A Bulk Queue Model for the Evaluation of Impact of Headway Variations and Passenger Waiting Behavior on Public Transit Performance

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***Abstract*—This paper demonstrates a model developed using the Markov chain technique, to ascertain the performance of public transit systems and examine the effects of stochastic variations in passenger arrival, waiting, boarding, and alighting behaviors on the regularity of headway along the route. The model addresses situations in which passengers abandon the system after a certain amount of waiting time. This accounts for the existence of a finite allowance of waiting time from the viewpoint of the passengers. The numerical examples included offer insights into factors that affect the reliability of public transit systems and presented analysis of the system performance measures such as mean counts of passengers served by transit systems, abandoned passengers, and unused space on vehicles. The impact of variability of departure headway on the utilization of public transit systems is illustrated. This model can be used as an analysis tool by transit planners to evaluate selection of system attributes.**

***Index Terms*—Headway variability, passenger waiting behavior, stochastic model, vehicle sizes.**

I. Introduction

User surveys reveal that reliability is an important attribute to passengers [1]–[4]. Reliability is also seen as a governing factor in selection of transport modes by users [5], [6]. In a survey conducted by the Independent Transport Safety and Reliability Regulator in Sydney in 2009 [7], 88% of the responses indicated that "*buses keeping to timetable*" is important or very important. Bus service punctuality was ranked first and third out of 30 attributes by rail passengers in a national survey in the United Kingdom conducted by the MAV consulting company in 2005 and 2006, respectively [8]. Thus, timetable reliability of public transport is the most important determinant of overall customer satisfaction [9].

Irregularities in bus headway discourage commuters to use public transit [6], [10]. In [11] and [12]; it was demonstrated that passengers are prone to change their transport mode as a result of changes in level of service reliability. Table I shows that a decrease in reliability adversely affects passengers' demand for public transit usage; in particular, occasional travelers appeared to be very susceptible with reliability.

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TABLE I
Effect of Changes on Mode Choice (After VRIJE Universiteit [11] and Van Oort [12])

Changes in reliability	Changes in passenger demand('+' and '- ' sign indicate increase and decrease in demand respectively)		
	Regular travellers	Occasional travellers	Non travellers
Increase in reliability (decrease in standard deviation of travel time)	(+) 9 %	(+) 22 %	(+) 9 %
Decrease in reliability (increase in standard deviation of travel time)	(-) 17 %	(-) 44%	-

Unreliability imposes uncertainty on the amount of time passengers need to wait at a transit stop for a vehicle. Behavioral scientists have found that there is a large impact on consumer attitudes on the value of time in which they need to wait to receive service with uncertainty since time is nontransferable, nonstorable, and nonperishable in nature. Leclerc *et al.* [13] suggested that the marginal value of waiting time is higher in the context of a short waiting time than a long one. When risk is involved in the outcome following a decision, consumers were found to be risk averse. In a normative theoretical analysis, Osuna [14] stated that uncertainty in waiting is a key cause of psychological pressure on consumers. It was shown that this psychological pressure begins to generate at the start of waiting, and is a result of the feeling of time being wasted and the uncertainty associated about the remaining waiting time. Therefore, passenger waiting time is an important criterion to assess the performance of public transport systems. The hypothesis adopted in this paper is that the addition of the duration of trip time, the variation of trip time, and the decrease in comfort (i.e., reduced probability of finding a seat on vehicle as it becomes crowded) caused by service unreliability may lead a passenger to respond in one of two ways: 1) continue to wait or 2) give up waiting for the bus either to find a different transportation mode or abandon the trip. Such behavior to abandon modes or trips will adversely affect productivity and consumer relations, and will ultimately lead to economic loss to transit operators through underutilization of vehicles, equipment, and workforce.

When bus services are frequent and passenger arrivals at bus stops are regular, passenger occupancy can fluctuate with the variations in bus headways. Irregular headways lead to uneven passenger occupancy on buses. In addition, variations in passenger counts result in some buses becoming full and being unable to serve certain stops. Bus capacity constraints lead to passengers being denied boarding when the bus is full. Rejected passengers need to wait for next available bus, and such a process results in elevation of demand for subsequent bus. Moreover, variability of bus dispatch headway amplifies this effect along the route. This paper develops an analytical tool to evaluate the impacts of passenger waiting behavior at transit stops on transit performance, particularly when passengers do not wait for more than a certain time threshold.

The remainder of this paper is organized as follows. The literature review presented in Section II. The modeling framework is described in Section III. Performance measures that are used for evaluation are presented in Section IV. The model is illustrated with a numerical example in Section V. Section VI, the final section, offers the conclusion.

II. Literature Review

There are several studies that have developed quantitative measures to evaluate reliability. Polus [10] proposed the inverse of standard deviation of travel time of an arterial route as a measure of reliability. Transport for London used *excess wait time* (EWT) as an indicator of service reliability from the perspective of passengers for high-frequency routes that operate on headway control. EWT is defined as the difference between average waiting time (AWT) that passengers actually wait for and the AWT based on the schedule (SWT) [15]. For high-frequency services, SWT is taken as half of the headway. The AWT is the waiting time of accumulated passengers that can be estimated by conducting surveys on actual arrival time of buses at stops. Mathematically, EWT = AWT − SWT.

Lin and Ruan [16] proposed a probability-based headway regularity metric as a measure of service reliability at stops. This measure was developed as a function of passenger activities (i.e., arrival, boarding, and alighting), dwell time at stops, number of stops, and expected headway of buses. The developed reliability measure was applied using automatic vehicle location data from the Chicago Transit Authority (CTA).

Trompet *et al.* [17] tested and described the strengths and weaknesses of four alternative performance indicators to compare variation of service regularity of high-frequency routes between urban transit operators. These four indicators are: 1) EWT; 2) standard deviation of difference between actual and scheduled headway; 3) wait assessment, which is ±2 min of scheduled headway; and 4) service regularity, which is ±20% of scheduled headway. They evaluated these indicators, and EWT was found to reflect customer perspective as it showed the average experience of all passengers in the data set. It was reported that, in comparison with EWT, the indicator that uses the difference of standard deviation between actual and scheduled headway reflected the experience of only 68% customers.

