A Strategic User Equilibrium Model Incorporating Both Demand and Capacity Uncertainty

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ABSTRACT:
In this paper, we present an extension of a novel equilibrium based assignment model originally proposed by Dixit et al (2013), which addresses day-to-day volatility in travel conditions resulting from short-term demand uncertainty, by incorporating the impact of capacity uncertainty. Specifically, the demand and capacity variability are represented independently using assumed known distributions. The proposed model is based on the premise that users gain knowledge of the demand and capacity distributions through their past travel experience. Using this knowledge, users seek to minimize their *expected* travel time and choose a strategy (i.e., travel route) accordingly. They then follow this strategy day-to-day, independent of the realized traffic demand and capacity. The network may therefore result in non-equilibrium assignment patterns, which is consistent with the lack of observed equilibrium in field networks. The model is mathematically formulated, expressions for the analytical link travel times and corresponding variability are derived and the uniqueness of the assignment solution regarding link flows is proven. Numerical analysis is conducted to demonstrate the performance and reliability of the model by comparing the analytical results with simulation based assignment.

Keywords: strategic user equilibrium, demand and capacity uncertainties, network modeling

1. INTRODUCTION

Realism and computational complexity are both critical factors in transportation planning models and must be equally considered to determine a model's practical applicability. Further, a major complication in transportation modelling is the ability to properly account for the inherent uncertainties regarding demand (*1, 2*) and capacity levels (*3, 4*). Furthermore, as has been noted, uncertainty surrounding these variables directly impacts route choice behaviour (*5*). It is, therefore, necessary to incorporate these stochastic elements into models to ensure robust planning capabilities but to do so in a manner that maintains computational tractability.

In this paper, we present an extension of the strategic user equilibrium model (StrUE) originally proposed by Dixit et al, which incorporated travel trip variability (*6*). This work extends the StrUE model to incorporate capacity variability in addition to day-to-day demand variability.

The StrUE model is based on the premise revealed by previous research that users tend to be "sticky" with regards to routing decisions – once they have selected an optimal route for their regular commute they tend to follow that route day-to-day. The StrUE model attempts to capture this behaviour in the resultant assignment pattern by assuming commuters and other road users are knowledgeable of the traffic conditions and that they are rational decision makers. The model relies on users minimizing their expected travel time based on the previous trip experiences in which they have gathered knowledge on demand (daily trips), and now also capacity variability (availability due to incidents, weather, etc.). The user's knowledge of each can be represented by a given distribution, with a known expected value and variance; a separate distribution is generated for demand and for capacity. Based on these known distributions, each user selects a travel route, to minimize their expected travel time subject to Wardrop's UE conditions – for each origin-destination (OD) pair the expected travel time on any used path is equal and less than the expected travel time on any unused path (*7*). Users then follow their chosen strategy (i.e., route) independent of the day-to-day realized demand and capacity. We define the resultant strategy using a set of link proportions – the proportion of total demand travelling on each link in the network. This strategy, or set of link proportions, remains fixed and independent of daily traffic conditions. However, the actual link flows on any given day, which are computed as the product of these link proportions and the total realized demand, will vary day-to-day, dependent on the realized demand. Therefore the travel patterns produced by the StrUE model on any given day need not conform to a state of equilibrium. In fact, the most common observed link flows for all days will likely be in a state of dis-equilibrium. This is consistent with the lack of observed equilibrium in real networks.

A literature review of the previous relevant research is presented in section 2. Section 3 defines the mathematical model and includes a derivation for the analytical solution to assignment strategy. Numerical analysis is demonstrated in section 4, conclusion and future research is presented in section 5.

2. LITERATURE REVIEW:

Transportation agencies are increasingly concerned with ensuring system reliability. In part, this has come about due to the finding that road users tend to value reliability at about the same magnitude as delays (*6, 8, 9*). Stochastic variations in transportation systems can be attributed to variations in demand, capacity or behaviour (*10-12*), hence it is important to incorporate the critical realities of both demand and capacity uncertainties in our transportation models for travel behaviour, so that consistent decisions can be made based on cost-benefit analysis associated to improving reliability by affecting variations in demand or capacity.

Improper consideration of demand variability in planning models can result in gross underestimation of travel time (*13*). Numerous research efforts have focused on the impact of day-to-day stochasticity regarding demand and capacity through specific model variations. For example, Clark and Watling (*14*) proposed an assignment model with stochastic demand to determine the impact on variance in total system travel time. Additionally, Lo and Tung considered stochastic capacity, and proposed the probabilistic user equilibrium (*15*). The concept of considering strategy in a user equilibrium context was proposed by Nguyen and Pallottino (*16*), specifically in the context of transit assignment. Later this work was extended to traffic assignment , where strategy reflected users changing routes due to gaining information at a node strategy is to change routes from intermediate information(*17*) and was further developed to the dynamic strategic model(*18*). Dixit et al proposed the strategic user equilibrium under demand uncertainty (*6*), which Waller et al. further expanded the linear programming formulation for dynamic traffic assignment, by dividing it into strategy stage and realized demand stage (*19*).

