VALIDATION OF LINK PROPORTIONALITY ASSUMPTIONS ACCOUNTING FOR DAY-TO-DAY VARIABILITY

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ABSTRACT

Transport systems are inherently uncertain, posing a challenge to network modellers and planners. Newer models (probabilistic user equilibrium, strategic user equilibrium) attempt to capture this uncertainty by explicitly acknowledging day-to-day randomness in the network, and solve for *strategic equilibrium* conditions based on the concept of travellers optimizing their expected performance. Strategic models are based on the assumption that the realized conditions on each network link vary proportionally with network-wide variation. While intuitive, this assumption is yet to be validated. Therefore, the objective of this work is twofold; to use observed network conditions to quantify the day-to-day variability and to validate the link proportionality assumption intrinsic to the novel strategic traffic assignment models. The analysis is conducted using measured 15-minute flows on a simple corridor-shaped network in Sydney, Australia. We find that day-to-day variation dominates time-of-day variation, and that variation in link demand can be explained by changes in network demand.

Keywords: network reliability, traffic assignment, demand variability, modelling assumptions.

1. INTRODUCTION

Traffic network equilibrium remains a critical component in the transportation planning process despite the day-to-day variance observed in flows and travel times which are often interpreted as violations of network equilibrium (Watling and Hazleton, 2003). Variability of link travel time in a network can be attributed to variability in traffic demand (Waller *et al.*, 2001; Stopher *et al*., 2008), capacity or supply (Brilon *et al*., 2005; Wu et al. 2010), route choice behaviour (Gordon *et al.*, 2001; Hamdouch *et al*. 2004) and departure time choice behaviour (Noland and Polak 2002). Furthermore, network performance variability can be attributed to multiple causes including day-to-day changes as well as time-of-day patterns. Whatever the reason, the lack of observable equilibrium represents an inconsistency between theory and reality that requires fundamental reconsideration of underlying assumptions for traffic equilibrium.

Moreover, travel time variability has become increasingly important in transport planning as evident in its inclusion in recent United States federal transportation legislation (MAP21) and the slew of Strategic Highways Research Program 2 reports. The value of reliability to travelers has been demonstrated using land values (Waddell & Foti, 2013) and as a component of travel costs (Lam & Small, 2001; Bates *et al*., 2001; Fosgerau *et al.*, 2008; Goodchild *et al.*, 2008; Carrion-Madera *et al.*, 2011; De Jong and Bliemer, 2015) which indicate that drivers value travel time reliability somewhere between 50 and 80 percent (Small *et al.*, 2005) as much as they value travel time.

Furthermore, researchers (Fosgerau, 2010 and Stogios, 2013) have empirically found that the standard deviation and mean of travel time are positively correlated. This effect has been explained based on stochastic demand and capacity from a departure time choice perspective (Fosgerau, 2010), as well as the stochastic nature of the traffic and capacity (Gayah *et al*. 2013). However, there is a gap between current network models and their ability to replicate these observed relationships between mean and variance of travel time.

While modelers are well aware of these inherent system uncertainties (Bell and Iida, 1997; Al-Deek and Emam, 2006; Emam and Al-Deek, 2006; Chen et al., 2007) and their ensuing impact on travel choices, traditional equilibrium models which predict link flows such as Stochastic User Equilibrium (SUE) (Daganzo and Sheffi, 1977; Sheffi and Powell, 1982; Mirchandani and Soroush, 1987; Maher and Hughes, 1997) and Deterministic User Equilibrium (Wardrop, 1952) are unable to consistently consider the impact of uncertainty in demand and supply. Despite this, most planning models utilize traditional equilibrium theories due to their important mathematical properties (e.g. solution consistency and convergence) and ability to capture the inherent self-centered nature of travelers.

To address these limitations, new network models have been explored that account for the inherent system uncertainties yet also maintain important convergence and mathematical properties. For example, the impact of variations to supply side reductions in capacity resulting from traffic incidents or adverse weather conditions on travel time has been studied by Asakura and Kashiwadani (1991), as well as Clark and Watling (2002, 2005). Lo and Tung (2000, 2003) proposed a concept of Probabilistic User Equilibrium (PUE) under uncertain travel times, which are a manifestation of stochastic fluctuations in capacity drawn from a uniform distribution. More recently, in an attempt to incorporate day-to-day demand variability, Dixit *et al.* (2013) proposed a strategic user equilibrium (StrUE) network assignment model. In this model, there exist expected link flow conditions with a proven uniqueness for link flow proportion. This work was extended to include capacity uncertainty (Wen *et al.,* 2014) and applied within a system-pricing context (Duell *et al.*, 2014). In addition, StrUE was shown to replicate the relationship between mean and variance of travel time on links.

It is critical to point out that the *proportionality* assumption (to incorporate supply and demand uncertainties) in PUE and StrUE is fundamentally different from the assumption of choice probabilities in SUE. Traditional SUE assumes static demand with perception errors resulting in choice probabilities. These choice probabilities could change based on different demand and supply realizations. On the other hand, PUE and StrUE assume path proportionality is based on expected costs and remains constant irrespective of daily demand or supply realizations. Though the path proportionality may be significantly different based on the assumed assignment framework, i.e. PUE or StrUE, a fundamental assumption that remains is that the proportion of traffic using a link from day-to-day should be constant. This study tests this particular assumption in these models using real data.

