Game Theoretic Model for Lane Changing: Incorporating Collision Risks

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ABSTRACT

The study employs a Quantal Response Equilibrium framework to model lane changing manoeuvres. In a Quantal Response Equilibrium payoffs for driver actions are probabilistic, whereby drivers on average have correct beliefs about other drivers' decisions however these beliefs are subject to error. The stochastic formulation reflects drivers having imperfect judgement or vision of others. Prior game theoretic studies in lane changing have pre-eminently assumed Nash equilibrium solutions with deterministic payoffs for actions.

The study method involves developing expected utility models for drivers' merge and give-way decisions. These utility models incorporate explanatory variables representing driver trajectories during a lane changing manoeuvre. The model parameters are calibrated against lane changing data at a freeway on-ramp, and estimated using a maximum likelihood estimation procedure. The calibration data used is the vehicle trajectory dataset collected under the Next Generation simulation (NGSIM) program.

The study was able to develop, calibrate and test a lane changing model with a Quantal Response Equilibrium game solution. It demonstrates QRE as a suitable formulation to model interaction in driver manoeuvres, accounting for drivers' errors in perception. Given this, the QRE interaction framework appears promising to model the efficacy of emerging V2V and V2I communication technologies which provide information to drivers to align their perceptions of stimuli with reality.

Keywords: Game Theory; Quantal Response Equilibrium; Lane Changing; Risk Perception; Driver Behavior

1. INTRODUCTION

In recent years, there has been an increasing focus towards using game theory to model the interdependence of manoeuvres between conflicting drivers in traffic (Barmpounakis et al., 2016; Chatterjee & Davis, 2013; Elvik, 2014; Kita 1999; Liu et al. 2007; Luo et al., 2015; Meng et al. 2016; Talebpour et al. 2015; Wang et al. 2015). The game-theoretic approach assigns a utility to each combination of driver decisions instead of only their disparate individual decisions. This focus on interaction ultimately leads to further insights in traffic safety and operations, in particular the quantification of behavioural norms and moral hazards of interaction.

The dominant assumption in game solutions of driver manoeuvres presented in prior literature is a mathematical Nash equilibrium of interactive behaviour. These studies include those models calibrated against data of observed field interactions (Kita 1999; Liu et al. 2007; Talebpour et al. 2015), those models with arbitrarily specified incentives for choices (Chatterjee & Davis, 2013; Meng et. al. 2016; Prentice, 1974) and purely theoretical models (Pedersen, 2003).

However Nash equilibrium solutions assume drivers have correct anticipations or beliefs of other drivers' decisions. A Quantal Response Equilibrium game solution on the other hand assumes drivers' beliefs are correct on average, however make errors according to a probability distribution (McKelvey and Palfrey, 1995). This formulation may greater reflect real driving behaviour, as it acknowledges errors in perception arising from mistakes in judgement or having imperfect vision of others. In particular, accounting for drivers' stochastic errors in perception may improve modelling of mean and variance in driver interactions. Dixit and Denant-Boemont (2014) showed that Strategic User Equilibrium (analogous to Quantal Response Equilibrium for route choice decisions) is able to accurately model mean and variability in strategic route choice decisions. Outside the driving context, McKelvey and Palfrey (1995) were able to produce Quantal Response Equilibrium estimates of strategies more accurate than Nash Equilibrium estimates.

Apaper by Barmpounakis et al. (2016) presents a Quantal Response Equilibrium in a sequential game abstraction for overtaking manoeuvres. However the study in this paper adopts a different approach to Barmpounakis et al. by inter-relating the decision payoff functions of game players to explicitly account for interactions. Further, when calibrating decision payoffs against observed interactions, the parameters in payoff functions for decisions are calculated simultaneously as game solutions are arrived. The authors of this paper believe it is integral to calculate game payoffs simultaneously with game solutions in order for payoffs to explicitly reflect interactions and not individual decisions.

The study in this paper investigates the efficacy of a Quantal Response Equilibrium solution for lane changing manoeuvres, by first defining the game structure: a simultaneous two-player, non-cooperative, non-zero sum game. Expected utility decision models for merging and give-way behaviour are accordingly developed. A probability distribution is specified for drivers' anticipation for payoffs in the decision models; this anticipation is probabilistic in Quantal Response Equilibrium but deterministic in Nash Equilibrium.