The strategic user equilibrium proposed by Dixit et al. (*6*) is a situation where users equilibrate to minimize their expected travel time based on the demand distribution, and the expected travel cost is less than the cost on any unused paths. An important aspect of this model is that the strategy remains fixed regardless of the specific day-to-day realized demand thus the prediction of link flow will likely vary with the realized demand.

The risk of variation of travel demand and its effects on route choice are explored by Uchida and Iida (*5*), who developed a new assignment model to consider the impact of variation in travel time. Furthermore, it should be noted that the impact of not considering these uncertainties leads to significant biases and errors, which was demonstrated by Duthie et al (*20*).

Capacity has been found to be a random variable with stochastic variations that are affected by driver behaviour and adverse weather conditions (*21, 22*). For this reason, it is extremely critical to incorporate stochastic capacity in planning models. Stochastic variations in capacity from the mean were found to be normally distributed under different traffic flow conditions (*4*). In fact, there is also extensive evidence that stochastic variations in capacity is found to follow a gamma distribution fairly well (*3*). Lo and Tung introduced the stochastic variation in capacity in the probabilistic user equilibrium, but assumed a simplistic uniform distribution (*15*).

This study provides a mathematical framework to incorporate demand and capacity uncertainty into the strategic user equilibrium paradigm. The next section presents the problem formulation and its equivalence to StrUE.

3. PROBLEM FORMULATION:

This section defines the mathematical concept of strategic user equilibrium, Table 1 explains the notation used in this section.

TABLE 1 Summary of notation.

N	Node (index) set
A	Link (index) set
K_{RS}	Path set
f_a	Proportion of total demand on link b; $f = (\ldots, f_a\ )$
t_a	Travel time on link a; $t = (\ldots, t_a, \ldots)$
p_k^{rs}	Proportion of flow on path k, connecting OD pair r-s, must be non-negative; $p^{rs} = (\ldots, p_k^{rs},\ \ldots)$; $p = (\ldots, p^{rs}, \ldots)$
c_k^{rs}	Travel time on path k connecting O-D pair r-s; $c^{rs} = (\ldots, c_k^{rs}\ )$; $c = (\ldots, c^{rs}, \ldots)$
q_{rs}	Fraction of total trips that are between OD pair r-s; $1 = \sum_{\forall rs} q_{rs}$
T	Random variable for total trips with probability distribution $g(T)$
g	Lognormal probability distribution for total demand of the network

C	Random variable representing the capacity on a link
ρ	Inverse gamma probability distribution for the inverse of the capacity $\frac{1}{C}$
G	Gamma probability distribution for the capacity
C_a	The capacity on link a
$\delta_{a,k}^{rs}$	Indicator variable $\delta_{a,k}^{rs} = \begin{cases} 1\ if\ a\ is\ included\ in\ path\ k \\ 0\ otherwise \end{cases}$ $(\Delta^{rs})_{a,k} = \delta_{a,k}^{rs};$ $\Delta = (\dots, \Delta^{rs}, \dots)$
$E[]$	The expected value of a variable
$Var[]$	The variance of a variable
$Std[]$	The standard deviation of a variable

The specific variation of strategic user equilibrium which is employed here was first introduced by Dixit et al (*6*) and aims to find a set of path proportions to minimize the cost on a path, these costs are the same on all used paths and smaller than costs on any other unused paths. The proportion of total demand q_{rs} is a constant and does not change, while the total demand is stochastic. The strategy of the users is related to the path proportions and users will not change the strategy once they select it. The total demand is a random variable with a known lognormal distribution while the capacity on a link is a random variable with a known gamma distribution (*3*). An important aspect to recognize is that if the capacity has a gamma distribution then the inverse of the capacity has an inverse gamma distribution. The link travel time function is assumed to be the Bureau of Public Roads function (*23*):

$$t_a(flow) = t_{af}\left[1 + \alpha t_{af}\left(\frac{flow}{C_a}\right)^{\beta}\right] \quad [1]$$

Where α and β are parameters of the BPR function and t_{af} is the free flow travel time on link a. Note, the BPR travel time function depends on the power or the inverse of capacity, and thus we use the inverse of capacity ($C_{inv} = 1/C$).

Therefore, the fraction of total trips on a link is only a function of the path proportions p_k^{rs}. The equilibrium is reached when the following objective function with respect to f is satisfied, both capacity and demand distribution are included when users making their routing decision:

$$Minimize\ z(f) = \sum_{a \epsilon A} \int_0^{f_a} \int_0^{\infty} \int_0^{\infty} t_a(fT)\ \ g(T)\ \ \rho(C_{inv})\ dT\ \ dC_{inv}\ \ df \quad [2]$$

Subject to:

$$\sum_k p_k^{rs} = q_{rs} \qquad \forall r, s \quad [3]$$

$$p_k^{rs} \geq 0 \qquad \forall r, s \quad [4]$$

$$f_a = \sum_r \sum_s \sum_k p_k^{rs} \delta_{a,k}^{rs} \qquad \forall r, s \quad [5]$$

Equation 2 is the objective function which is later shown to be equivalent to finding an assignment solution for StrUE. Equation 3 represents the conservation of OD flows, i.e., the sum of proportion of total demand on all paths from origin *r* to destination s should be equal to the fraction of total demand for OD pair rs. Equation 5 ensures that the fraction of total trips on a link is the sum of all path proportions using the link, where $\delta_{a,k}^{rs}$ is an indicator variable that indicates whether path k between OD pair rs uses link a, which is determined by the network topology.