The incorporation of the concept of proportionality of link flows (for PUE and StrUE) has been the key insight for models to incorporate the impact of day-to-day demand and supply variability to study system uncertainty in transport networks. Although the theories and simulations provide reasonable support, there is a need to validate the proportionality assumption based on real-world traffic to support future use of the models for long-term planning purposes. Therefore the objectives of this work are twofold. First, we aim to demonstrate that day-to-day variability in network conditions contributes significantly to expected network performance, and its inclusion is necessary in order to accurately model traffic behavior. Second, we demonstrate that the path proportionality assumptions employed in strategic models such as StrUE and PUE are consistent with the observed network conditions.

The next section introduces the data sources and methods used to compare the assumptions with reality. Section 3 presents the results of the analysis showing variable demand and constant link proportions. Finally, the implications of the findings are discussed in Section 4 as well as future work.

2. DATA AND METHODS

The adopted approach utilizes real world traffic data to test the validity of the StrUE and PUE models' assumption regarding constant path proportions under variable demand. The models output expected demand proportions and variability at the link level (rather than the path level), therefore the validation conducted will also utilize link level analysis.

Link-based traffic volumes from a 20.1 km section of the M4, the Western Motorway connecting Sydney and the Blue Mountains, are used to empirically validate the assumption. The observations are taken from buried loop detectors. By sensing when a vehicle is present over the detector, the system gathers vehicle counts approximately every 0.5 km along the test section. Each monitor site location is defined as a unique link, and a road segment between ramps can be composed of as many as 11 links. The study segment is measured at 44 mainline detectors, 5 on-ramps and 4 off-ramps between Lapstone (monitor site ms90b) and Eastern Creek, New South Wales (monitor site ms47b) as shown in Figure 1. The observations are aggregated into 15 min averages over the month of February 2013—this temporal resolution is sufficient since most traffic assignment models are applied generally to the peak period.

In the corridor network, trips are implicitly tied to on-ramp origins and off-ramp destinations. In the study area, travellers can move between these origins and destinations on the M4 or on a parallel major arterial, A44, the Great Western Highway. Data for A44 is not available. The model assumption of link proportionality reflects real-world behavior of the system regarding how demand is distributed across the network on different days. If link proportionality is found, it implies that, when demand is low on the network, use of the network changes proportionally across all the O-D pairs and routes.

For example, consider a rainy day when demand is especially low on the M4 corridor-network. One feasible explanation is that trips originating at the far western end are a bit lower than usual because people with longer trips choose not to make them in bad weather, but the usual number of trips are originating in the middle and eastern parts of the road because these trips are shorter and more manageable in the bad weather. This situation would violate the assumption of link proportionality because the fraction of network-wide vehicles traveling on the western links would decrease compared to an average-demand day. If we observe link proportionality in the data, we must reject this and similar explanations for the dominant mechanism for variation in network demand. An explanation that is consistent with link proportionality might be that, on a low-demand rainy day, trips from all origins decrease proportionally because they reflect the spatial distribution of activity centres, which does not vary day to day.

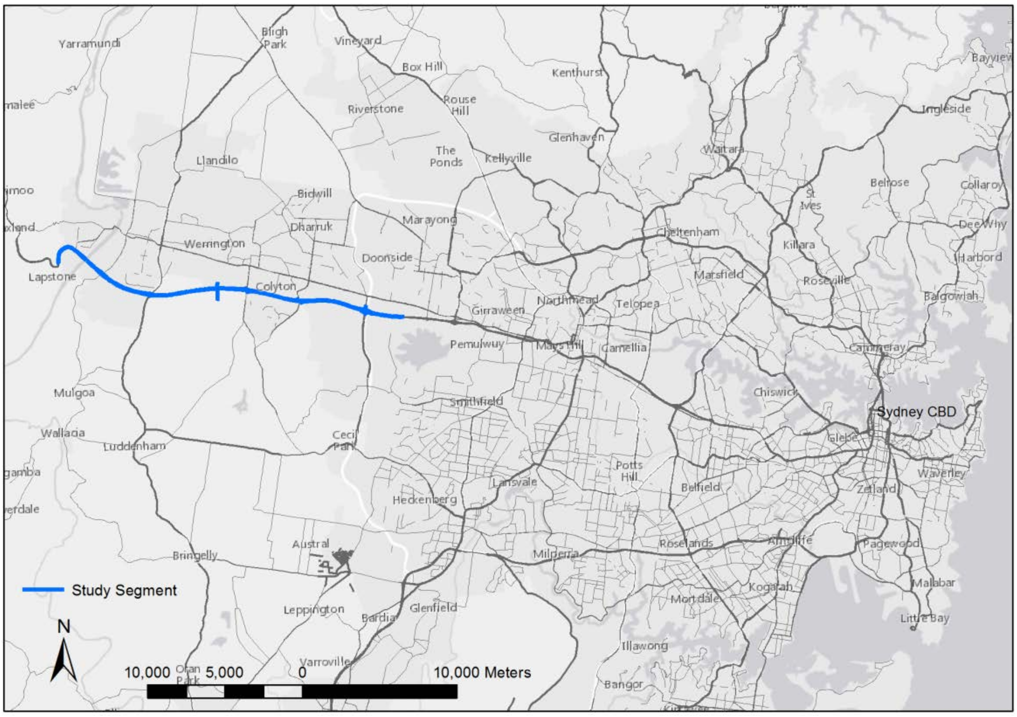

Figure 1 Map of Sydney showing the location of the study segment at the western end of the M4. A44 runs roughly parallel to the M4 slightly to the north.