To analyze the performance of transit systems, a technique widely used in queueing theory, namely Markov chain, is used in this paper. In queueing theory, serving more than one customer at a time is known as a bulk service. In transport systems, an analogy for customers and servers can be made with passengers and vehicles, respectively. Bailey [18] was one of the first to conduct a mathematical investigation of bulk queues formed by random arrival of passengers served in batches under a fixed vehicle capacity. Jaiswal [19] extended Bailey's model for a series of stops where the number of passengers served was different. Using embedded Markov chain and semi-regenerative technique, Tadj and Ke [20] examined control policy for bulk-service queue system where the server starts to work only when the waiting count is larger than some predefined number. Tadj *et al.* [21] and Tadj *et al.* [22] developed a bulk queue model considering that customers leave the system once they fail to receive service due to limited capacity. However, these models did not deal with the situation where passengers abandon their interest to receive transit service after waiting for some random amount of time and become unwilling to wait further.

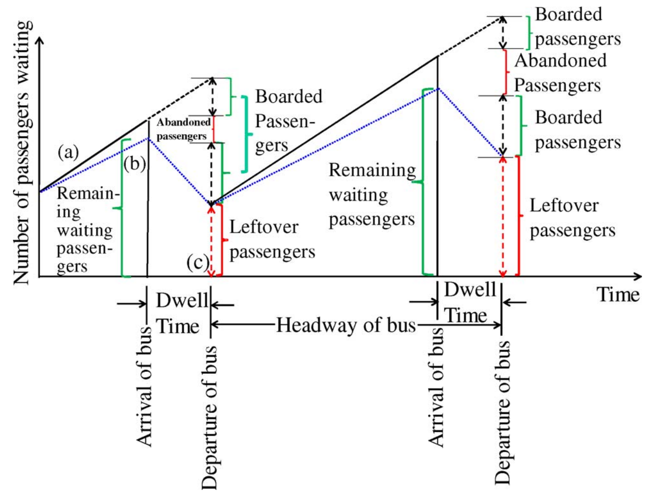

Fig. 1. Schematic presentation of the dynamic interaction between a bus and passengers at stops.

This paper describes a probabilistic model developed to capture the impacts of passengers waiting behavior on public transit systems adopting the Markov chain technique. This paper also provides a numerical application for the analytical model developed in this paper, which is then illustrated using an example.

III. Modeling Framework

The various components of the model, the theory, and the procedure followed to evaluate performances of bus transit systems in a probabilistic setting are presented here.

A. Dynamic Interaction of Buses and Passengers at Stops

The dynamic operation of bus transit in a route that contains multiple stops is illustrated with the aid of a schematic shown in Fig. 1. Vuchic [23] presented a diagram for a deterministic setting with different selection of axes. In bus transit services, the overall travel time of passengers depends on: 1) dwelling time at stop for serving passengers; and 2) travel time along the route. From the point of view of a bus, empty buses dispatched from the dispatch station and traveled along the route, allowing passengers to alight and board at stops. This process of serving, boarding, and alighting passengers continues stop after stop. As shown in Fig. 1, there are three events that relate to passengers. These events are: a) passenger arriving at the bus stops; b) passengers boarding the bus if space is available upon arrival of bus at the stop; and c) bus departing with passengers for the next stop. Moreover, passengers wait after they join the queue for the arrival of the bus at the stop, and they abandon waiting once a certain waiting time budget is spent. This behavior is shown as a certain withdrawal rate, removing passengers from the waiting queue. This withdrawal process can occur before or after the bus arrival depending on an individual passenger waiting time budget. The schematic also indicates that it is possible for some passengers to be left behind at the stop when the bus has insufficient space to accommodate all waiting passengers. Before formalizing the theoretical framework to evaluate reliability of bus transit, the model assumptions are stated.

B. Assumptions

The model assumptions are as follows.

1) Operational:

- Bus size: Buses dispatched in this service are assumed of the same size.
- Service process: Buses serve passengers up to its passenger capacity after passengers alight from the bus at a given stop. Buses skip a stop if there is no passenger boarding at the stop.
- Capacity of stop: The capacity of the bus stop to accumulate arriving passengers is assumed infinite.
- Headway of bus: Bus headway, as well as dwell time, depends on the number of alighting and boarding passengers. Bus headway is assumed to follow gamma distribution. This assumption is based on findings by Lin and Ruan [16], who used bus data from the CTA to show that the headway followed a gamma distribution. In addition, the bus headways are assumed less than 10–12 min to justify adoption of random properties for passenger arrivals at bus stops since earlier studies have found that the passenger arrival rate is random if the headways are lower than 10–12 min.
- Number of alighting passengers: No passenger disembarks from the bus at the first stop as the bus arrives empty at the first stop. At other stops except the last one, the number of alighting passengers depends on the arrival occupancy. The passenger alighting probability is assumed the same during the period of study. Passenger behavior is assumed independent of each other. Thus, the number of alighting passengers at each stop is assumed to follow a binomial distribution, as suggested by Andersson and Scalia-Tomba [24]. At the last stop, all onboard passengers alight to ensure that the bus is empty. However, it should be noted that alighting behavior can be influenced by the capacity constraint of buses and the abandon mechanism through changes in the origin–destination (OD) pattern. For example, if there is not enough space available on the bus to accommodate passengers at the origin stop for a particular destination stop, as a consequence, there will be no alighting at the destination stop.
- Alighting and boarding process: It is assumed that boarding of passengers on the bus starts after the completion of alighting of passengers. A fraction of onboard passengers alight upon arrival at a stop, the bus then picks up waiting passengers until available space is filled.
- Travel time: Travel time between two stops is assumed to have a random distribution with a specified mean and variance. There is no condition ensuring that buses do not overtake each other. It is assumed that the travel-time variance reflects the stochastic elements of traffic flow on roads, such as congestion effects and driver behavior.
- Boarding and alighting times: The boarding and alighting time for each passenger is assumed as constant.

2) Demand:

- Arrival of passengers: It is assumed that passengers arrive randomly at a stop according to a Poisson process. The number of passengers waiting is a function of the arrival rate of passengers at that stop and the time interval between two consecutive arrivals of buses. Furthermore, arrival of passengers at one stop is independent from arrivals at any other stop. It is presumed that passenger demand does not change over the period of study.
- Passenger waiting behavior: The assumption made in this paper is that passengers abandon waiting for the bus if they do not receive service before a threshold waiting time. The abandonment of passengers is assumed to follow a Poisson process with a mean time equivalent to what they are willing to wait (MTWW) before deciding to leave the stop. Hence, passengers who left the stop without being served are considered lost from the system. It is worth to mention that the expected number of passengers abandoning their trips during a certain time interval at a stop does not depend on the rate of arrival of passengers at that very stop.