The optimization model [1]-[4] can be re-written as a Lagrangian problem (*24*):

$$L(p, u) = z[f(p)] + \sum_{rs} u_{rs}(q_{rs} - \sum_k p_k^{rs}) \quad [6]$$

The first order optimality conditions of the Lagrangian are:

$$p_k^{rs}\frac{\partial L(p,u)}{\partial p_k^{rs}} = 0 \qquad and \qquad \frac{\partial L(p,u)}{\partial p_k^{rs}} \geq 0 \qquad \forall k, r, s \qquad [7]$$

$$\frac{\partial L(p,u)}{\partial u_{rs}} = 0 \qquad [8]$$

For equation [7],

$$\frac{\partial L(p,u)}{\partial p_k^{rs}} = \underbrace{\frac{\partial z[f(p)]}{\partial p_k^{rs}}}_{\text{1st part}} + \underbrace{\frac{\partial}{\partial p_k^{rs}}\sum_{rs} u_{rs}\left(q_{rs} - \sum_k p_k^{rs}\right)}_{\text{2nd part}} \qquad [9]$$

We use the chain rule to decompose the first part of [9] as follows:

$$\frac{\partial z[f(p)]}{\partial p_k^{rs}} = \sum_{a\epsilon A}\frac{\partial z(f)}{\partial f_a}\frac{\partial f_a}{\partial p_k^{rs}} \qquad [10]$$

Due to the fact that $z(f)$ is the integral of f from 0 to f_a, the partial derivative of $z(f)$ with respect to f_a is:

$$\frac{\partial z(f)}{\partial f_a} = \frac{\partial}{\partial f_a}\sum_{a\epsilon A}\int_0^{f_a}\int_0^{\infty}\int_0^{\infty} t_a(fT)\, g(T)\, \rho(C_{inv})\, dT\, dC_{inv}\, df$$

$$= \int_0^{\infty}\int_0^{\infty} t_a(f_aT) g(T) \rho(C_{inv}) dT dC_{inv} \qquad [11]$$

Equation [11] implies that the partial derivative of the objective function with respect to link flows is equal to the expected travel time on the link. And from equation 5 we know:

$$\frac{\partial f_a}{\partial p_k^{rs}} = \delta_{b,l}^{rs} \qquad [12]$$

When the results of [11] and [12] are substituted into equation [10], the first part of equation [9] can be rewritten as:

$$\frac{\partial z[f(p)]}{\partial p_k^{rs}} = c_k^{rs} \qquad [13]$$

Where c_k^{rs} is the expected travel time on a path k between OD pair *rs*. Regarding the second part of equation [9]:

$$\frac{\partial}{\partial p_k^{rs}}\sum_{rs} u_{rs}\left(q_{rs} - \sum_k p_k^{rs}\right) = -u_{rs} \qquad [14]$$

When the results from the first and second parts of [9] are combined, the optimality conditions can be written as:

$$p_k^{rs}(c_k^{rs} - u_{rs}) = 0 \qquad and \qquad (c_k^{rs} - u_{rs}) \geq 0 \quad \forall k, r, s \qquad [15]$$

$$\sum_k p_k^{rs} = q_{rs} \qquad \forall k, r, s \qquad [16]$$

$$p_k^{rs} \geq 0 \qquad \forall k, r, s \qquad [17]$$

Equations [15] and [16] are the flow conservation and non-negativity constraints for the proportions of flow, respectively. Equation [16] holds for all paths connecting each OD pair in the network. When these conditions are met the expected cost of any used path between OD pair *rs* will be equal and minimal, where a used path is defined as any path that has a non-zero proportion of flow. If the proportion on a path between *rs* is zero, then the expected travel time on the path is greater than the expected travel time at equilibrium. This shows the equivalence between the optimization model (Equations 2-5) and StrUE.

An important property for any traffic assignment problem is uniqueness in link performance so that project evaluation can be conducted in a consistent manner. This is possible if the Hessian matrix of the Lagrangian function is positive definite. The elements of the Hessian matrix can be represented as below:

$$\frac{\partial^2 z(f)}{\partial f_a \partial f_b} = \int_0^\infty \int_0^\infty \frac{\partial t_a(f_a T)}{\partial f_b} g(T)\rho(C_{in}) dT dC_{inv} = \begin{cases} \int_0^\infty \int_0^\infty \frac{\partial t_a(fT)}{\partial f_b} g(T)\rho C_{inv} dT\, dC_{inv} & if a = b \\ 0 & otherwise \end{cases} \quad [18]$$

