DYNAMIC USER OPTIMAL TRAFFIC ASSIGNMENT WITH RECOURSE

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ABSTRACT

Limitations of static network equilibrium models have led to numerous research efforts in predicting the temporal and spatial traffic conditions throughout road networks. Static models do not account for the dynamic nature of traffic. Accordingly, new approaches to depict these scenarios have been formulated, such as Dynamic User Optimal (DUO). This study extends DUO to DUO with recourse (DUOR) where a user can also alter their journey en-route dependent on the traffic conditions and the available information through ITS technology.

This study proposes the modelling framework Dynamic User Optimal with Recourse using a Cell Transmission Model (DUOR-CTM). The model focuses on initially understanding whether a Dynamic User Optimal with Recourse (DUOR) solution arises and also determines the impact of information on the user optimal travel cost. The study presents the results of a sample network and highlights the need to account for information in a dynamic context.

Keywords: Dynamic User Optimal with Recourse, Information, Incidents, Simulation, Cell Transmission Model

Subject area: (Please put a "X" as appropriate, you may choose more than one)

	a)	Transportation Infrastructure and Built Environment
	b)	Sustainability Issues in Transportation
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X	d)	Travel Behavior Modeling
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	Yes	X	No

1. INTRODUCTION

Limitations of static network equilibrium models have led to numerous research efforts in predicting the temporal and spatial traffic conditions throughout road networks. Static models do not account for the dynamic nature of traffic. For instance, the presence of a disruption to a network, such as a breakdown or accident, will transform users' route choice decision making when compared to day-to-day decision making under normal conditions. The improved depiction of these scenarios will assist in developing incident management and disaster mitigation strategies as well as having the potential to improve the utilization of intelligent transport systems (ITS). Accordingly, new approaches to depict these scenarios have been formulated, such as Dynamic User Optimal (DUO). DUO assumes that each driver determines their best route from origin to destination based on the prevailing traffic conditions. However DUO can be extended to DUO with recourse where a user can also alter their journey en-route dependent on the traffic conditions.

Within this study, it is further considered that road users' route choice is adaptive and dependent on the traffic state of the current route and the perceived states of the alternative routes available between an origin and destination pair. Users will change route if they perceive an alternative route will minimise their travel cost. This means that instead of traversing on the shortest path, a user will traverse on the least cost routing policy. A routing policy may include a number of paths which are perceived to be the shortest depending on the state of the network. Furthermore intelligent transport systems (ITS) are becoming increasingly prevalent within transport networks and provide users with information regarding the state of the network. These information sources affect users' route selection. Accordingly, it is important to account for not only the adaptive behaviour of users but also the impact of en-route information.

This study proposes the Dynamic User Optimal with Recourse using a Cell Transmission Model (DUOR-CTM). DUOR-CTM incorporates information sources into the cell transmission model (CTM) to understand the impact of information on network performance in a dynamic context. CTMs provide an opportunity to dynamically simulate a traffic network, resulting in a prediction of the macroscopic traffic behaviour. The states of the network have a specified probability of occurrence that is depicted using multiple cell transmission models with alterations to the capacity of specific cells within a network. The simulation is conducted with individual users being assigned a specific routing policy, given a set of pre-defined network parameters, where expected travel time would be determined across all possible states. Iteration of the assigned policy is then conducted to determine the policy structure which will minimise the expected travel time for each individual vehicle, resulting in the determination of the Dynamic User Optimal with Recourse (DUOR) solution. The study presents the results of a sample network and highlights the need to account for information in a dynamic context.

2. BACKGROUND

Road networks experience uncertainty as a result of traffic incidents such as break downs and accidents, poor weather conditions, natural disasters and unscheduled maintenance and construction works. As a result of these disruptions to a network, users experience extended delays, higher fuel costs, and greater levels of emissions in the atmosphere, thus creating an increased economic and social burden on the community (Downs, 2005). Hence the need to model and assess these scenarios is essential to the provision of sustainable transport networks.

