucks and a wide range of trucks is typically available within the fleet. RMCDP can be characterized as a generalized Vehicle Routing Problem (VRP) and also it was analytically and empirically studied that large scale RMCDPs are NP-hard (Lu et al., 2003, Maghrebi et al., 2014a, Maghrebi et al., 2014c, Maghrebi et al., 2014d, Maghrebi et al., 2013a, Maghrebi et al., 2013b, Schmid et al., 2010, Wang, 2001, Yan and Lai, 2007, Maghrebi et al., 2014b).
One of the early RMCDP formulation was introduced by Feng and Wu (2000) which was a single-depot model and solved heuristically. One of the most advanced and heavy formulation was introduced by Naso et al. (2007). They introduced a multi-objective model for multi-depot RMCDP with a homogeneous fleet. However, the main issue with their formulation is the large number of decision variables as well as side constraints. A wide variant of RMC formulation were introduced by Yan and his colleagues; such as when the overtime is considered (Yan and Lai, 2007) or covering the incidents (Yan et al., 2012) and also associating stochastic travel times (Yan et al., 2012). Lin et al. (2010) presented a formulation that introduces uncertainty in the demand for RMC and minimizes the total waiting time. The authors represent the RMC dispatching problem as a job shop problem where the construction site represents a job and trucks represent workstations. This model was designed for a single-depot dispatching problem with a heterogeneous fleet. Another model in this context was presented by Schmid et al. (2009) for a single depot with a heterogeneous fleet. Their model forces MIP to avoid unsupplied customers by penalizing the unsatisfied customers in the objective function. In an effort to address the case of multi-depot RMCDP, Asbach et al. (2009) introduced a novel model whose structure is much simpler than that

of other introduced models and which can be used for modelling multi-depots and a heterogeneous fleet. In this formulation, a depot is divided into a set of sub-depots based on the number of possible loadings at that depot. Similarly, a customer is divided into a set of sub-customers according to the number of required deliveries. According to these assumptions, their formulation is no longer needed to deal with customers who require more than one delivery, which results in a reduction in the number of side constraints. However, as a result of replacing the customers with deliveries, the number of decision variables is increased. Kinable et al. (2014) recently introduced a new formulation similar to (Asbach et al., 2009) but solved using constraint programming. It must be noted that in this paper it is only focused on statistic networks not the dynamic routing problem such as (Fajardo and Waller, 2012a, Fajardo and Waller, 2012b).

In this paper we address the planning and logistics problem of delivering RMC to a set of customers from a single depot. In this dispatching problem, schedule conflicts must be avoided, i.e. no two trucks can be assigned to deliver RMC at a customer at the same time. Further, we examine the case where deliveries must respect timing restrictions induced by the available time windows at each customer.

3. Mathematical Formulation

In this section, we present the mathematical formulation proposed to solve the Single-depot RMC Dispatch Problem (S-RMCDP). The objective of the S-RMCDP is to supply a maximum number of customers while minimizing transportation costs. This approach has been widely used in the literature to represent real-world problem configurations. In this paper, we focus on the case where a single RMC depot is available and time windows at the depot and at customer nodes must be respected throughout the delivery problem. To represent the scheduling constraints that arise when two or more trucks are using the same depot to load RMC or supply the same customer, we propose a novel formulation which jointly optimizes the scheduling and routing decisions herein.

The main sets used within the paper are:

depot node index	0
set of customer nodes	C
set of final nodes	F
set of vehicles	K

Note that unlike multi-depot RMC models where a set of start nodes are used to represent the location where each delivery truck begins its journey, in the case of a single depot delivery problem, all vehicles must visit the same depot before visiting any other node in order to load RMC; hence we use the node index 0 to represent the depot as well as the start point of all delivery trucks. We hereby refer to the set $\{0\} \cup \mathrm{C}$ as the set of service nodes. We assume that no trip between any two customers can be made, hence the set of arcs corresponding to potential trips for concrete delivery is assumed to be composed of three types of trips (see Figure. 1):

1. **Depot–Customer:** trips from the depot to customer nodes (the vehicle carry RMC).
2. **Customer–Depot:** trips from the customer nodes to the depot (the vehicle is empty and will be loaded at the depot).
3. **Customer–Final:** trips from the customer nodes to the final nodes.