The model is calibrated and tested against a large trajectory dataset collected under the NGSIM program (Federal Highway Administration, 2006). 45 minutes of vehicle trajectory data is used, describing positional information along a section of the Interstate 80 in Emeryville, California. Moridpour et al. (2010) mentions the importance of using large trajectory datasets to improve development of lane changing models.

The study focuses on merging and give-way interactions at freeway on-ramps.

Once the theoretical model is developed and data sample is prepared, the study utilises a maximum likelihood procedure to estimate the Quantal Response Equilibrium model parameters against the observed field interactions. In particular, the study allows driver decision payoff parameters to be heterogeneous across vehicle class types and traffic conditions experienced. This allows for further interaction insights for these subgroups. A fixed point algorithm is used to converge to QRE game solutions. The calibrated QRE models in the study are cross-validated against separate test datasets.

2. GAME THEORETIC REPRESENTATION

2.1 Type of game

The lane changing interaction is modelled as a simultaneous two-player, non-cooperative, non-zero sum game. The game-theoretic studies of Kita (1999), Liu et al. (2007), Meng et. al. (2016) and Talebpour et al. (2015) likewise adopt this structure.

In this study, lane changing interactions between one mainline driver and one on-ramp driver is modelled. Each driver can make either one of two manoeuvres. The on-ramp driver can choose to either ‘merge’ or ‘do not merge’, whilst the mainline driver can choose to ‘give-way’ or ‘do not give-way’. The interaction between these two drivers is modelled, as it is considered dominant over the interaction with any other surrounding vehicles (Kita, 1999).

The lane changing interaction at the on-ramp is represented as a simultaneous game. That is the on-ramp and the mainline players both decide their manoeuvre at the same time. The simultaneous representation is considered because there is a limited amount of time for each player to make their decision given the stimuli provided by each other and surrounding vehicles.

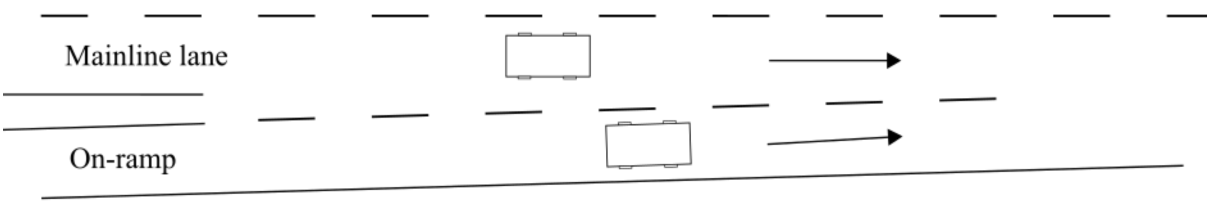

FIGURE 1 Schematic representation of the Powell Street on-ramp along Interstate 80 in Emeryville California; the location of the data collection

2.2 Decision timing

Every instance when the longitudinal coordinate of the rear of an on-ramp vehicle passes the longitudinal coordinate of the front of an adjacent mainline lane vehicle with respect to the direction of travel, is considered as an interaction in this paper (see Figure 2). Hence the instance when the rear of the on-ramp player's vehicle passes the front of the mainline player's vehicle is taken as the decision time. At this time and vehicle positioning it is assumed that these conflicting drivers have already formulated anticipations of each other.

Having a definition for interactions allows for a consistent calibration dataset for the decision models. The traffic conditions at the decision time are input into the lane changing decision models.

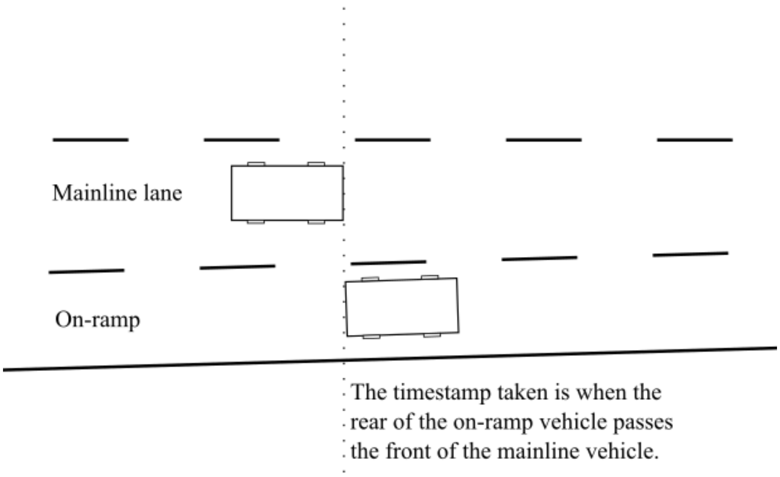

FIGURE 2 Schematic representation of the interacting players.