A traveller makes route choice decisions based on the knowledge obtained through personal experience and exogenous information (Balakrishna et al., 2013). Personal experience is limited by the lack of knowledge of all available options and is impacted by the severity of the uncertainties described above. The provision of exogenous information through ITS is intended to alleviate the cognitive stress of decision making and improve the quality of the decision for the user as well as the network (Balakrishna et al., 2013). The route choice behaviour of travellers in the presence of an ITS are dynamic and contrast those which would be observed within deterministic traffic assignment models (Gao, 2012, Unnikrishnan and Waller, 2009). Conventional static traffic assignment modelling assumes that users select paths from origin to destination with no en-route adaptive route choice behaviour. In contrast, recent research (Gao, 2012, Unnikrishnan and Waller, 2009) have investigated adaptive routing as individuals having a routing policy (or sometimes known as a hyper-path), a series of possible paths that he or she would take given the state of the network. This study aims to contribute to this key area of understanding by developing a dynamic network modelling framework that accounts for the acquisition of en route information and the potential for adaptive routing.

The concept of optimal strategic routing, or sometimes referred to as optimal adaptive routing, by individual travellers has been studied by a number of researchers (Hall, 1986, Polychronopoulos and Tsitsiklis, 1996, Pretolani, 2000, Waller and Ziliaskopoulos, 2002) and a detailed literature review of recent work can be found in (Gao, 2005). Studies such as Waller and Ziliaskopoulos (2002) have formulated the acquisition of en-route information within the context of shortest path determination. A majority of these studies investigate flow-independent stochastic time-dependent networks and presented little improvements in user performance metrics when considering the optimal strategic routing decisions. In addition to understanding behaviour at an individual level it is also important to assess the impact of adaptive routing at a network wide level. This has generally been completed by considering equilibrium models. In a time dependent network context there have been a number of studies which have investigated the presence of Dynamic User Equilibrium (DUE) and Dynamic User Optimal (DUO) solutions (Mahmassani and Herman, 1984, Friesz et al., 1993, Papageorgiou, 1990). DUE and DUO have been used interchangeably throughout a number of studies which focused on representing an equilibrium state on a time-varying road network (Ge et al., 2012). Ge et al. (2012) provides a thorough definition of the field, defining DUE and DUO under different equilibrium tolerance scenarios and highlighting that DUE is a special case of DUO with zero tolerance. The definition of DUO (Waller and Ziliaskopoulos, 2006, Golani and Waller, 2004) used for this particular study is as follows; the situation where each individual user travelling between a specific origin and destination cannot unilaterally change their path to reduce their travel cost whilst accounting for the prevailing traffic conditions. Using this as the basis, Dynamic User Optimal with Recourse (DUOR) was defined as; *"the situation where each individual user travelling between a specific origin and destination cannot unilaterally change their routing policy whilst accounting for the prevailing traffic conditions"*. The key difference between conventional DUO and DUOR is that a user can also alter their journey en-route dependent on the traffic conditions. The primary aim of the study is to identify if DUOR conditions are present within a transport network.

It is also important to note the recent research conducted regarding network assignment models that account for adaptive en-route behaviour. Traditionally these models have centred on transit assignment and capacitated networks (Nguyen and Pallottino, 1989, Marcotte et al., 2004). Recently there have been a couple of studies that have investigated strategic routing in a static framework (Ukkusuri and Patil, 2007, Unnikrishnan and Waller, 2009). This particular study incorporates the concepts of User Equilibrium with Recourse (UER) presented by Unnikrishnan and Waller (2009). UER is a static equilibrium that accounts for en-route route choice in the presence of information. Investigation of adaptive routing within networks in a dynamic context have resulted in some simulation based approaches, for example Gao and Chabini (2006) present dynamic user optimal traffic assignment in which a user is assigned to links dependent on the state of the link. In 2012, Gao extends the previous study to consider the impacts of real-time information on the users' routing decisions and the system cost in a stochastic time-dependent traffic network under generalised equilibrium conditions. These studies have focused on link level equilibration, however the study presented in this paper considers path or routing policy level user optimal conditions.