2.1 Observed flow versus inferred demand

Most traffic assignment models can predict link volumes to be greater than the capacity, and therefore these volumes are interpreted as demand for using the link. Therefore, all analysis is undertaken considering the demand.

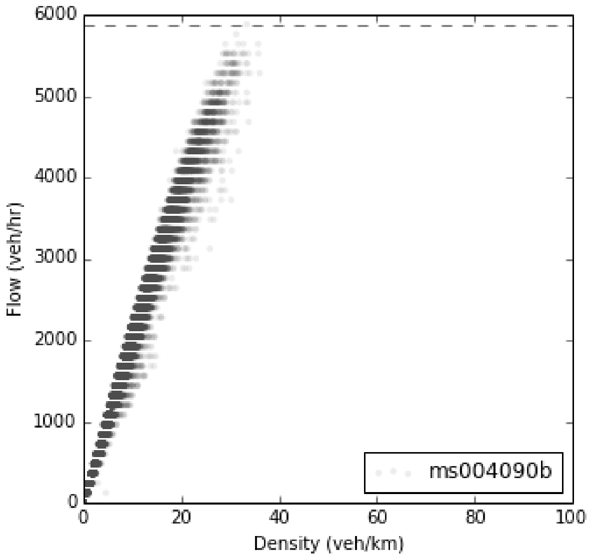

Figure 2 Observed fundamental diagram for the first upstream monitor site showing that conditions are always uncongested so flow and demand are equivalent. Maximum flow is shown with a dashed line.

Under totally uncongested conditions, the observed flows represent the demand at that location. However, when a link is congested or if congestion at an upstream bottleneck restricts traffic, the observed vehicle counts will underestimate the link demand. For consistency, demand on each link is estimated, regardless of congestion status, by summing flows from upstream links including mainlines and ramps. The calculation is initialized at the upstream end—the first monitor site, ms90b, is always uncongested (see Figure 2) so the observed flow is equal to the demand. Moving downstream, the demand at each detector is equal to the immediately upstream mainline demand plus immediately upstream on-ramp flow and minus any immediately upstream off-ramp flow where relevant. The schematic in Figure 3 shows how the monitor sites are positioned on one subsection and how demand can be inferred from upstream monitor sites. For example, demand at ms89b is equal to the flow observed at ms90b, but the demand at ms88b is equal to the demand at ms89b (*i.e.* the flow at ms90b) minus the flow observed at the ms88x off-ramp. All ramps are assumed to be uncongested. This calculation is essential in congested conditions where the observed flow is strictly less than the demand.

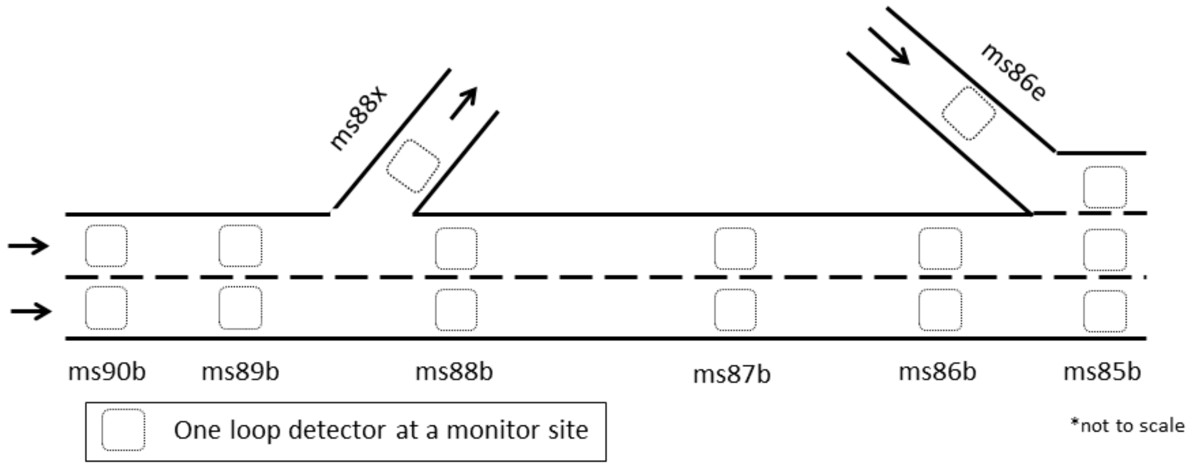

Figure 3 Schematic of a subsection of the study area showing how link demand can be estimated for each link (monitor site) by starting at the upstream end (ms90b) and letting demand for each link equal the sum of demand on the upstream link and any relevant ramp flows.

To estimate the link proportions, each estimate of the link demand needs to be paired with the corresponding network demand during that period. The network demand was calculated by summing the outflow demand along the entire corridor. The outflow includes measured flows on each off-ramp plus the estimated demand on the final downstream link (monitor site ms47b). Equivalently, total inflow could have been used as the proxy for network demand, but outflow was found to be more robust because off-ramps are not metered.

2.2 Method for quantifying and categorizing variability in demand

Variation in demand during the peak period is one shortcoming of static, deterministic traffic assignment modeling. One approach is to introduce time-dependent deterministic demand which accounts for fluctuations in typical demand levels during the modeling period. However, demand variation from day to day (for example, comparing demand across weekdays at 7AM) is an inherent trait of the network and can only be addressed with stochastic treatments of demand (*i.e.* where demand is represented as a random variable with a known distribution) such as the StrUE model. In this work, the importance of variable demand is tested by examining the change in demand within the peak period. The approach here is to categorize and quantify the contributions to demand variability in the peak period from time-of-day patterns versus day-to-day randomness. A stochastic-demand model is required if day-to-day effects are a significant component of the variability of demand.