And the Hessian matrix is shown below:

$$H[f(p)] = \begin{pmatrix} \frac{\partial^2 z(f)}{\partial f_1 \partial f_1} & \frac{\partial^2 z(f)}{\partial f_1 \partial f_2} & \cdots & \frac{\partial^2 z(f)}{\partial f_1 \partial f_n} \\ \frac{\partial^2 z(f)}{\partial f_1 \partial f_2} & \frac{\partial^2 z(f)}{\partial f_2 \partial f_2} & \cdots & \frac{\partial^2 z(f)}{\partial f_2 \partial f_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial^2 z(f)}{\partial f_n \partial f_1} & \cdots & \frac{\partial^2 z(f)}{\partial f_n \partial f_{n-1}} & \frac{\partial^2 z(f)}{\partial f_n \partial f_n} \end{pmatrix} \quad [19]$$

In the Hessian matrix, the diagonal elements are positive, which implies that the Hessian matrix is positive definite. Therefore the objective function is strictly convex, and hence there exists a unique solution to the optimization problem. Note that the mathematical proof is only correct when each O-D pair is proportional to the total demand and the proportions of each O-D pair is fixed, which is consistent with our assumptions.

As mentioned earlier, $g(T)$ is considered to be a lognormal distribution with mean $E[T]$ and variance $Var[T]$, which determines the parameters of the lognormal distribution:

$$g(T) = Lognormal \sim (\mu, v) \quad [20]$$

Where the parameters μ and v are defined as:

$$\mu = ln(E[T]) - \frac{1}{2} v$$

$$v = ln(1 + \frac{Var[T]}{(E[T])^2}) \quad [21]$$

The capacity of the links is assumed to follow a gamma distribution (*3*) with scale parameters k and θ that describe the shape of the distribution.

$$G(C) = Gamma \sim (k, \theta) \quad [22]$$

Therefore the mean and variance of the capacity is defined as:

$$\begin{cases} E(C_a) = k_a * \theta_a \\ Var(C_a) = k_a * \theta_a^2 \end{cases} \quad \forall a \epsilon A \quad [23]$$

If the capacity has a gamma distribution, the inverse of capacity (C_{inv}) has an inverse gamma distribution, with the following parameters:

$$\rho(C_{inv}) = Inv\_Gamma\left(k, \frac{1}{\theta}\right) \quad [24]$$

Therefore, the objective function in Equation 2 can be solved and re-written as:

$$z(f) = \sum_{a \epsilon A} t_{af} [f_a + \frac{\alpha}{\beta+1} L_\beta(a) M_\beta f_a^{\beta+1}] \quad [25]$$

Where M_β is the βth moment of the lognormal demand distribution and is defined in [27] and $L_\beta(a)$ inverse gamma distribution for capacity on link a as defined in [26].

$$L_\beta(\mathrm{a}) = \frac{(\frac{1}{\theta_a})^\beta}{(k_a - 1) \ldots (k_a - \beta)} \quad [26]$$

$$M_\beta = e^{\beta\mu+\frac{1}{2}v\beta^2} \quad [27]$$

Using this information, the expected travel time and variance of link travel times can be evaluated based on the probability distributions of demand and capacity. The expected travel time on a link is defined in [28], while the variance of travel time on a link is defined in [29]

$$E(t_a) = t_{af} + \alpha t_{af} f_a^{\beta} M_\beta L_\beta(a) \quad [28]$$

$$var(t_a) = E(t_a^2) - [E(t_a)]^2$$

$$var(t_a) = \alpha^2 t_{af}^2 f_a^{2\beta}[M_{2\beta}L_{2\beta}(a) - M_\beta^2 L_\beta(a)^2] \quad [29]$$

Therefore the expected total system travel time is the weighted sum of the expected link travel times, which may be defined as:

$$E(TSTT) = E\sum_a [f_a T * t_a(f_a T)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_a f_a T[t_{af} + \alpha t_{af} T^\beta \left(\frac{f_a}{C}\right)^\beta]g(T)\rho\left(\frac{1}{c}\right) dTd\left(\frac{1}{c}\right)$$

$$E(TSTT) = \sum_a f_a t_{af} M_1 + \alpha t_{af} f_a^{\beta+1} M_{\beta+1} L_\beta(a) \quad [30]$$

Using this information the variance of total system travel time is calculated as:

$$var(TSTT) = E\left[(\textstyle\sum_a [f_a T * t_a(f_a T)])^2\right] - (E[\textstyle\sum_a f_a T * t_a(f_a T)])^2 \quad [31]$$

One of the strengths of this model is that we have tied back the variability in travel time to the variability in demand and capacity through a simple analytical expression.

4. NUMERICAL RESULTS AND ANALYSIS:

The StrUE model is implemented to evaluate network performance under a range of demand and capacity uncertainty levels. Analysis is conducted in the Sioux Falls network(*25*) with 24 nodes and 76 links (see Figure 1). The total demand T is the summation of all O-D demands, and MT is the mean total demand for all O-D pairs, and ST is the standard deviation of total demand. The O-D demand is specified as proportions of the total network demand, therefore the demand for a given O-D is the O-D proportion multiplied by the total demand, namely: $D_{rs} = MT * q_{rs}$. The mean capacity and standard deviation of a link a is MC_a and SC_a, respectively. Free flow travel time on a link is given by t_{af}. The BPR parameters α and β are equal to 0.15 and 4, respectively. The network properties can be found in the Sioux falls network properties data.