From the previously defined sets, we can define the set of all nodes and the set of arcs induced by these nodes. We denote these sets as follows:

set of nodes $N = \{0\} \cup C \cup F$

set of arcs $A = \{\{(0,j): j \in C\} \cup \{(i,0): i \in C\} \cup \{(i,j): i \in C, j \in F\}\}$

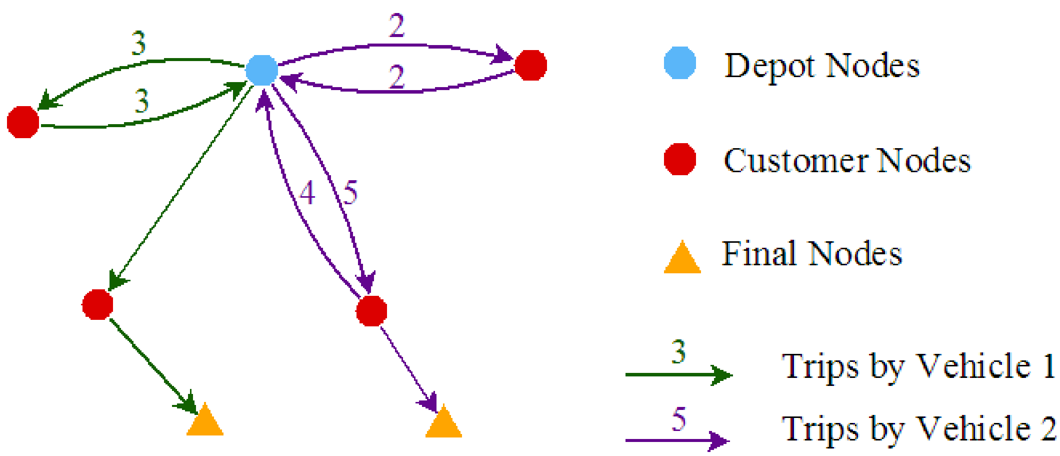

Figure. 1 – An example of a solution for the S-RMCDP with two vehicles; the number besides an arc indicates if this trip is repeated more than once.

We define the following parameters of the model:

q_i	demand of customer $i \in C$
c_k	capacity of vehicle $k \in K$
$[a_i, b_i]$	feasible time window for $i \in \{0\} \cup C$
σ_i	service time for $i \in \{0\} \cup C$
β_u	penalty for not servicing customer $u \in \bar{C}$
z_{ijk}	travel cost on arc $(i,j) \in A$ for vehicle $k \in K$
γ	maximum concrete haul time

In order to incorporate the time windows restrictions in the model, we first determine the number of available service slots for each node of the set $\{0\} \cup C$. Let η_i be defined as:

$$\eta_i \equiv \left\lfloor \frac{b_i - a_i}{\sigma_i} \right\rfloor, \qquad \forall i \in \{0\} \cup C \tag{1}$$

Let $S_i \equiv \{0, \dots, \eta_i - 1\}$ *be* the set of service (loading or unloading) slots at node $i \in \{0\} \cup C$. Our formulation of the RMC dispatching problem requires three types of decision variables, customer satisfaction, assignment and routing variables which are defined as follows:

- Customer satisfaction variables: $y_i \equiv \begin{cases} 1 \text{ if customer } i \in C \text{ is fully supplied;} \\ \quad 0 \text{ otherwise.} \end{cases}$

- Assignment variables: $s_{ikm} \equiv \begin{cases} 1 \text{ if node } i \in \{0\} \cup C \text{ is assigned to slot } m \in S_i \text{ by } k \in K; \\ \quad 0 \text{ otherwise.} \end{cases}$

- Routing variables: $x_{ijk} \geq 0, integer;$ for every trip planned on arc $(i,j) \in A$, by $k \in K$

The proposed formulation uses integer variables to represent the number of trips made along an arc of the network and account for repeated service; unlike most vehicle routing models which do not generally require multiple visits and use binary variables.