This kind of model approximates behavior where people have a low tolerance to waiting. Passengers who look for an alternative mode of transportation to be on time can be considered impatient passengers.

C. Notations

The following notations have been adopted for the formulations:

N	total number of stops along the route;
n	index number of stops to be served;
λ^n	passenger arrival rate at stop n;
D^n	dwell time of bus at stop n;
L^n	occupancy of bus departing stop n;
H^n	headway of bus at stop n (i.e., time between two consecutive buses at stop n);
A^{*n}	number of alighted passengers at stop n;
B^{*n}	number of boarded passengers at stop n;
b_A	alighting time for each passenger;
b_B	boarding time for each passenger;
C	bus size in terms of seats and standing passengers allowed;
d^n	alighting proportion of passengers at stop n;
τ^n	mean time that passengers are willing to wait at stop n;

x_k	$\{0, 1, 2 \ldots, C\}$, i.e., number of passengers in the bus at stop n;
p^n	probability vector for the number of onboard passengers at the instant of departure from a stop n after boarding;
$p^{n'}$	probability vector for the number of passengers remaining on board on the bus after alighting at stop n;
s^n	probability vector of space available on the bus at the instant the boarding process starts at stop n;
q^n	probability vector of number of passenger waiting for bus at stop n;
ρ^n	demand intensity at stop n;
W^n	expected value of passenger waiting time at stop n;
$\text{Var}R^n$	variance of bus travel time between stops n and $n-1$;
CVH^n	coefficient of variation of headway of bus at stop n;
$\text{Var}H^n$	variance of bus headway at stop n;
$\text{Var}L^n$	variance of bus occupancy at stop n;
$\text{Covar}H_i^n L_i^n$	covariance of headway and occupancy of buses at stop n;
$\text{Covar}H_i^n L_{I-1}^n$	covariance of headway and occupancy of buses at stop n with respect to the previous bus;
$\text{Covar}L_I^n L_{i-1}^n$	covariance of occupancy of buses at stop n with respect to the previous bus;
$\text{Covar}H_I^n H_{i-1}^n$	covariance of headway at stop n with respect to the previous bus.

D. Stochastic Model for Public Transit Operations

The number of passengers waiting at a stop can be considered a random/stochastic sequence that can be modeled using an embedded Markov chain. This approach can determine the probability of the number of passengers waiting at a stop in steady-state conditions. Due to the stochastic nature of the passengers alighting at each stop, the number of passengers and the space available on the bus are random variable. When the alighting process finishes at a stop, the boarding process begins. The bus can only accept passengers until it has available space. The passengers board based on the first-in–first-out rule. There are four sets of probabilities required to describe the model related to transit route operation. These probabilities are listed as follows.

Probability vector at a stop representing the number of:

1) onboard passengers after the alighting process has completed;
2) remaining capacity in the bus at the instant the boarding process begins;
3) passengers waiting to board when the bus arrives;
4) onboard passengers after the boarding process has completed.

Details of these vectors and the various steps involved in the proposed model are described in the following.

a) Probability vector representing the number of onboard passengers at a stop after the alighting process has completed: When a bus reaches a stop carrying passengers from previous stops, some onboard passengers may intend to alight at the stop. On the other hand, other passengers remain on board as they have not yet reached their destinations. This process is modeled as a Markov chain, which results in the following propositions.

The transition probability matrix representing the transition of the number of onboard passengers on the bus during the alighting process upon arrival at a stop n is represented by $A^n = [a_{ij}^n]$, which is a $(C+1) \times (C+1)$ matrix. Each element in A^n is represented by a_{ij}^n that denotes the conditional probability that j number of passengers will remain on board at stop n given that the bus arrived from an earlier stop (n-1) to the present stop n with i number of onboard passengers. In other words, a_{ij}^n is the conditional probability that j passengers remain on board after alighting of $(i-j)$ number of passengers at stop n. This is the cell entry for row i, column j in matrix A^n. Hence, the following proposition can be proposed.

Proposition 1: The transition probabilities of matrix A^n can be given by

$$a_{ij}^n = \begin{cases} 1, & \text{for } i = 0, j = 0 \\ \binom{i}{i-j}(d^n)^{i-j}(1-d^n)^j & \text{for } i \geq j \\ 0, & \text{for } i < j. \end{cases}$$

The derivation for which is provided in Appendix A.

The probability vector for the number of onboard passengers at the instant of departure from a stop n after boarding is given by vector $p^n = (p_0^n, p_1^n, p_2^n, \ldots, p_C^n)$. The probability vector for the number of passengers remaining on board at the end of completion of the alighting process at stop n is given by $p^{n'} = p^{n-1}.A^n$. The derivation of $p^{n'}$ is trivial and is carried out using the Chapman–Kolmogorov equations [25].

b) Probability vector of space available on the bus at the instant the boarding process starts: Once it is possible to calculate vector p^n, the probability vector of space available on the bus at the instant the boarding process starts can be derived using vector $s^n = (s_0^n, s_1^n, s_2^n, \ldots, s_C^n)$, where s_i^n represents the probability that i spaces are available at the instant of starting the passenger boarding process. The element of vector S^n is given as

$$s_i^n = p_{C-i}^{n'} \qquad \forall\, i.$$

The derivation of vector s^n is shown in Appendix B.

c) Probability vector of number of passengers waiting to board on the bus at stops: The probability vector of the number of passengers waiting to board on a bus at stops can be determined using Proposition 2. This proposition proposes a probability-generating function associated with the queueing process for passengers waiting at a stop n after the alighting process has finished and the boarding process starts.

The probability vector of the number of passenger waiting for bus at stop n is given by the vector $q^n = (q_0^n, q_1^n, q_2^n, \ldots, q_\infty^n)$, where q_j^n represents the probability that j number of passenger waiting for bus at stop n. Here, j could be any number between zero to infinity. In order to follow the procedure to evaluate steady-state vectors mentioned in Section III-G, it is sufficient to determine $C+1$ number of unknown probabilities, i.e., $q_0^n, q_1^n, q_2^n, \ldots, q_C^n$ of vector q^n. To determine these unknown probabilities, a polynomial equation in a complex plane was derived. Hence, the following proposition is proposed.