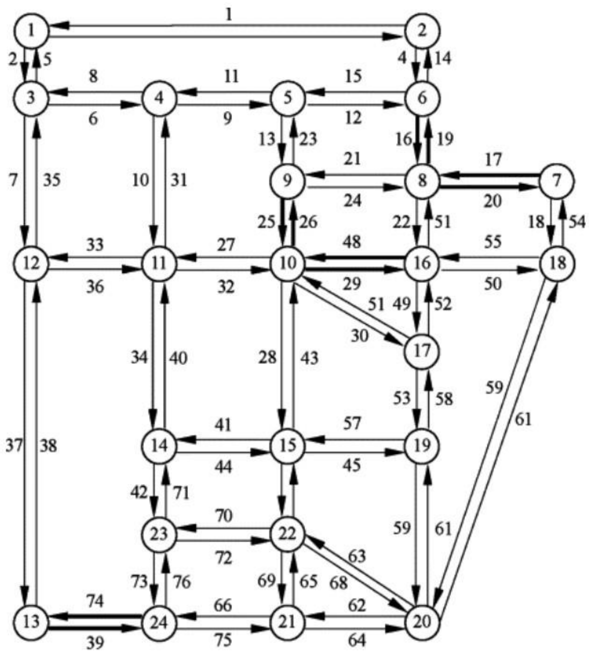

FIGURE 1 Sioux Falls network.

The level of uncertainty is quantified for the total aggregate demand and capacity of each link, and is described by the coefficient of variation (CV), where $CV(Total\ demand) = \frac{E[T]}{Std[T]}$, and $CV(Capacity) = \frac{E[C_a]}{Std[C_a]}$. The same capacity uncertainty level applies to all links in the network, representative of a reduction is total capacity on a network, for instance due to weather, however each link capacity is sampled independently. The CV for both total demand and link capacity may vary from 0% to 20% with an increment of 2.5%, and 0% represents the deterministic case where total demand or capacity is a constant instead of a variable. The two forms of uncertainty are treated independently.

The mean and standard deviation (Std) of link travel times can be computed for the model in two ways: i) estimated from the analytical model using equations 26 and 27, and ii) empirically through simulation. For the simulation analysis, Monte Carlo sampling is implemented to generate demands from a set of log normal demand distributions (i.e. each distribution was defined by a mean and variance of the total trip distribution), and link capacities are sampled from a Gamma distribution. The two samples are independent. For each realized demand and capacity combination, the StrUE link flows are calculated based on the set of link proportions identified by the Frank-Wolfe algorithm, from which the corresponding link travel times are computed. The mean and standard deviation of link travel times are estimated based on the entire demand sample set, equal to 1000 demand-capacity scenarios. For the remainder of the analysis the estimated mean and standard deviation calculated using equations 26 and 27 are referred to as Estimated E[TT] and Estimated Std[TT]. The empirically computed mean and standard deviation are computed using the simulation procedure described above, and referred to as Simulated E[TT] and Simulated Std[TT]. Analogous to the link travel time evaluation, the E[TSTT] and Std[TSTT] are estimated using equations 28 and 29 from the analytical model, hereby referred to as Estimated E[TSTT] and Estimated Std[TSTT]. Similarly, the mean and standard deviation computed via simulation based on the sampled set are referred to as Simulated E[TSTT] and Simulated Std[TSTT].

Figure 2 provides the E[TSTT] for the Sioux Falls network from both the analytically estimated and simulated StrUE evaluation approaches for different combinations of demand

and capacity uncertainty. The CV ranges from 2.5% to 20% for both demand and capacity variables. In total, 64 different combinations of demand-capacity distribution scenarios are evaluated using each approach. In figure 2, the R squared value is approximately 0.98, which indicates that the simulated E[TSTT] closely approximates the estimated E[TSTT]. Based on these results the analytical model is used for evaluation purposes in Figure 3.

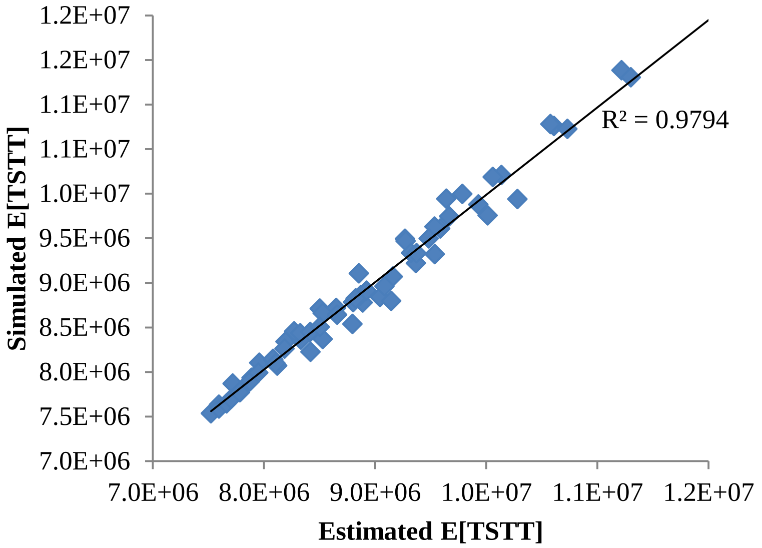

FIGURE 2 Simulated E[TSTT] compared to Estimated E[TSTT].