Determination of DUOR solutions was carried out using simulation through a Cell Transmission Model (CTM). CTMs can be applied to develop cell based dynamic equilibrium models which are a class of dynamic traffic assignment (DTA) model. These models can capture equilibrium conditions and realistic traffic dynamics, such as queue spillback, queue formation and dissipation (Szeto, 2013). CTMs have been used in developing system optimal DTA frameworks (Li et al., 1999, Li et al., 2003, Ziliaskopoulos, 2000) and furthermore in deriving equilibrium models (Lo and Szeto, 2002, Lo, 1999, Ukkusuri and Waller, 2008). These studies have enhanced the understanding of driving behaviour and its implications on network performance. This is emphasised by a study conducted by Lo and Szeto (2004) who highlighted the value of dynamic modelling, utilising tools such as the CTM, by suggesting that project rankings and policy prioritisation are significantly different between using static models and DTA models. This can potentially lead to misinformed decision making and an exacerbation of traffic and transport problems.

The review of the recent literature indicates that a considerable effort has been made to understand the dynamic nature of traffic using simulation and analytical methods which have been essential to transport planners and traffic engineers alike. This particular study simulates the time dependent incorporation of en route information acquisition within a cell transmission model.

3. MODELLING FRAMEWORK

The proposed Dynamic User Optimal with Recourse using a Cell Transmission Model (DUOR-CTM) applies the concepts of User Equilibrium with Recourse (UER) (Unnikrishnan and Waller, 2009) to the general CTM (Daganzo, 1994). The model developed provides a time dependent simulation model accounting for adaptive routing in the presence of en route information. Accordingly, the modelling framework uses the CTM to dynamically simulate potential route choice scenarios to determine if a user optimal condition is present.

As detailed within Section 2, a CTM provides the ideal platform to dynamically simulate a traffic network to understand the travel behaviour of individual vehicles. The CTM simulation process applied within this study is described briefly, however for a thorough understanding of the mechanics of a general CTM model refer to Daganzo (1994) and Daganzo (1995). A road network consists of nodes and links, where a link constitutes a road segment. The CTM defines road links as a series of cells and vehicles are transmitted through the cells from each origin to each destination. In the CTM, a specified discretised time interval is initially assumed to simulate the vehicles. A road segment is divided into cells spatially where the length of the cells is determined by the distance travelled by a vehicle travelling at free-flow speed (v_f) during one time interval. Each cell can contain a certain number of vehicles, N, which is dependent on the jam density. The cell can transmit a specific number of vehicles during each time interval, Q, which is dependent on the capacity of the road segment. Vehicles progress through the network considering the following flow propagation equations, which are discrete approximations to hydrodynamic flow theory;

$$n_i^{t+1} = n_i^t + y_i^t - y_{i+1}^t \qquad (1)$$

$$y_i^t = \min(n_{i-1}^t, Q_i^t, \delta(N_i^t - n_i^t)) \qquad (2)$$

In equations (1) and (2), the subscript i refers to a cell i within a road segment, *(i -1)* refers to a cell upstream of cell i and *(i +1)* refers to a cell downstream of cell i. The variables n_i^t, y_i^t, Q_i^t, δ and N_i^t denote the number of vehicles in cell i, inflow of cell i, transmission flow capacity (where the minimum of the current and preceding cell capacities are considered), the ratio of the free-flow speed and backward propagation speed, and maximum holding vehicles of cell i (set equal to 1), respectively. Equation (1) depicts the conservation of flow, where the cell occupancy at time *(t+1)* adds the inflow and deducts the outflow at time *(t)*. Equation (2) defines the propagation of flow to the next upstream cell by considering the capacity constraints. A first-in-first-out (FIFO) policy is maintained throughout the modelling.

Given the above method of simulation, a framework for Dynamic User Optimal with Recourse was devised based on the static traffic assignment equilibrium model, UER. The model accounts for one-step local information and user recourse on account of gaining that information as users traverse the network (Wijayaratna et al., 2013). A UER solution assumes that the links within a network experience multiple states with an associated probability of occurrence reflecting different traffic conditions that those users will encounter. The users' route choice is dependent on the state of the link, and thus instead of selecting a single optimal path from origin to destination, a user would select a path dependent on the state of the network. The alternatives will arise for a user at every divergent point within a network where a user has a choice about the next link to take, and if en-route information is provided to the user he or she will make a state dependent choice and have the opportunity to adapt his or her route. Thus a user will have a set of alternative paths dependent on the state of the network which are known as hyper-paths or routing policies. Equilibrium flows within a network occur when the expected costs of all used routing policies are minimum and equal. The advantage of the UER methodology is that it accounts for the presence of information in the user route decision-making process (Wijayaratna et al., 2013). Figure 1 presents the utilisation of the concepts of UER to develop the DUOR framework.