The approach for testing the importance of day-to-day variability starts by estimating the typical variance in demand every 15 minutes across the 20 weekdays in February 2013. This gives the day-to-day variability by quantifying how much demand can be expected to change when looking at the most specific time interval in the data. In contrast, variability within the peak period measures how much the typical demand at each time interval varies during the morning peak period. Both types of demand variability contribute to the motivation for traffic assignment models that include uncertainty, but if day-to-day variation is significant, there is a strong argument in favor of models that incorporate demand distributions.

2.3 Method for testing link proportionality

The assumption of link proportionality can be rephrased as the characterization of the relationship between link and network demand for each monitor site. Proportionality implies that variation in link demand is completely determined by variation in network demand. This relationship is illustrated for each monitor site in the study segment in Figure 4—the linear relationship between link and network demand implies constant link proportionality. By fitting a linear, bivariate model to each location, it is possible to estimate the link-specific proportion (*i.e.* the coefficient of network demand, $\hat{\beta}_{ND}$) and the percent of variation in link demand that can be explained by changes in network demand (*i.e.* the coefficient of determination, R^2). The approach here is to determine the proportions for each link and show that the unexplained variation in link demand is insignificant.

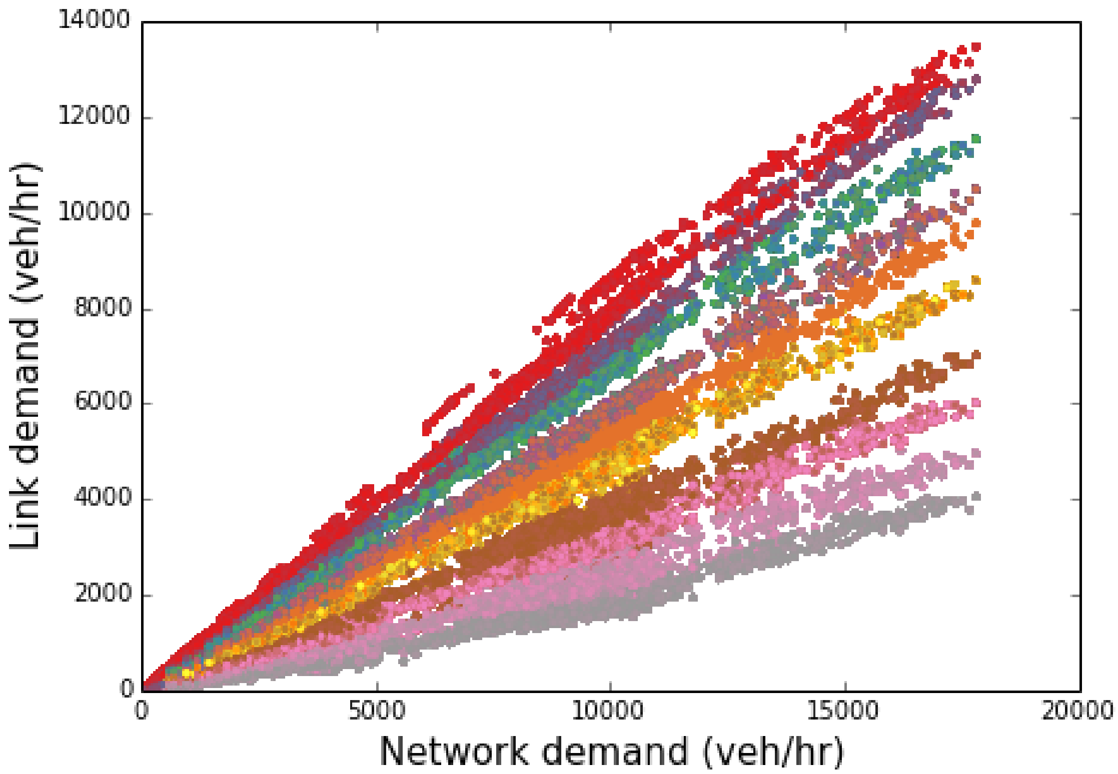

Figure 4 The relationship between link and network demand showing constant proportions (slopes) for each location (color)

3. RESULTS

3.1 Day-to-day variation in demand dominates time-of-day variation in demand

Validation of the proportionality assumption associated to variability of demand requires known link demands and total travel demand for the network (i.e. facility). However, link demand cannot be directly observed in congested conditions. For the purposes of this work, the method described in

Section 2 is used to estimate link and network demands based on the observed flows and known topology of the network.

A comparison of the observed flow and the inferred demand is illustrated in Figure 5 for monitor site 4067b which is located in the middle of the facility. Each dot in the figure represents an observed link flow at a given time on a given day. The flows are plotted in 15 minute increments over the entire day (24 hr. period) for each of the 28 days in February, 2013. The figure shows that the relationship between link flow and demand changes throughout the day, and demand is generally greater than or equal to flow. Prior to the morning peak period (red dots), the flow is closely related to demand. However, as the demand builds to capacity at morning rush hour, the demand deviates from (exceeds) the flow. Throughout the course of the day the demand decreases, and by the end of the day (blue dots), both flow and demand are low again.