In Figure 3 the E[TSTT] under different uncertainty levels are presented. The CV of capacity is represented on the x-axis and each series corresponds to a different CV of total demand, as indicated by the legend. The CV ranges from 0% to 20% with an increment of 2.5%.

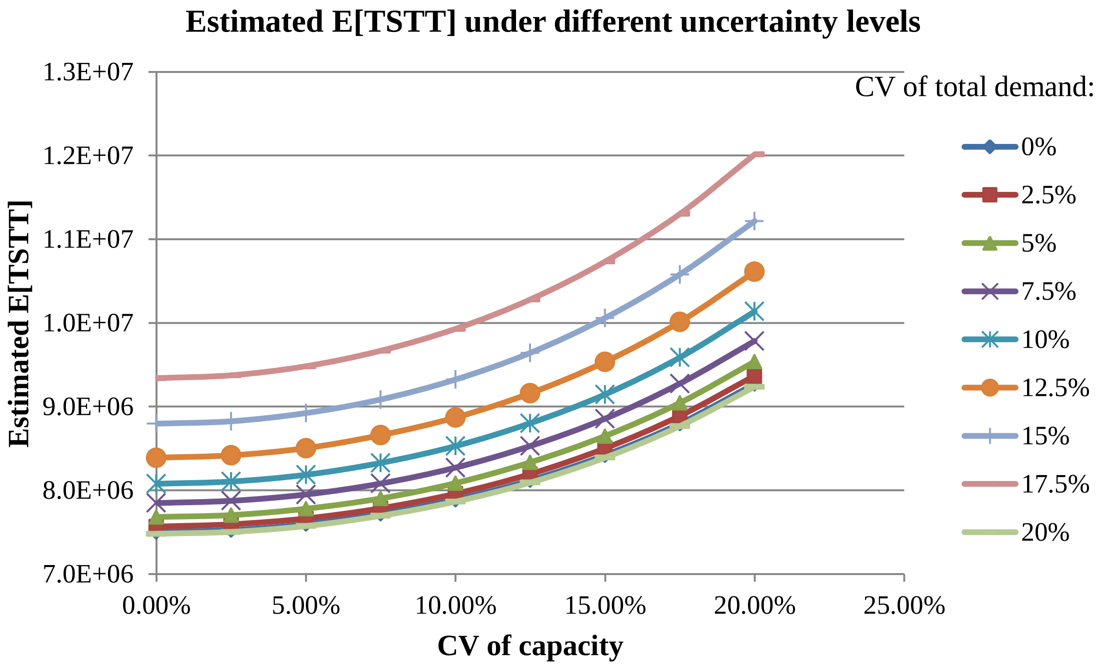

FIGURE 3 Estimated E[TSTT] under varying uncertainty.

The E[TSTT] rises more than 20% when the CV of capacity increases from 0% to 20%. Similarly, an increase in the CV of total demand also results in an increased E[TSTT]. The results illustrate both demand and capacity uncertainty corresponds to a decrease in system performance, which increases with the level of uncertainty. The effect is further exaggerated when both types of uncertainty are accounted for. It is shown in the figure that when both capacity and total demand are deterministic, the TSTT is significantly smaller than the case when the variations of both total demand and capacity are high. Therefore incorporating both demand and capacity uncertainty into the StrUE model can help avoid underestimating future system performance.

Figure 4 provides the Std[TSTT] for the Sioux Falls network from both the analytically estimated and simulated StrUE evaluation approaches for different combinations of demand and capacity uncertainty. Similarly to Figure 2, the CV ranges from 2.5% to 20% for both demand and capacity variables, and 64 different combinations of demand-capacity distribution scenarios are evaluated using each approach. In figure 4, the R squared value is 0.99, which indicates that the simulated Std[TSTT] closely approximates the estimated Std[TSTT]. Based on these results the analytical model is used for evaluation purposes in Figure 5.

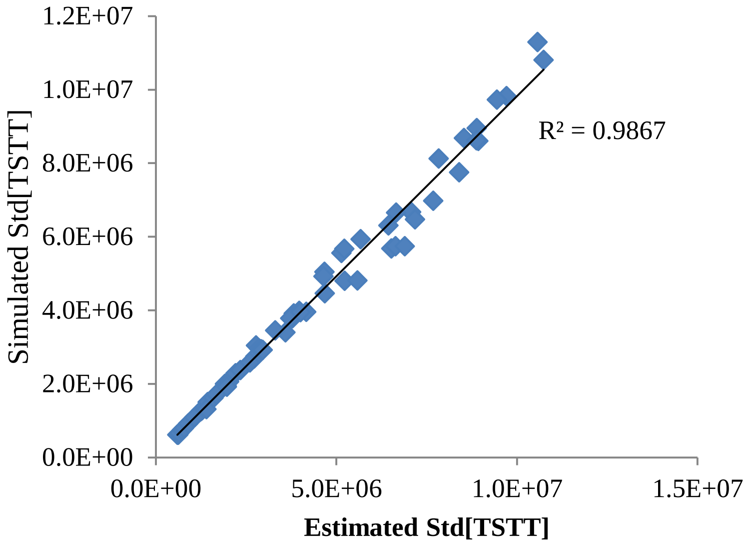

FIGURE 4 Simulated Std[TSTT] compared to Estimated Std[TSTT]