Proposition 2: The probability-generating function associated with the probability vector of the number of passenger waiting for bus at stop n is represented by $Q(z)$ and is given by

$$Q(z) = \frac{\sum_{r=0}^{C} s_r H(z) \sum_{i=0}^{r-1} q_i \left[\sum_{k=0}^{i} G_i(z) z^C - z^{C-r+i}\right]}{z^C - \sum_{r=1}^{C} s_r H(z) z^{C-r}} \quad (1)$$

where $G_i(z)$ represents the probability-generating function of abandoned passengers and given by

$$G_i(z) = \sum_{k=0}^{i} \frac{\left(\frac{H(z)}{\tau}\right)^k e^{-\frac{H}{\tau}}}{k!}$$

and $H(z)$ represents the probability-generating function of headway distribution at a stop and is given by

$$H(z) = \frac{1}{\left[1 + (CVH)^2 H\left(\lambda - \frac{1}{\tau}\right)(1-z)\right]^{(CVH)^2}}.$$

The derivation of (1) is shown in Appendix C. The denominator of (1) has C number of roots, where C is the capacity of the bus, and the coefficient of variation of bus headways (CVH) is a necessary input to determine these roots. One of these roots is unity. The rest of $C-1$ roots are nonunity. Using these $C-1$ roots in the numerator, $C-1$ number of linear equations can be found. One more linear equation can be found using the normalizing condition $Q(1) = 1$. By solving this C number of linear equations, unknown probabilities, i.e., $q_0^n, q_1^n, q_2^n, \ldots, q_C^n$, can be determined.

d) Probability vector of onboard passengers after the boarding process has completed: After evaluating unknown probabilities q_0^n, q_1^n, $q_2^n, \ldots, q_C^n$, it is possible to evaluate the probability vector of the onboard passengers when the boarding process has completed. Proposition 3 proposes a transition probability matrix associated with passengers boarding on the bus at a stop n after the alighting process has completed.

The transition probability matrix associated with the boarding passenger counts for a bus at stop n is represented by $B^n = [b_{kl}^n]$, which is a $(C+1) \times (C+1)$ matrix. Each element in B^n is represented by b_{kl}^n, which denotes the conditional probability that $(l-k)$ new passengers will board the bus given that the bus already has k passengers on board after the alighting process has finished. This is the cell entry for row k, column l of the matrix B^n. Hence, the following proposition can be proposed.

Proposition 3: The transition probabilities of matrix B^n are shown as

$$b_{kl}^n = \begin{cases} q_{l-k}^n, & \text{for } 0 \le k \le l < C \\ 1 - \sum_{l=0}^{C-1} q_{l-k}^n & \text{for } l = C \text{ and } 0 \le k < C \\ 0 & \text{for } l < k < C \\ 1 & \text{for } l = k = C. \end{cases}$$

The derivation for which is provided in Appendix D.

Given that the probability vector for the number of passengers remaining on board on the bus after alighting at stop n is given by $p^{n'}$, the probability vector of the number of onboard passengers at the instant of departure from a stop n is given by

$$p^n = p^{n'} \cdot B^n = p^{n-1} \cdot A^n \cdot B^n.$$

The derivation for this is based on the Chapman–Kolmogorov equations [25]. The probability vector $p^{n'}$ at a stop n can be computed using Proposition 1.

E. Stability Condition

To ensure stability, the maximum of the mean number of passengers that can be served at a stop during a visit of a bus can be given by the mean of space available on the bus at the start of boarding process, i.e.,

$$\overline{C}^n = \sum_{i=0}^{C} i s_i^n \quad \text{for} \quad i = 0, 1, 2, \ldots, C. \quad (2)$$

Therefore, for a stable system, the number of arrivals at a stop should satisfy the following condition:

$$\rho^n = \frac{(\lambda^n - 1/\tau^n) H^n}{\overline{C^n}} \le 1 \quad (3)$$

where ρ^n can be termed as demand intensity at a stop n. It can be shown using (3) that demand intensity a stop n is inversely proportional to the mean value of space available on the bus at the instant the passenger boarding begins and directly proportional to the arrival rate of passengers and headway of bus at a stop n. This term reflects the demand for transit service at a stop in relation to supply.

F. Model for Headway and Variance of Occupancy Level for Limited Bus Size

As mentioned in Section III-D, the CVH is an input to evaluate (1). Therefore, it is necessary to find the headway variation at stops. To determine the headway variation at bus stops, the mode developed by Hickman [26] for an unlimited capacity of a bus was modified to incorporate capacity constraints. As mentioned in Section III, the total travel time of the bus along a route is the sum of: 1) dwelling time at stops; and 2) the travel time between stops. Dwelling time is the sum of time spent to serve passengers for alighting and boarding at stops. Thus, dwelling time of buses at stops is influenced by the demand for services at the stops and hence affects the commercial speed of buses. Mathematically, following a similar derivation process of Hickman [26] and Marguier [27], for a particular bus i at stop n, this can be represented as

$$D_i^n = b_A A_i^{*n} + b_B B_i^{*n}. \quad (4)$$

The number of alighting passengers and boarding passengers can be calculated by

$$A_i^{*n} = d^n L_i^{n-1} \quad (5)$$

$$B_i^{*n} = (\lambda^n - 1/\tau^n) H_i^{n-1}. \quad (6)$$

The expected headway with a particular bus i at stop n is

$$H_i^n = H_i^{n-1} + D_i^n - D_{i-1}^n. \tag{7}$$

The expected occupancy for a particular bus i at stop n is

$$L_i^n = L_i^{n-1} - A_i^{*n} + B_i^{*n}. \tag{8}$$

The system dynamics of headway and occupancy can be succinctly expressed in the matrix form as follows:

$$M_i^n = F^n M_i^{n-1} + G^n M_{i-1}^{n-1} \tag{9}$$

where

$$M_i^n = \begin{bmatrix} H_i^n \\ L_i^n \end{bmatrix}$$
$$F^n = \begin{bmatrix} 1 + b_B(\lambda^n - 1/\tau^n) & b_A d^n \\ (\lambda^n - 1/\tau^n) & 1 - d^n \end{bmatrix}$$
$$G^n = \begin{bmatrix} -b_B(\lambda^n - 1/\tau^n) & -b_A d^n \\ 0 & 0 \end{bmatrix}.$$