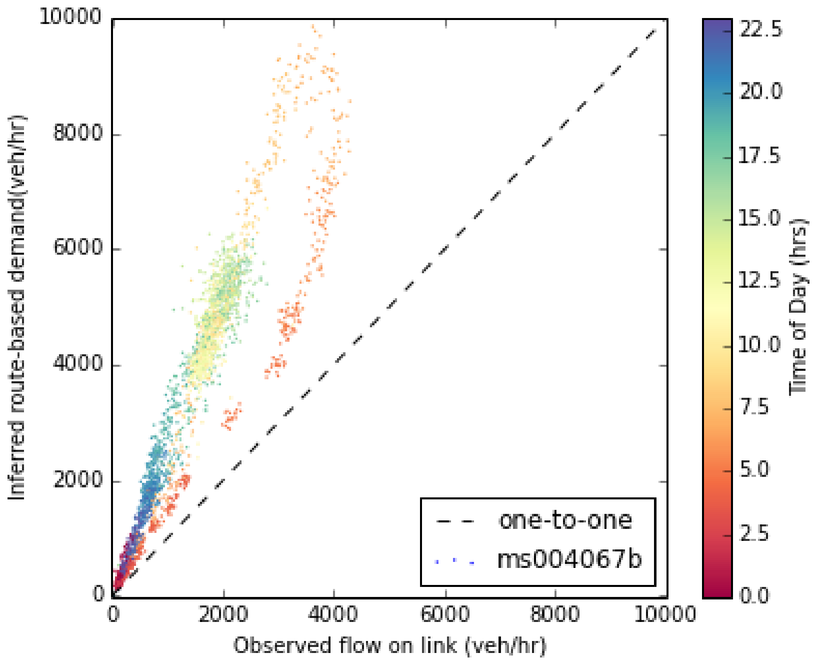

Figure 5 Comparison of flow and demand showing time-of-day patterns at monitor site 4067b

Figure 5 highlights the change in travel demand over the course of a single day. Of specific interest in this work is the day-to-day variability in travel demand, and the impact of that variability on traveler's route choice behavior. Figure 6 illustrates both the variability throughout the day and between days, further underscoring the importance of incorporating demand variability into traffic assignment models. The blue and orange lines correspond to two detector locations, at both the far end of the network (monitor site ms89b, reds) and nearer the CBD (monitor site ms50b, blues). Each line corresponds to a single day in February 2013, and the lighter color lines correspond to the weekend days. Over the course of the day, demand varies considerably with similar patterns visible at both ends of the facility. For example, the region bounded by dashed lines demonstrates how much demand changes within the morning peak period (6-9AM), in which many traffic assignment models assume that demand is constant. Additionally, a different pattern is seen for weekdays and weekend days, with obvious morning and evening peaks during the week, and higher midday demands which remain relatively stable during the weekend days. In the same figure the demand is also shown to vary significantly day-to-day. For example, even when limited to weekdays at a specific time (for example, 7:30am), demand can vary day to day by about 2000 vehicles per hour (up to ~20% of the peak-period demand). This day-to-day variability is especially evident in the more congested section of the facility (blue lines). These results confirm that deterministic models with constant demand are unlikely to reflect reality. As such, it is imperative to incorporate the impact of day-to-day variability into traffic assignment models.

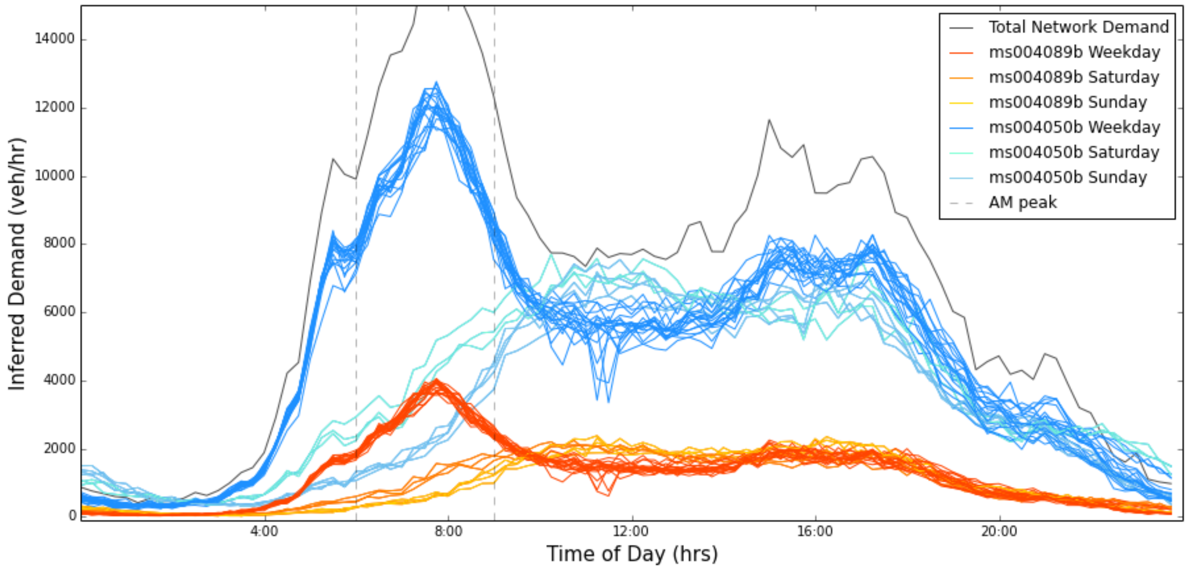

Figure 6 Demand at two locations in February 2013 showing weekend and weekday time-of-day patterns and day-to-day variations