2.3 Definition of manoeuvres

Mainline player ‘give way’ and ‘do not give way’ decisions are defined based on their acceleration behaviour at the time and positioning in Figure 2; accelerations $\leq -0.25ms^{-2}$ are defined as ‘give way’ manoeuvres, whereas accelerations > $-0.25ms^{-2}$ are defined as ‘do not give way’ manoeuvres. The threshold value $-0.25ms^{-2}$ is taken instead of $0ms^{-2}$ to hedge against noise in acceleration values in the model calibration and verification datasets.

On-ramp player ‘merge’ and ‘do not merge’ decisions are defined on whether they take the gap in front of the mainline player. This lane changing decision by on-ramp vehicles to merge or to not merge is latent in nature, hence taking the earliest interaction timing when the on-ramp vehicle can physically merge in front of the mainline vehicle (as in Section 2.2) allows to account for all potential merge manoeuvres after passing the putative follower, whether they are done immediately, later, or not done.

The normal form of the game is presented in Table 1. The payoff equations are a function of vehicle trajectories, and are discussed in the section following.

TABLE 1 Payoff matrix for the two players

[On-ramp driver payoff], {Mainline driver payoff}

		Mainline driver action	
		Give-way	Do not give-way
On-ramp driver action	Merge	$[0]$, $\{b_0 + b_2.d_{mainline} + b_3.\Delta V_{lm} + b_4.\Delta V_{om}\}$	$[a_1.v_{onramp}^2]$, $\{b_1.v_{mainline}^2\}$
	Do not merge	$[a_0 + a_2.d_{onramp}]$, $\{b_0 + b_2.d_{mainline} + b_3.\Delta V_{lm} + b_4.\Delta V_{om}\}$	$[a_0 + a_2.d_{onramp}]$, $\{0\}$

3. METHODOLOGY

3.1 Payoff model formulation

In Quantal Response Equilibrium drivers make decisions with the lowest perceived costs. These perceptions are subject to error however, and hence drivers behave stochastically against rational expectations.

The models for merge and give-way decisions in this study follow Expected Utility Theory, whereby drivers have decision utilities dependent upon expected actions of conflicting drivers. In this way payoff functions between conflicting drivers are inter-related, and interactions are explicitly accounted for.

The expected utility decision models are shown in equations 1 to 4.

The coefficients of the co-decision utilities are the anticipations or beliefs of the other drivers' decisions, p_{merge} and $p_{giveway}$. p_{merge} is the mainline player's anticipation that the on-ramp player will merge, and $p_{giveway}$ is the on-ramp player's anticipation that the mainline player will give way. The anticipations are probabilities that lie between 0 and 1 inclusive: $0 \leq p_{merge} \leq 1$, $0 \leq p_{giveway} \leq 1$.

Game theoretic decision models for on-ramp player utilities:

$$EU_{merge} = \left(1 - p_{giveway}\right) \times \left(a_1 . v_{onramp}^2\right) \quad [1]$$

$$EU_{donotmerge} = a_0 + a_2 . d_{onramp} \quad [2]$$

Game theoretic decision models for mainline player utilities:

$$EU_{giveway} = b_0 + b_2 . d_{mainline} + b_3 . \Delta V_{lm} + b_4 . \Delta V_{om} \quad [3]$$

$$EU_{donotgiveway} = p_{merge} \times (b_1 . v_{mainline}^2) \quad [4]$$

Where,

v_{onramp}: Velocity of the on-ramp player at decision time

$v_{mainline}$: Velocity of the mainline lane player at decision time

ΔV_{lm}: Velocity difference between the putative leading vehicle on the mainline and the mainline lane player, at decision time

ΔV_{om}: Velocity difference between the on-ramp player and the mainline player, at decision time

d_{onramp}: remaining distance to the end of the acceleration lane for the on-ramp player, at decision time

$d_{mainline}$: remaining distance to the end of the acceleration lane for the mainline lane player, at decision time

$a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4$: parameters to be estimated

The above explanatory variables for player decisions have a Pearson correlation between them of less than 0.5 in the dataset, ensuring a level of independence amongst model variables.