The proposed model for the S-RMCDP is represented by Equations (2)-(11).

$$\min \sum_{(i,j)\in A}\sum_{k\in K} z_{ijk}x_{ijk} + \sum_{i\in C}(1-y_i)\beta_i \tag{2}$$

Subject to:

$$\sum_{m\in S_i}\sum_{k\in K} s_{ikm}c_k \geq y_i q_i \qquad \forall\, i\in C \tag{3}$$

$$\sum_{k\in K} s_{ikm} \leq 1 \qquad \forall i\in C, \forall m\in S_i \tag{4}$$

$$\sum_{m\in S_j} s_{jkm} = x_{0jk} \qquad \forall k\in K, \forall j\in C \tag{5}$$

$$\sum_{m\in S_0} s_{0km} = \sum_{i\in C} x_{i0k} \qquad \forall k\in K \tag{6}$$

$$\sum_{i\in C}\sum_{j\in F} x_{ijk} \leq 1 \qquad \forall\, k\in K \tag{7}$$

$$\sum_{j\in N} x_{ijk} - \sum_{j\in N} x_{jik} = 0 \qquad \forall\, k\in K\ ,\forall i\in C \tag{8}$$

$$y_i \in \{0,1\} \qquad \forall i\in C \tag{9}$$

$$s_{ikm} \in \{0,1\} \qquad \forall k\in K, \forall i\in \{0\}\cup C, \forall m\in S_i \tag{10}$$

$$x_{ijk} \in \mathbb{Z}^+ \qquad \forall\,(i,j)\in A, \forall k\in K \tag{11}$$

The objective function (2) seeks to minimize the transportation costs while supplying a maximum number of customers. Constraint (3) is the customer satisfaction constraint and allows variable y_i to be equal to 1 at the condition that the demand of the customer i is fully supplied. Constraint (4) is the schedule conflict constraint and ensures that no service slot is assigned to more than one vehicle. Constraints (5) and (6) link the scheduling variables with routing variables for the trips between the depot and customer nodes. Constraints (7) and (8) are flow constraints which ensure that no more than 1 final trip is planned for each vehicle and that flow is conserved at every customer node. Finally, Constraints (9), (10) and (11) define the domain of each decision variable.

The model represented by Equations (2)-(11) is a Mixed-Integer Linear Program (MILP) which can be solved by enumerative algorithms such as Branch-and-Bound and/or Branch-and-Cut which are widely implemented in off-the-shelf optimization software.

4. Computational Results

The proposed formulation is tested by field data which was extracted from a broad dataset that representative of an active RMC dispatching problem in the region of Adelaide, Australia. In the selected RMCDP instance, 7 delivery trucks are available to deliver readymade concrete to 8 customers each of which has a predetermined RMC demand. The depot is used as the final node to organize the journey of the RMC delivery trucks. The available time windows of each customer and the capacity of the trucks are given in Table 1. Figure 2 depicts a map of the original instance where the size of the customer nodes is proportional to their demand. In this dataset, many customers have a tight available time window and a moderate demand in RMC. In order to examine the robustness of the model, we consider two modifications based on the original instance described in Table 1:

- *Improved time windows:* in this instance, the width of each time window with less than 10delivery slots is increased such that at least 10 per customer are available.
- *Inflated demand:* in this instance, the demand of each customer is doubled.

Table 1 – Instance data for the S-RMCDP; these values were obtained after processing a realistic dataset representative of an active concrete delivery problem in the region of Adelaide, Australia.

Customer	Demand	Time Window	Service Time	Number of Slots
1	1.6	[6.75,7]	0.15	1
2	76.4	[6.9,14.1]	0.15	48
3	10	[7,7.8]	0.15	5
4	45.8	[7.5,13.1]	0.15	37
5	8	[9.9,10.75]	0.15	5
6	4.6	[10.9,11.2]	0.15	1
7	2	[12.9,13.15]	0.15	1
8	7	[13.5,13.8]	0.15	2