In order to highlight the importance of day-to-day demand stochasticity in traffic assignment models, the variance in day-to-day demand is compared with the time-of-day variance during the peak period. Focusing on the weekday AM peak illustrated in Figure 6, the demand is measured 20 times (the 20 weekdays in February 2013 are each represented by a line in the figure) at 13 times of day within the morning peak period (every 15 min between 6-9AM inclusive). For each location, the day-to-day variability during the peak period is the mean of the N=13 time of day variances, σ_i^2, as defined in equation 1,

$$\sigma^2{}_{DtD} = \frac{1}{N-1}\sum_{i=1}^{N}\sigma_i^2. \qquad [1]$$

To measure time-of-day variability, the mean of each of the (N=13) groups of 20 (day-specific) demands, μ_i is calculated. The variance of the means, $\sigma^2{}_{ToD}$, quantifies the contribution to variability from the shape of the demand profile, and is defined in equation 2,

$$\sigma^2{}_{ToD} = \frac{1}{N-1}\sum_{i=1}^{N}(\mu_i - \bar{\mu})^2 \qquad [2]$$

The results in Table 1 show that day-to-day variance is higher than time-of-day variance at every single location. Note that most locations do not have unique variances because their demand is identical—demand only changes on consecutive links if vehicles are added or subtracted by a ramp. The ratio of day-to-day variance to time-of-day variance indicates that variation in demand is dominated by day-to-day randomness, and therefore time-specific models are not sufficient to accurately capture the realized variability in network performance.

Table 1 Relative sizes of day to day and time of day variances—day-to-day is always more important.

Monitor Site ID	Time of Day Variance	Day to Day Variance	DtD/ToD	Monitor Site ID	Time of Day Variance	Day to Day Variance	DtD/ToD
ms004089b	334217.8	433353.8	1.297	ms004067b	1496528.5	2611423.3	1.745
ms004088b	618345.6	623641.9	1.009	ms004066b	1390195.3	3156025.6	2.27
ms004087b	618345.6	623641.9	1.009	ms004065b	1390195.3	3156025.6	2.27
ms004086b	618345.6	623641.9	1.009	ms004064b	1390195.3	3156025.6	2.27
ms004085b	675331.5	1008184.9	1.493	ms004063b	1390195.3	3156025.6	2.27
ms004084b	675331.5	1008184.9	1.493	ms004062b	1390195.3	3156025.6	2.27
ms004083b	675331.5	1008184.9	1.493	ms004061b	1390195.3	3156025.6	2.27
ms004082b	675331.5	1008184.9	1.493	ms004060b	1390195.3	3156025.6	2.27
ms004081b	675331.5	1008184.9	1.493	ms004059b	1247652.9	4070605.6	3.263
ms004080b	970003.9	1299703.4	1.34	ms004058b	1247652.9	4070605.6	3.263
ms004079b	970003.9	1299703.4	1.34	ms004057b	1247652.9	4070605.6	3.263
ms004078b	958170.6	2088901.8	2.18	ms004056b	1247652.9	4070605.6	3.263
ms004077b	958170.6	2088901.8	2.18	ms004055b	1247652.9	4070605.6	3.263
ms004076b	958170.6	2088901.8	2.18	ms004054b	1247652.9	4070605.6	3.263
ms004075b	958170.6	2088901.8	2.18	ms004053b	1247652.9	4070605.6	3.263
ms004074b	958170.6	2088901.8	2.18	ms004052b	1247652.9	4070605.6	3.263
ms004073b	958170.6	2088901.8	2.18	ms004051b	1672950.1	4112956.8	2.459
ms004072b	958170.6	2088901.8	2.18	ms004050b	1672950.1	4112956.8	2.459
ms004071b	958170.6	2088901.8	2.18	ms004049b	1672950.1	4112956.8	2.459
ms004070b	958170.6	2088901.8	2.18	ms004048b	1651524.1	4089913.4	2.476
ms004069b	958170.6	2088901.8	2.18	ms004047b	1651524.1	4089913.4	2.476
ms004068b	958170.6	2088901.8	2.18	**Average**			**1.747682**

3.1 Link proportionality is constant by time of day

To address the deficiencies attributed to deterministic demand models, the StrUE model assumes an implicit relationship between link level demand and total network demand, in which the uncertainty in link travel time is purely a function of day-to-day travel demand variability. More precisely, variations in the link demand are analytically defined as a function of the variation in network demand. To validate this assumed relationship, link-level demand and network-level demand should display a linear relationship during a given period (typically morning peak). This hypothesis is tested below, and illustrated to hold true in Figure 7, Figure 8 and Table 2.

Breaking the data down by monitor site location, we can estimate link proportions using bivariate linear regression. The coefficient of variation, R^2, quantifies how much of the variation in link demand is determined by network demand. The result of this process is shown for two example locations in Figure 7. At monitor site 50b where demand is generally higher, the link ratio (slope of the fit) is fairly constant throughout the day and over 99% of the variation in link demand is explained by variation in network demand. Closer to the Blue Mountains where demand is lower (monitor site 89b) the link ratio is lower. The ratio shown in black is the overall fit throughout the day, but the morning observations show a steeper relationship than those in the afternoon. This location illustrates the relevance of time-of-day specific ratios.