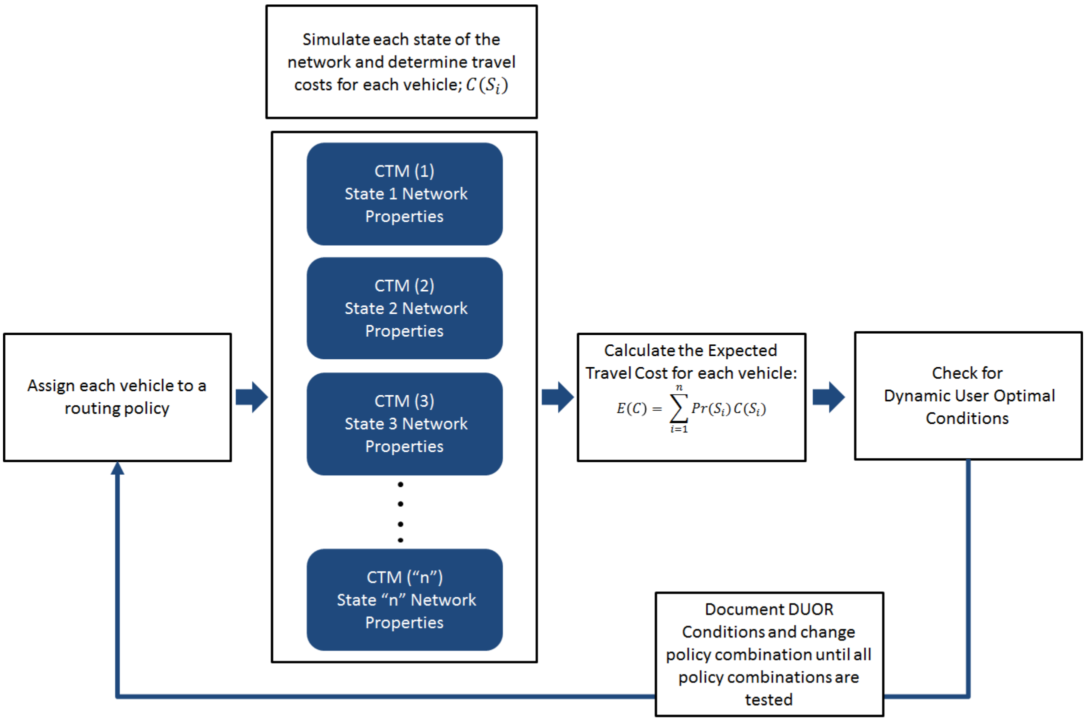


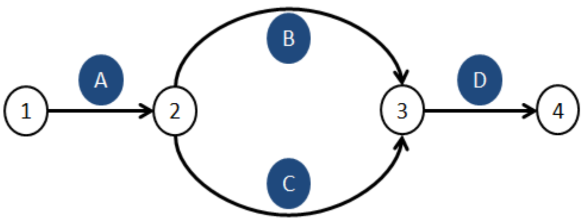
Figure 1: Dynamic User Optimal with Recourse Framework

An iterative simulation approach is applied to determine the DUOR solution for a network. Considering a single origin-destination pair with a fixed demand and *"n"* states with a specified probability of occurrence, the simulation process begins by initially assigning all the vehicles to any one of the feasible routing policies available for the network. Individual cell transmission models depict each state of the network described in the UER framework by applying alterations to the cell capacity and cell propagation capacity. The costs (travel time) experienced by each vehicle for each state are computed with each CTM simulation. These costs were used to then calculate the expected travel cost for each vehicle considering all *"n"* states of the network. The combination of routing policies was then tested to see if DUOR conditions were satisfied. This was completed by simulating alternate policy combinations and checking to see if a user is "unable to switch to an alternative policy to reduce their expected travel cost". If this is the case then DUOR is achieved. Iteration of different policy combinations was tested to determine all feasible DUOR solutions for the network.