In Figure 5 the impact of demand and capacity uncertainty on system variability is illustrated. The CV of capacity is represented on the x-axis and each series corresponds to a different CV of total demand, as indicated by the legend. The CV ranges from 0% to 20%.

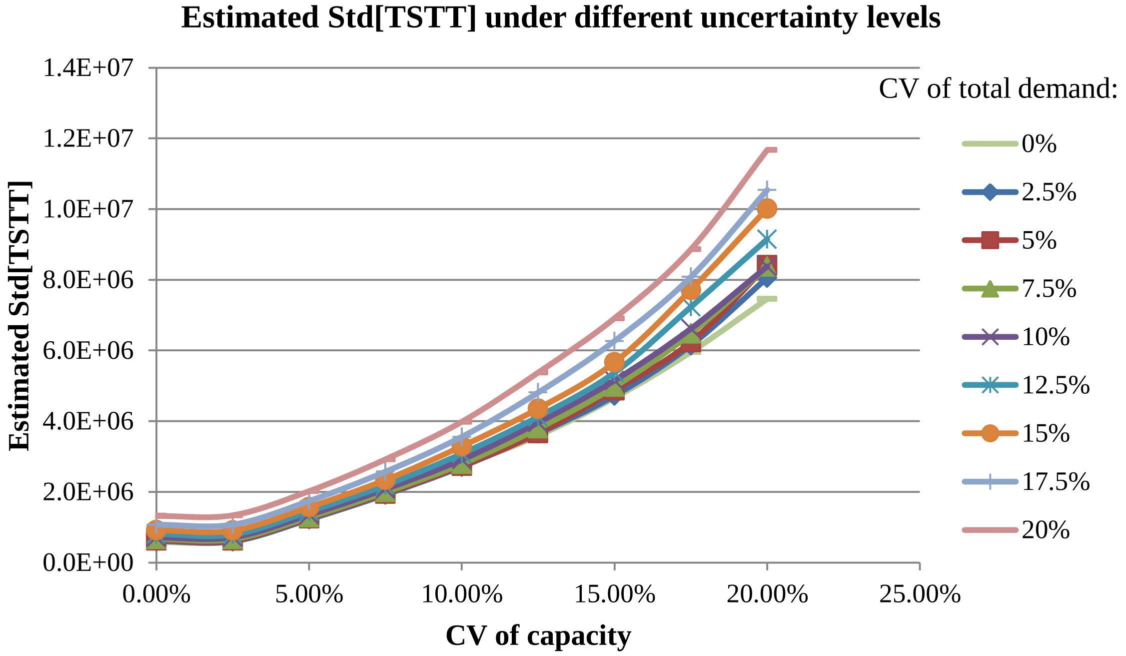

FIGURE 5 Estimated Std[TSTT]

Figure 5 illustrates the increased variability in system performance that results from an increase in demand and capacity uncertainty. For each CV of total demand, the Std[TSTT] increases at least 700%, while when CV of capacity equals 20%, the estimated Std[TSTT] only increases 50%, from 8 million to 12 million. So in contrast to the expected system performance, the capacity uncertainty has a more exaggerated impact on Std[TSTT] than demand uncertainty. This can be explained by the model assumptions; the O-D proportions remain fixed, and the aggregate demand is treated as the random variable, whereas the link capacities are sampled independently of one another. However, the Std[TSTT] increases with both types of uncertainty, therefore neglecting either demand or capacity uncertainty will lead to an overly robust estimation of system performance.

Figure 6(a) further supports the StrUE model predictions by comparing the estimated and simulated E[TT] and Std[TT] for 2 links, link 10 and link 15 of Sioux Falls. The results are representative of the other links in the network. The same 64 combinations of demand and capacity uncertainty levels as in Figures 2 and 4 are evaluated using both the estimated and simulated evaluation methods. In Figure 6(a) the x-axis represents the simulated E[TT] and the y-axis represents the estimated E[TT]. In Figure 6(b) the x-axis represents the simulated Std[TT] and the y-axis represents the estimated Std[TT].