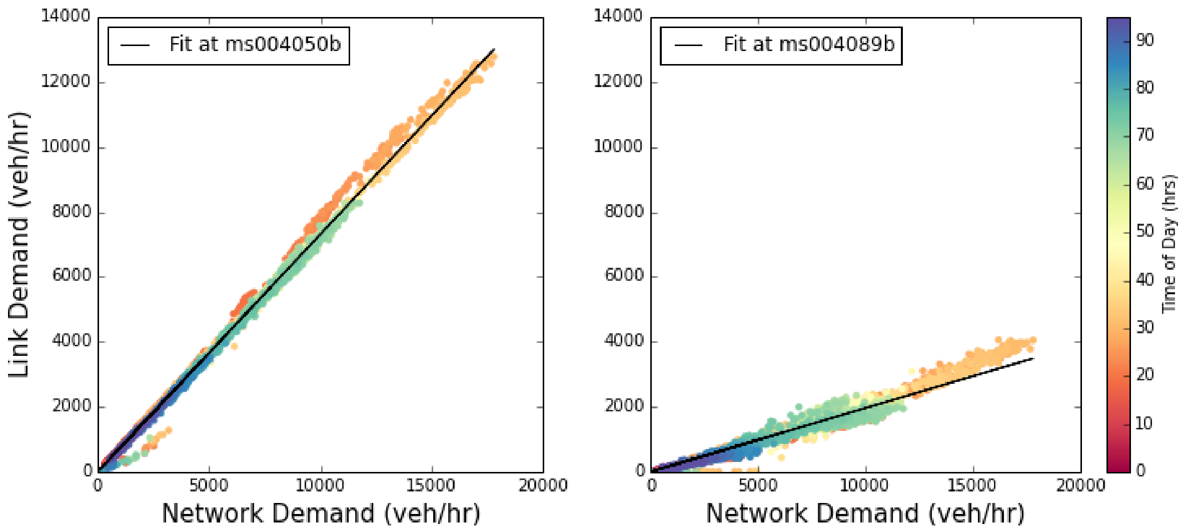

Figure 7 The relationship between link and network demand at two locations showing constant ratio fits and how the ratio changes with time of day

The relationship between link and network demand can also be visualized through the time of day pattern of their quotient. Figure 8 shows that the link proportions vary slightly over the day, with some patterns associated with the middle of the night and the morning peak. There is also a clear difference between weekend and weekday patterns. The horizontal dotted lines show the overall fitted value—even though the proportions vary around this line, at each time of day from 4:00AM to 8:00PM, the scatter is tight.

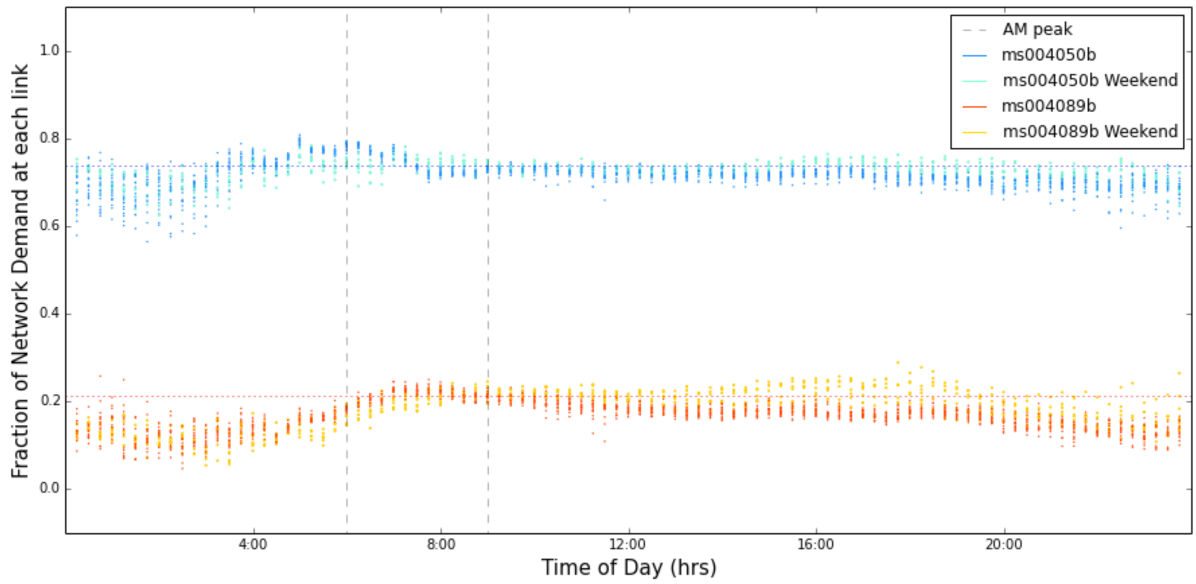

Figure 8 Link proportions by time of day for two locations showing how the relationship between link and network demand changes over the day.

In order to quantify the time-of-day variation in the link proportions and the completeness of the link-to-network-demand relationship, three separate linear models fit the ratios: all times of day, the 6-9AM morning peak and between 7:30-7:45AM. The results from these models are shown in Table 2—all of the models indicate that over 95% of the variation in link demand can be explained by changes in network demand. Locations near the Blue Mts (near 89b) have stronger changes in the link proportion over the day and more specific time windows tend to improve the coefficient of determination.