Collision risk and magnitude are characterised by the decision probabilities $p_{giveway}$ and p_{merge}, displacement variable $d_{mainline}$ and the kinetic energy variables $v_{mainline}^2$, v_{onramp}^2. $d_{mainline}$ indicates the risk mitigation employed by the mainline lane driver to cautiously give way to the on-ramp driver in the case they merge earlier than expected. The kinetic energy variables are used to explain the ‘merge’ and ‘do not give way’ co-decision (equations 1 and 4). Kinetic energy is thus used to represent the magnitude of crash consequences.

The payoff functions assume that interacting drivers are also motivated by time savings in addition to minimising collision risks and consequences with each other. Motivation for time savings amongst mainline players is captured by the velocity differential variable ΔV_{lm}, describing the mainline lane vehicle's desire to achieve a suitable car following velocity for its current leader. It is also accounted amongst on-ramp players through d_{onramp}, whereby on-ramp drivers may choose to merge later to reach the front of mainline queues. Therefore as the interacting drivers are motivated beyond collision risks and consequences which affect both players, the game equilibrium is non-trivial (Liu et al. 2007).

The decision models use remaining distance to the end of the acceleration lane as an explanatory variable, similar to Kita (1999) and Liu et al. (2007) who use remaining time. Remaining distance was chosen instead for this study as a large proportion of interaction observations had invalid remaining time values. That is, vehicles had deceleration values at decision time such that a complete stop would be achieved before reaching the end of the acceleration lane, and hence had infinite remaining time to reach the end of the acceleration lane.

3.2 Model estimation and cross validation

The expected utilities for decisions are used as arguments in logit functions to calculate the probability for choices (equations 5 and 6).

$$p_{merge} = \frac{e^{-E[u_{merge}(1-p_{giveway})]}}{e^{-E[u_{donotmerge}]} + e^{-E[u_{merge}(1-p_{giveway})]}} \quad [5]$$

$$p_{giveway} = \frac{e^{-E[u_{giveway}]}}{e^{-E[u_{donotgiveway}(p_{merge})]} + e^{-E[u_{giveway}]}} \quad [6]$$

The anticipations p_{merge} and $p_{giveway}$ in the expected costs (equations 1 and 4) are the same as the choice probabilities estimated by the logit models (equations 5 and 6). This is the premise behind quantal response equilibrium; the perceived probability of other drivers' choices are equal probability of drivers' choices on average, however are subject to some error.

Statistically computing p_{merge} and $p_{giveway}$ are thus fixed point problems of the type p_{merge}= F($p_{giveway}$) and $p_{giveway}$ = H(p_{merge}), where F and H are functions. The probabilities p_{merge} and $p_{giveway}$ are solved iteratively, with seed values of p_{merge} and $p_{giveway}$ used in the expected utility functions to generate the first set of parameter estimates. The first set of parameter estimates are used as inputs for the logit models to generate new values of p_{merge} and $p_{giveway}$. These generated values of p_{merge} and $p_{giveway}$ are then used to update estimates of the parameters, and this method is iterated until the values of p_{merge} and $p_{giveway}$ are converged.

The convergence in this case is a Logit QRE (McKelvey and Palfrey, 1995). Errors in driver perception against choices follow an extreme value distribution. In a Nash Equilibrium, the driver anticipations are correct and equal driver choices with no error.

This logit fixed point QRE convergence method is also displayed in McKelvey and Palfrey (1995), Offerman et al. (1998) and Rogers et al. (2009).

Maximum likelihood estimation is used to solve equations 5 and 6. The most likely parameter values of $a_0, a_1, a_2, b_0, b_1, b_2, b_3$ and b_4 in the EU models are estimated jointly by maximising their fit against observed decisions in the calibration dataset.

The maximum likelihood estimation procedure involved constructing expected utility indices (∇EU) from the above logit models. This index was the difference in expected utility between each of the binary choices.