The passage of the bus along the route is represented by matrices F^n and G^n. To incorporate the capacity constraint in the model developed by Hickman [26], (9) was extended to derive the system dynamics for headway variance and occupancy variance, which is given as

$$\begin{aligned} V_i^n =& 2F^n S^n L^n + 2G^n S^n N^n - F^n S^n N^n \\ & - (F^n S^n)^T N^n + F^n V_i^{n-1} X^n + G^n V_{i-1}^{n-1} B'^n \\ & + F^n W_i^{n-1} U^n + G^n W_{i-1}^{n-1} A'^n + F^n Q_i^{n-1} Z^n \\ & + E^n Q_i^{n-1} Z^n + \overline{F^n M_i}^{n-1} D'^n + \overline{G}^n \overline{M}_{i-1}^{n-1} D'^n \end{aligned} \tag{10}$$

where

$$Q_i^n = \begin{bmatrix} \text{Covar} H_i^n H_{i-1}^n & \text{Covar} H_i^n L_{i-1}^n \\ \text{Covar} H_{i-1}^n L_i^n & \text{Covar} L_i^n L_{i-1}^n \end{bmatrix}$$
$$\overline{F}_j = \begin{bmatrix} b_B(\lambda^n / - 1/\tau^n) & -b_A d^n(1 - d^n) \\ (\lambda^n - 1/\tau^n) & d^n(1 - d^n) \end{bmatrix}$$
$$\overline{G}^n = \begin{bmatrix} b_B(\lambda^n - 1/\tau^n) & -b_A d^n(1 - d^n) \\ 0 & 0 \end{bmatrix}$$
$$L'^n = \begin{bmatrix} 1 + b_B(\lambda^n - 1/\tau^n) \\ (\lambda^n - 1/\tau^n) \end{bmatrix}$$
$$E^n = \begin{bmatrix} 1 + b_B(\lambda^n - 1/\tau^n) & b_A d^n \\ 0 & 0 \end{bmatrix}$$
$$\overline{M}_i^n = \begin{bmatrix} H_i^n & 0 \\ 0 & L_i^n \end{bmatrix} \quad V_i^n = \begin{bmatrix} \text{Var } H_i^n \\ \text{Covar} H_i^n L_i^n \end{bmatrix}$$
$$N^n = \begin{bmatrix} -b_B(\lambda^n - 1/\tau^n) \\ (\lambda^n - 1/\tau^n) \end{bmatrix} \quad D'^n = \begin{bmatrix} b_B \\ -b_A \end{bmatrix}$$
$$Z^n = \begin{bmatrix} -b_B(\lambda^n - 1/\tau^n) \\ -b_A d^n \end{bmatrix} \quad W_i^j = \begin{bmatrix} \text{Covar } H_i^n L_i^n \\ \text{Var } L_i^n \end{bmatrix}$$
$$S^n = \begin{bmatrix} \text{Var} R^n & 0 \\ 0 & 0 \end{bmatrix} \quad U^n = [b_A \; d^n]$$
$$A'^n = [-b_A \; d^n] \quad B'^n = [-b_B(\lambda^n - 1/\tau^n)]$$
$$X^n = [1 + b_B(\lambda^n - 1/\tau^n)] \quad H'^n = [b_A d^n \; (1 - d^n)].$$

The derivations for these are similar to that provided by Hickman [26] and Marguire [27] and are shown in Appendix E. The effect of travel-time variance on the headway variance is represented by the first four terms of (10). It is expected that high variability in travel time would lead to high variances in the headways. The Fifth to eighth terms represent the propagation of variability along the route due to variances of headway and occupancy at the previous stop. The ninth and tenth terms represent correlation between headways and occupancies of adjacent vehicles. The last two terms represent expected value of headway and occupancy.

The matrix for covariances of bus headway that are required in the ninth and tenth terms of (10) can be expressed as

$$\begin{aligned} Q_i^n =& G^n V_{i-1}^n L'^n + G^n W_{i-1}^{j-1} H'^n + G^n Q_{i-1}^{n-1} G^{n^T} \\ & + F^n S^n F^{n^T} + F^n S^n G^{n^T} + (F^n S^n G^n)^T \\ & - F^n S^n F^{n^T} - G^n M_{i-1}^{n-1} F_0^T \end{aligned} \tag{11}$$

where

$$\overline{F}_0 = \begin{bmatrix} b_B & 0 \\ 1 & 1 \end{bmatrix}.$$

In (11), the first three terms show the propagation of covariance along the route. The fourth to sixth terms show the effect of travel-time variation on correlation of headway and occupancy, whereas the last term represents the effect of current headway and occupancy on correlation of two adjacent vehicles.

G. Procedure to Evaluate Steady State Vectors

This section provides a step-by-step procedure to enumerate the steady-state vectors using the model framework described in Section III-D and F.

For the first stop:

1) Define a "passenger vector" that identifies the probability of number of passengers waiting at the first stop as

$$q^1 = \left(q_0^1, q_1^1, q_2^1, \ldots, q_\infty^1\right)$$

where q_j^1 denotes the probability of j passengers waiting at the first stop at the instant boarding process starts.

2) Define a "space vector" for a bus at the first stop at the instant the boarding starts as $s^1 = (s_0^1, s_1^1, s_2^1, \ldots, s_C^1)$. Since the first stop always starts by picking passengers with C number of spaces available, the space vector always has the form: $s^1 = (0, 0, 0, \ldots, 1)$.
3) Set a desired value for the coefficient of variation of dispatch headway (CVDH) (e.g., 0, 0.1, 0.2 etc) at the first stop.
4) Determine the unknown probabilities $q_0^n, q_1^n, q_2^n, \ldots, q_C^n$ at the first stop using (1). The process for determining these probabilities is shown in Appendix F with an example.