Table 2 Coefficients and R2 for the three models for each monitor site

Monitor Site ID	Overall model			6-9AM model			7:30AM model		
	$\widehat{\beta_{ND}}$	$\widehat{\beta_0}$	Adj. R^2	$\widehat{\beta_{ND}}$	$\widehat{\beta_0}$	Adj. R^2	$\widehat{\beta_{ND}}$	$\widehat{\beta_0}$	Adj. R^2
ms004089b	0.213	-152.2	0.960	0.238	-281.7	0.972	0.241	-263.4	0.989
ms004088b	0.269	-175.7	0.955	0.288	-348.4	0.949	0.284	-282.9	0.988
ms004087b	0.269	-175.7	0.955	0.288	-348.4	0.949	0.284	-282.9	0.988
ms004086b	0.269	-175.7	0.955	0.288	-348.4	0.949	0.284	-282.9	0.988
ms004085b	0.347	-160.1	0.975	0.359	-333.5	0.979	0.359	-337.8	0.991
ms004084b	0.347	-160.1	0.975	0.359	-333.5	0.979	0.359	-337.8	0.991
ms004083b	0.347	-160.1	0.975	0.359	-333.5	0.979	0.359	-337.8	0.991
ms004082b	0.347	-160.1	0.975	0.359	-333.5	0.979	0.359	-337.8	0.991
ms004081b	0.347	-160.1	0.975	0.359	-333.5	0.979	0.359	-337.8	0.991
ms004080b	0.395	-177.0	0.973	0.409	-412.8	0.978	0.411	-407.7	0.991
ms004079b	0.395	-177.0	0.973	0.409	-412.8	0.978	0.411	-407.7	0.991
ms004078b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004077b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004076b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004075b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004074b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004073b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004072b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004071b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004070b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004069b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004068b	0.497	-155.3	0.989	0.500	-240.6	0.987	0.499	-280.2	0.990
ms004067b	0.553	-169.1	0.990	0.568	-398.9	0.989	0.571	-380.3	0.990
ms004066b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004065b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004064b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004063b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004062b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004061b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004060b	0.603	-141.9	0.993	0.609	-251.8	0.989	0.615	-309.6	0.992
ms004059b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004058b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004057b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004056b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004055b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004054b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004053b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004052b	0.684	-58.9	0.994	0.671	-2.1	0.983	0.671	-139.1	0.992
ms004051b	0.739	-76.6	0.997	0.740	0.2	0.992	0.741	-101.0	0.998
ms004050b	0.739	-76.6	0.997	0.740	0.2	0.992	0.741	-101.0	0.998
ms004049b	0.739	-76.6	0.997	0.740	0.2	0.992	0.741	-101.0	0.998
ms004048b	0.785	71.9	0.996	0.762	302.0	0.991	0.764	151.3	0.999
ms004047b	0.785	71.9	0.996	0.762	302.0	0.991	0.764	151.3	0.999

4. DISCUSSION

Variability is important because it has inherent value and it contributes to the discourse on network modelling equilibrium. A complex array of sources of variability (demand, supply, route and destination choice) and type (time of day patterns versus day-to-day random fluctuations) has resulted in a multitude of modelling approaches. Certain newer approaches explicitly acknowledge day-to-day randomness in the network (probabilistic user equilibrium, strategic user equilibrium). The implicit assumption made by these models is that the realized conditions on each link vary proportionally with network-wide variation. The foundation for this assumption is that travellers behave strategically, *i.e.* their route and destination choice will remain constant regardless of the travel decisions of other network users. This collection of behaviours represents a time-of-day strategy that stays constant over different total demand realizations. The Strategic User Equilibrium (StrUE) model operationalizes this assumption by maintaining link proportionality while varying overall demand.

This work uses observed network conditions for a motorway in Sydney, Australia to demonstrate both the importance of incorporating day-to-day variability in travel models and validate the assumed link proportionality. The results indicate that both day-to-day and time-of-day demand vary significantly, which contributes to the unrealistic results of deterministic user equilibrium models. The analysis also illustrates that day-to-day variation in demand is more significant than time-of-day variation within the peak period, suggesting that time-specific models are not sufficient for modelling purposes— day-to-day demand must be treated stochastically in order to accurately model the behaviour of the network.

After establishing the significance of day-to-day demand variability, the validity of link proportionality (*i.e.* the constant ratio of link demand to network demand) is tested. The relationship between link flow and total network flow is modelled at 44 locations for the entire month. A bivariate linear regression allows us to quantify the value of the link proportion (the coefficient of network demand, $\hat{\beta}_{ND}$) and the amount of link variation that can be explained by variation in the network demand (the coefficient of determination, R^2). The data indicate that the link proportionality assumption is valid, especially when considering specific times of day. At 7:30AM, variation in network demand explains at least 99% of the variation in link demand. Even aggregating across an entire day, the link proportionality assumption leaves less than 5% of the variation in link demand unexplained. These findings help to validate the assumptions of link proportionality in StrUE, PUE and related models. However, there still needs to be significant effort undertaken to evaluate which assignment framework (PUE or StrUE) best replicate the distribution of the proportion of traffic on links.

These findings open new avenues of research into demand reliability in transport networks. The assumptions tested above should be validated on more complex networks. Testing link proportionality in a network with data from multiple routes between an origin-destination pair would provide stronger evidence for or against these assumptions. The same data source used here could also be adapted to validate the results rather than the assumptions of a day-to-day stochastic-demand traffic assignment model such as StrUE. In particular, we wish to confirm that StrUE's predicted link flows and travel times better match reality than their deterministic user equilibrium (DUE) counterparts. A fully validated, stochastic-demand model will provide a valuable tool for transportation planners.

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