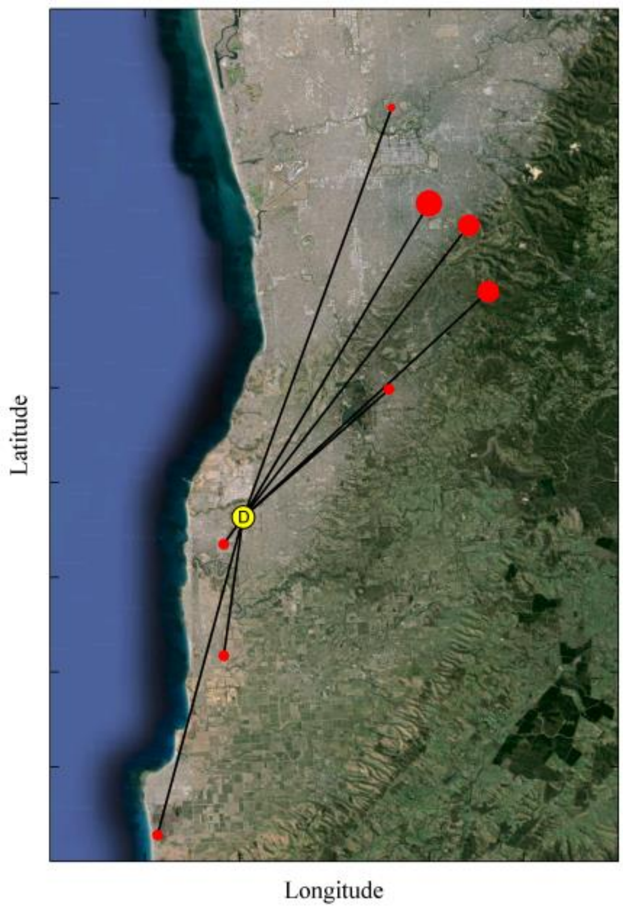

Fig. 2 – Map of the original instance for the single-depot RMC dispatching problem in the region of Adelaide, Australia.

For each instance, we use the Euclidean distance to represent the distance between nodes together with a vehicle speed of 30km/h to estimate the travel cost matrix $[z_{ijk}]$.

The model is implemented on a 64-bit Windows computer with 8Gb of RAM using the AMPL/CPLEX package. The results are detailed in Tables 2 and 3.

Table 2 – Results of the implementation of the proposed model for the S-RMCDP on four instances: one realistic dataset and three modified datasets.

Instance	Number of customer fully supplied	Total distance travelled (km)	Total Number of deliveries
Original	8	68.4686	52
Improved time windows	8	68.4686	52
Inflated demand	7	105.58	88
Improved time windows + inflated demand	8	117.6	92

Table 3 – Assignment of delivery trucks to customers nodes for the original instance.

		Customer ID							
		1	2	3	4	5	6	7	8
Truck ID	1							1	1
	2			1					
	3					1			
	4		3		2				
	5		6		1		1		
	6	1	2	3					
	7					1			
Total		1	11	4	3	2	1	1	1

The results show that the model is able to solve efficiently as each instance is solved in less than 5 seconds. In the original instance, all the customers are fully supplied and the total distance travelled is about 68.5 km with a total of 52deliveries. Improving the flexibility of the time windows does not change the optimal solution, nor the computation time required to solve the problem. However, inflating the demand in RMC without improving the time windows reduces the number of customer fully supplied to 7 and increases the total distance travelled to 105.6 km and the number of deliveries to 88. After improving the time windows, the remaining unsatisfied customer can be fully supplied at the expense an additional 12 km and 4 deliveries.

5. Conclusion

In this paper we have presented a novel formulation for the RMC dispatching problem that seeks to find the cheapest routes to service a maximum number of customers while respecting routing and time windows induced constraints. In particular, the proposed formulation relaxes the scheduling restrictions and provides the optimal assignment of delivery vehicles to customer so as to meet the available time windows at each customer. The model has been implemented on realistic instances representative of an active RMC dispatching problem in the region of Adelaide, Australia. Our results show that the model can be implemented on relatively large instances. Future research will be focused on the development of a scheduling-oriented model, which is able to provide the actual schedule of trucks. In particular if we assume that vehicles can wait upon arrival at the depot or at a customer node if there is no available slot; the service time of vehicle k at node i for service slot m can be determined by $\alpha_{ikm} = m\sigma_i + a_i,\ \forall i \in \{0\} \cup C, \forall m \in S_i, \forall k \in K$and a heuristic method can be used to find the best scheduling so as to minimize the idle time at customer nodes given the vehicle assignment and routes obtained after solving the proposed model.

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