5) Determine the variance of occupancy at the first stop using the following [25]:

$$\mathrm{Var}\left(L_i^n\right) = \sum_{k=0}^{C} p_k^n \left(x_k - L_i^n\right)^2. \tag{12}$$

6) Determine the transition probability matrix associated with passenger boarding at the first stop that is represented by B^1.
7) Determine the probability vector p^1 of the bus leaving the first stop as $p^1 = p^{1'} \cdot B^1$. Since no passengers alight at the first stop, the probability vector of passengers remaining on board at the first stop can be written as $p^{1'} = (1, 0, 0, \ldots, 0)$.

Determining the probability vector q^n at the second and subsequent stops:

8) Determine the transition probability matrix associated to the alighting passengers at the second stop, which is represented by A^2.
9) Determine the probability vector $p^{2'}$ of the bus at the second stop by multiplying the onboard passenger vector of the bus leaving the first stop by the alighting transition matrix of the second stop A^2, based on Proposition 1, i.e., $p^{2'} = p^1 \cdot A^2$, where p_1 is the probability vector for onboard passengers at the instant the bus departs from the first stop.
10) Determine the probability vector of the available space on the bus at the second stop, when boarding process starts. Based on Proposition 1 (b)

$$\begin{aligned} s^2 &= \left(s_0^2, s_1^2, s_2^2, \ldots, s_C^2\right) \\ &= \left(p_C^{2'}, p_{C-1}^{2'}, p_{C-2}^{2'}, \ldots, p_0^{2'}\right), \qquad n \geq 2. \end{aligned}$$

11) Determine Q_i^2 using (11).
12) Determine V_i^2 using (10). The first entry of V_i^2 gives the variance of headway at stop 2.
13) Calculate the CVH at stop 2 using (13).
14) Determine the probability vector q_j^2 at the second stop using (1) and a similar process as step 4.
15) Determine the transition probability matrix associated with passenger boarding at the second stop that is represented by B^2.
16) Determine the probability vector p^2 of bus leaving the second stop using $p^2 = p^{2'} \cdot B^2$.
17) Determine the variance of occupancy at the second stop using (12).
18) Repeat steps 8–17 for all successive stops, substituting the vector and matrix subscript "1" with n and "2" with $(n+1)$.

IV. Performance Measures

Using the steady-state probabilities derived earlier, various measures of performance can be calculated.

1) Coefficient of Variation of Headway at Stop n: CVH is defined as the ratio of standard deviation of headway at a stop to mean headway at the same stop [28]. Mathematically

$$\mathrm{CVH}^n = \frac{\sqrt{\mathrm{Var}(H^n)}}{H^n}. \tag{13}$$

2) Estimated Mean Waiting Time at Stop n: The estimated mean waiting time (EMWT) is the mean time that passengers wait for buses with limited carrying capacity at stop n estimated as follows [29]:

$$\mathrm{EMWT}^n = \frac{\mathrm{MNWP}^n}{\left(\lambda^n - \frac{1}{\tau^n}\right)} - \frac{H^n}{2} \tag{14}$$

where MNWP^n represents the mean number of waiting passengers at a stop n and is given by (15), shown at the bottom of the page. The derivation of (15) is shown in Appendix G.

3) Excess Waiting Time at Stop n:

$$\mathrm{EWT} = \mathrm{EMWT} - \mathrm{SWT} \tag{16}$$

where SWT indicates the scheduled waiting time of passengers, which is half of the published headway of buses.

4) Ratio of Lost Work to Demanded Work: A new performance measure, namely, the *ratio of lost work to demanded work* (Lw/Dw) is proposed to explore the impact of bus size and frequency of service on the performance of the transit system. Passengers are intended to travel along the route based on their origin and destination stop. Passengers per kilometer that demanded to travel along the route can be defined as "Demanded Work (Dw)." However, due to unwillingness to wait for the bus, some passengers may leave the system that was also intended to travel along the route based on their origin and destination stop. The passengers per kilometer that was anticipated to travel by leaving passengers can be termed as "Lost Work (Lw)." Thus, Lw/Dw presents how much work is lost from the system for the transit operator in relation to how much work were waiting for service in the system. The higher value of this ratio indicates that a higher amount of work is lost from the system and *vice versa*. Analytically, it is obvious that a

$$\mathrm{MNWP}^n = \frac{\left[\overline{C}^n - \left(\lambda^n - \frac{1}{\tau^n}\right)H^n\right]\left[\sum_{r=0}^{C} s_r^n (C-r)(C-r+1) - C(C-1) + 2\left\{\mathrm{CVH}^n\left(\lambda^n - \frac{1}{\tau^n}\right)H^n\right\}^2 + 2C\left(\lambda^n - \frac{1}{\tau^n}\right)H^n - \left\{\mathrm{CVH}^n\left(\lambda^n - \frac{1}{\tau^n}\right)H^n\right\}^2\left\{1 + \frac{1}{(\mathrm{CVH}^n)^2}\right\}\right]}{2\left[\sum_{r=0}^{C} s_r^n (C-r) - C + \left(\lambda^n - \frac{1}{\tau^n}\right)H^n\right]}$$

$$+ \sum_{i=0}^{C-1} \frac{1}{1 - z_i^n} + \mathrm{CVH}^n\left(\lambda^n - \frac{1}{\tau^n}\right)\frac{H^n}{2} \tag{15}$$

TABLE II
PARAMETERS OF BENCHMARK EXAMPLE BUS ROUTE (HICKMAN, [26])

Stop	Arrival rate (Passenger/ minutes)	Alighting proportion	Travel time (minutes)	Travel time variance ($minutes^2$)
1	0.75	0	-	-
2	1.5	0	5	0.8
3	0.75	0.1	5	0.2
4	3	0.25	5	1
5	1.5	0.25	5	0.4
6	1	0.5	5	0.4
7	0.75	0.5	5	0.4
8	0.5	0.1	5	0.1
9	0	0.75	5	0.6
10	0	1	-	-

lower value of Lw/Dw can be achieved by providing large-size buses. However, providing large-size buses is unproductive for the transit operator since it increases unused capacity of bus.