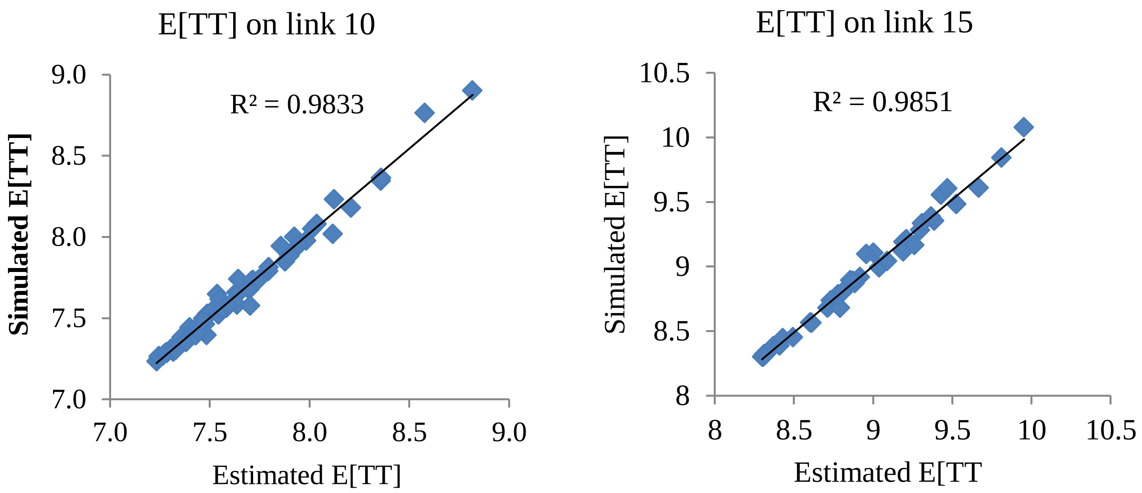

FIGURE 6(a) Simulated E[TT] compared to Estimated E[TT] on links 10 and 15.

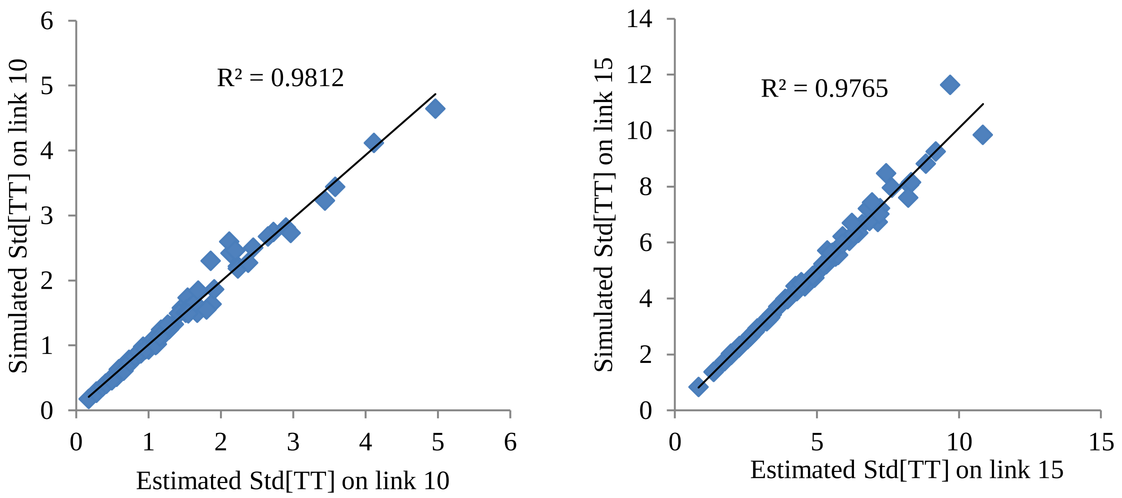

FIGURE 6(b) Simulated Std[TT] compared to Estimated Std[TT] for links 10 and 15.

The R values are close to 1, which suggest the estimated link travel time and corresponding variability closely approximates the simulated results, which supports the reliability of the analytical expressions for E[TT] and Std[TT] from the StrUE model. These results also illustrate the ability of the StrUE model to capture both link level and system level performance. Note that a higher variation in demand/capacity will result in a greater variance between estimated and simulated results because of the random generation process of the Monte-Carlo simulation.

5. CONCLUSION:

This paper incorporated capacity uncertainty into a previously introduced strategic equilibrium model (6) which deals with day-to-day route assignment in the context of short-term travel demand uncertainty. The analytical solution to the strategic user equilibrium considering both capacity and demand uncertainty was derived mathematically and the uniqueness of the optimal solution was proven which ensures the model can be applied to transportation planning applications which require unique project rankings. Further, numerical analyses were conducted employing this model to explore the importance of considering demand and capacity uncertainties on transportation analysis. The results demonstrated that the analytical solution approximate the simulated results which support the use of the techniques as a computationally efficient method for considering traffic flow and network reliability in a planning context. Additionally, it was presented that both demand and capacity uncertainty levels can significantly increase E[TSTT] and Std[TSTT], thus incorporating them reduced the bias of future traffic prediction.

However, a key consideration is that due to the mathematical restrictions of the gamma distribution, the coefficient of variance of capacity must be within 34%, otherwise the solution will not be a natural number. The simulation method can be used to solve the problem in this situation. Numerous future research directions are valuable to further facilitate the usefulness of this work. For instance, additional probability distributions should be explored with consideration for empirical fit to both the demand and capacity uncertainties. Further, dynamic variations of this problem are critical to capture both the day-to-day volatility of conditions, use behaviour and the realism of congestion dynamics.

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