4. RESULTS AND ANALYSIS

This section demonstrates the proposed DUOR modelling framework that dynamically accounts for the impact of en route information. A hypothetical two path network was investigated to understand the presence of DUOR in addition to determining the impact of information acquisition within a network. The analysis was conducted to determine the presence of DUOR solutions within the sample networks as well as to understand the impact of information by comparing a "with information" scenario to a "no-information" scenario. The methodology described in the previous section was used to calculate the DUOR solution depicting the "with information" scenario. The "no-information" scenario was determined by removing the ability for users to re-route, thus resulting in an expected DUO solution which incorporates multiple states of a network. The performance metric used to compare the scenarios was Expected Total System Travel Time (ETSTT). Throughout the analyses it is assumed that there are two states present ("no incident" and "incident present") with a probability of 0.8 applied to state 1 and a probability of 0.2 applied to state 2 of both networks.

The two path network is presented in Figure 2. Included in Figure 2 are the network properties and feasible policies considered for the assessment. The network considers a higher capacity link (B) and a lower capacity link (C) with the presence of an incident resulting in a restriction in capacity of link B. Low demand levels of 4, 6, 8 and 12 vehicles were tested in order to minimise the computational effort of the brute force algorithm devised within the methodology.



Policy	P1	P2	P3	P4
State 1	A-B-D	A-B-D	A-C-D	A-C-D
State 2	A-B-D	A-C-D	A-C-D	A-B-D

Link	State 1			State 2		
	N (veh)	Q (veh)	V (km/hr)	N (veh)	Q (veh)	V (km/hr)
A	10	5	50	10	5	50
B	2	2	60	1	1	60
C	1	1	60	1	1	60
D	8	4	50	8	4	50

Figure 2: Two Path Network and properties

Table 1 presents the results of the Two Path Network. Multiple DUOR permutations, that is, an arrangement of routing policies where no individual can improve his or her travel time by using an alternative policy, were observed across the 4 demand scenarios tested. The path-based user optimal approach results in non-uniqueness which is not unexpected as it is discussed throughout literature (He et al., 2010). The numbers of permutations increases with greater levels of demand as there are more feasible permutations for greater numbers of vehicles. For example, the scenario simulating 12 vehicles has 4^{12} = 16,777,216 total policy permutations, and 5184 of these policy permutations resulted in DUOR solutions. It is interesting to note that the proportion of DUOR solutions relative to the total number of policy permutations decreases with increasing traffic volumes. This suggests that increased congestion levels limit the likelihood of optimal policies being included within an individual's choice set. These DUOR solutions were confirmed as optimal as there were no observed changes in ETSTT across all permutations calculated. The results also indicate that there is a positive impact of the presence of information within a network; ETSTT was equal to or lower in the "with information" scenario as compared to the "no information" scenario, especially considering greater demand levels (8 and 12 vehicles). Due to the lack of computational power, greater numbers of vehicles could not be tested and this will be scope for future refinement of the methodology and further scenario testing. It is also noted that the number of expected DUO permutations is significantly lower than DUOR permutations as the removal of information reduces the number of policies from the decision choice set.

Table1: Two Path Network Results

Number of Vehicles	With Information			No Information		
	Total Policy Permutations	Number of DUOR Solutions	ETSTT (s)	Total Policy Permutations	Number of DUOR Solutions	ETSTT (s)
4	256	24	1063.2	16	2	1063.2
6	4096	72	1599.6	64	2	1600.8
8	65536	432	2190	256	2	2192.4
12	16777216	5184	3358.8	4096	2	3368.4

5. CONCLUSION

This study proposes a modelling framework, Dynamic User Optimal with Recourse using a Cell Transmission Model (DUOR-CTM), which determines a path-based dynamic user optimal with recourse (DUOR) solution for a single origin destination pair. Demonstration of the model on a sample two path network indicates that there is the presence of multiple DUOR solutions for specific network structures. In addition, the presence of information was shown to improve the Expected Total System Travel Time in conditions with greater levels of congestion within the network. These results have implications for the decision to implement ITS systems throughout road transport networks. Future work will involve development of the DUOR-CTM model by reducing the computational burden of the current methodology as well as investigating the application of the model to larger, more realistic networks.

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