V. NUMERICAL EXAMPLE

To test the proposed model, this paper utilizes the numerical example presented by Hickman [26]. Suppose a planner wants to examine steady-state performance of a ten-stop system where each stop has an infinite waiting room, and each stop is equally spaced in travel time of 5 min, which results in a one-way travel time of 45 min. The last two columns in Table II give the expected value of travel time between stops and the variance of travel time (in minutes and square minutes, respectively). The frequency of buses is assumed ten buses per hour, resulting in a mean headway of 6 min. It is assumed that passenger arrivals at stops follow a Poisson distribution with mean arrival rate specified in column 2 of Table II. Table II also shows the proportion of passengers that alight in each stop. Moreover, the boarding time and alighting time for each passenger are assumed 0.05 (3 s) and 0.03 min (1.8 s), respectively. To investigate the role of bus sizes on the performance of the transit system, different bus sizes are considered. The CVDH is varied from 0 to 1 with an increment of 0.2. The effects of the bus size are investigated by changing bus capacity in a stepwise manner from 35- to 60-passenger capacity. Bus capacity here includes standing passenger spaces and available number of seats. As the value of MTWW is not known from literature, a sensitivity analysis was conducted to capture the implication of passenger waiting behavior on transit system performances. Values of MTWW were varied from 0 to 60 min with an increment of 2 min. In addition, the bus headway is assumed less than 12 min, where the passenger arrival rate is random [17], [30].

Computations have shown that the variation of bus headway increases as bus moves along the route. Fig. 2 shows that the value of CVH keeps increasing as the bus proceeds along the route. For the first departure stop, this value is CVDH. It is observed that the graphs do not intersect and the separation between graphs grow slightly as downstream stops are considered. Therefore, a high value of dispatch headway variability causes consistently high values of variation of headway at subsequent stops. This displays the knock on pattern of variance of headway as the buses proceed downstream. The lowermost

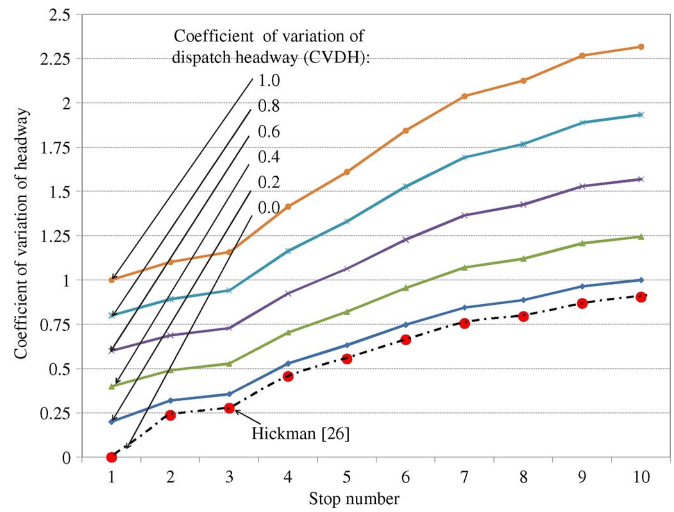

Fig. 2. CVH along the route for MTWW = 30 min and headway = 6 min.

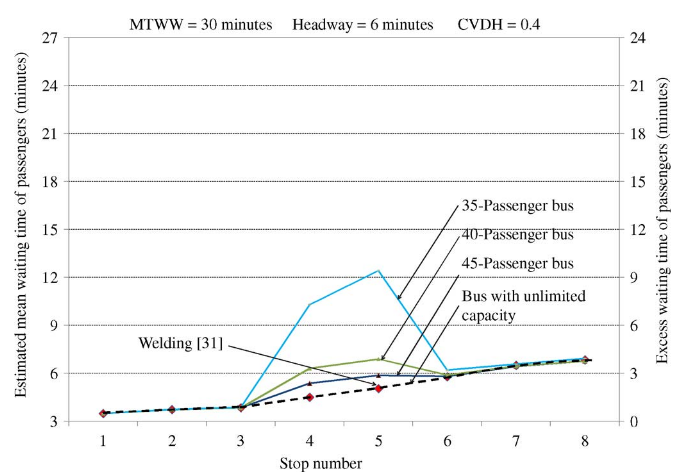

Fig. 3. EMWT and EWT of passengers along the route for MTWW = 30 min, CVDH = 0.4, and headway = 6 min.

graph in Fig. 2 is in agreement with the shape put forward by Hickman [26] for a service with CVDH = 0, i.e., no variation in dispatch headway. This suggests that Hickman [26] model is a specific submodel of the model presented in this paper.

Fig. 3 shows the EMWT and EWT at stops along the route for MTWW = 30 min with a headway of 6 min for a CVDH of 0.4. The EMWT increases at some stops as the bus size decreases. For example, at stop 5, which is the maximum load point, EMWT increases from 5.81 to 7.98 min when the passenger carrying capacity of the bus is reduced from 45 to 40 with CVDH = 0.4. The corresponding values of EMWT and EWT further increase for same bus sizes for an increased value of CVDH = 0.8, as shown in Appendix H. Appendix H also indicates that EMWT and the corresponding EWT increases downstream along the route with the increase in variability of dispatch headway.

The sharp increase in waiting time with low-capacity buses at certain parts of the route is intuitive, as those sections have some passengers being left behind at stops. Such passengers contribute heavily toward the AWT. In essence, the graphs show that the AWT is determined by variability of headway and

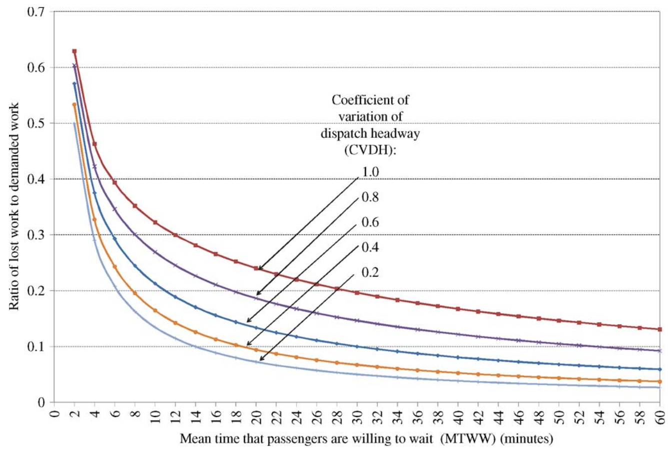

Fig. 4. Lw/Dw for different CVDHs for a 35-passenger bus and headway = 6 min.

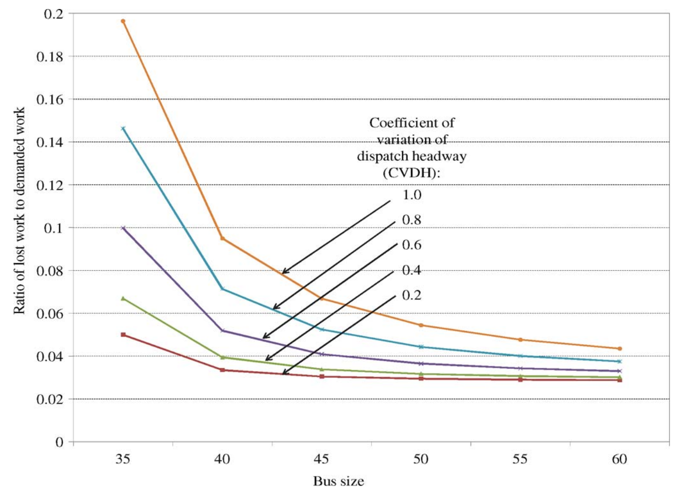

Fig. 5. Lw/Dw for different bus sizes for MTWW = 30 min and headway = 6 min.

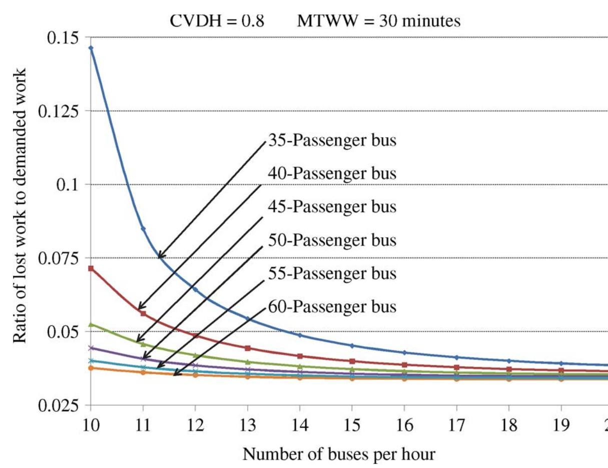

Fig. 6. Lw/Dw as a function of bus frequency for different bus sizes (CVDH = 0.8 and MTWW = 30 min).

denials of boarding when the buses arrive with insufficient capacity. However, the sudden decrease in AWT at a downstream stop, as shown in the figure, is difficult to conceptualize as the conventional wisdom is that improvement from deteriorated conditions of a transit route is usually a gradual process at best. However, the AWT from denial boarding reduces when buses reach a section with a low boarding demand and a high level of alighting. Yet, the AWT increases for downstream stops, as observed in the graphs.

It can be noted here that the EMWT for unlimited capacity of bus is a function of mean headway and headway variation as given by Cham [28] and Welding [31], i.e.,

$$W^n = H^n \times \left[1 + (\mathrm{CVH}^n)^2\right]/2. \tag{17}$$

The proposed model was tested for EMWT in a system where all waiting passengers were accommodated within a large bus; a comparison of the results between the proposed model and (17) did not show significant differences. This can be viewed as a verification of the proposed model with restricted conditions of service with infinite bus capacity. Further verification is provided in Appendix I.

Implication of MTWW on the utilization of the transit system using the Lw/Dw is demonstrated in this paper in three ways: 1) as a function of MTWW, shown in Fig. 4, 2) as a function of the bus size, shown in Fig. 5; and 3) as a function of bus frequency, shown in Fig. 6.

Fig. 4 shows the impact of MTWW on Lw/Dw for a 35-passenger bus with different values of CVDH. It shows that, for 35-passengers bus, the Lw/Dw decreases as the MTWW increases. A high value of MTWW means passengers are willing to wait more time, i.e., a fewer number of passengers abandoned the system. Thus, Lw/Dw acquires a low value with the indication of higher utilization of transit systems. However, it can be also seen that Lw/Dw becomes high for a high value of CVDH for the same value of MTWW. This means higher dispatch variability imposes high loss of passengers from the system.

Fig. 5 shows the impact of bus sizes on Lw/Dw for different values of CVDH for one particular value of MTWW = 30 min. It can be seen that Lw/Dw is low for large-size buses. Moreover, it can be also seen that the results in the graphs become closer to each other for different values of CVDH when the bus size increases. Therefore, it can be concluded that large-size buses can minimize loss of passengers due to unreliability more than the small-size buses.

Fig. 6 shows how the frequency of buses in the system affects the Lw/Dw. The values of Lw/Dw tend to reduce sharply for smaller size buses as the bus frequency is increased. For large-size buses, graphs are somewhat flat. For example, for CVDH = 0.8, the Lw/Dw reduces from 0.146 to 0.085 when the frequency of buses is increased from 10 to 11 buses per hour for a 35-passenger bus, whereas the reduction is small from 0.0376 to 0.0361 with a 60-passenger bus.

VI. Conclusion

The reliability analysis presented in this paper is based on a theoretical model of bus transit operations. The purpose of this paper is to demonstrate its application to evaluate performance of bus operations. In this paper, passengers have been considered to wait a certain amount of time and may look for alternative modes or abandon the trip once their patience run out. Based on passenger behavior, the analysis has investigated the effect of bus dispatch headway variability and effect of bus capacity on service reliability of a high-frequency bus route. The proposed model is able to vary the variability of departure headway of buses at the dispatch station. Numerical results have shown that, for selected route parameters, downstream headway variance increases along the route with a low level of sensitivity to changes of bus capacity. The effect of unreliability in headway of buses along with bus passenger carrying capacity on utilization of public transit systems is demonstrated. It was found that the increase in headway variances in a transit route increases passenger leftover at stops. This increase in number of passengers abandoned is further enhanced by the reduction of bus size. Thus, unreliability reduces utilization of transit systems. Implications of passenger carrying capacity are also taken into account. The model is useful to estimate the performance of transit systems by evaluating the bus headway regularity and passenger occupancy in a realistic manner.

One limitation of this presentation is the lack of observed measurements of MTWW before they abandon the system. Whereas literature demonstrates that transit systems lose patronage due to its unreliable operations, it is yet to quantify how long passengers are willing to wait in real cases. A sensitivity analysis is conducted to alleviate this drawback in this paper. How long passengers will be willing to wait, in real world, require field investigations, and Logit model may be applied to determine appropriate values of MTWW for stops and routes based on their catchment characteristics.

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