Expected utility index for the on-ramp driver decisions:

$$\nabla EU_{merge} = (EU_{merge} - EU_{donotmerge}) \quad [7]$$

The log-likelihood to be maximised for the on-ramp driver decisions is thus:

$$LL^{onramp} = \ln L(\, a_0, a_1, a_2 \colon y, X) \quad [8]$$

$$= \sum_i \left[\ln\left(\Phi(\nabla EU_{merge}) \times \boldsymbol{I}(y_i = 1) \right) + \ln\left(\left(1 - \Phi(\nabla EU_{merge})\right) \times \boldsymbol{I}(y_i = 0) \right) \right] \quad [9]$$

Expected utility index for the mainline driver decisions:

$$\nabla EU_{merge} = (EU_{merge} - EU_{donotmerge}) \quad [10]$$

The log-likelihood to be maximised for the mainline driver decisions is thus:

$$LL^{mainline} = \ln L(\, b_0, b_1, b_2, b_3, b_4 \colon y, X) \quad [11]$$

$$= \sum_j \left[\ln\left(\Phi(\nabla EU_{giveway}) \times \boldsymbol{I}(y_j = 1) \right) + \ln\left((1 - \Phi(\nabla EU_{giveway})\,) \times \boldsymbol{I}(y_j = 0) \right) \right] \quad [12]$$

Where y_i and y_j represent the binary choice of a player, ***I*** and ***J*** are indicator functions which take a value of 1 when the condition is satisfied and zero otherwise. ***X*** is a vector of traffic conditions during the lane changing interaction, derived from the NGSIM trajectory data.

The parameters $a_0, a_1, a_2, b_0, b_1, b_2, b_3$ and b_4 were all jointly estimated. The end result is a final log-likelihood to be maximised:

$$LL = LL^{onramp} + LL^{mainline} \quad [13]$$

To estimate the impact of surrounding traffic conditions upon the model parameters, the ML analysis was generalised to allow the core parameters $a_0, a_1, a_2, b_0, b_1, b_2, b_3$ and b_4 to be a linear function of them. The models are extended to be $a_n = a_{n0} + \beta_n \boldsymbol{X}$ and $b_n = b_{n0} + \alpha_n \boldsymbol{X}$, where a_{n0} and b_{n0} are fixed parameters, β_n and α_n are vectors of effects associated the traffic condition variables being represented by $\boldsymbol{X}$ (Table 2), and *n* = [0,1,2] for the on-ramp player and *n* = [0,1,2,3,4] for the mainline player.

Cross-validation was performed to test the efficacy of the QRE framework to model the interactive decisions. The sample was split 70/30; 70% of observations were used for model calibration whilst 30% were used for verification testing.

The fixed-point problem was first solved for the calibration sample. Once the probabilities p_{merge} and $p_{giveway}$ converged, the parameter values $a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4$ were used as inputs to the verification test sample. These $a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4$ values were made fixed and the p_{merge} and $p_{giveway}$ values were left to converge in the verification test sample. In this way, the QRE algorithm was followed for both the calibration and verification datasets.

Cross-validation testing was performed 10 times (the sample was split randomly 70/30 ten times). It is important to note that an equal number of mainline players and on-ramp players were included in each calibration and verification dataset.

4. DATA

The decision models were calibrated against empirical trajectory data of vehicles travelling along a section of the Interstate 80 in Emeryville, California. This data was collected under the Next Generation simulation (NGSIM) program in April 2005. 45 minutes of trajectory was collected, and the whole of this dataset was utilised for the study in this paper.

Figure 3 illustrates the data collection site, and provides a representation of the lane geometry. The on-ramp tapers to join the adjacent mainline lane, forming one lane. Adjacent to the on-ramp and outer mainline lane (not shown in Figure 3) is a shoulder lane.

It is important to note the trajectory data was smoothed according to Thiemann et al. (2008) to address noise in the positional information. In particular, the NGSIM I-80 trajectory data exhibits unrealistic velocity and acceleration distributions with spikes present. The smoothing post-processing was performed before any data analysis.

First, displacement values were differentiated to velocities and accelerations using symmetric difference quotients, then a symmetric exponential moving average filter was applied to these displacement, velocity and acceleration values. The smoothing times for displacement, velocity and acceleration were respectively $T_x = 0.5s$, $T_v = 1s$ and $T_a = 4s$ akin to Thiemann et al. (2008).

There were 735 instances where an on-ramp vehicle passed a mainline vehicle on the adjacent lane. All instances where at least one of these vehicles was travelling less than 10km per hour were removed, as the model was to be calibrated by interactions at speed. Any driver interactions under this 10km/hr threshold speed were assumed irrelevant to unsafe throughput or significant give-way behaviour. Thus a total sample of 397 defined interactions was ultimately used.

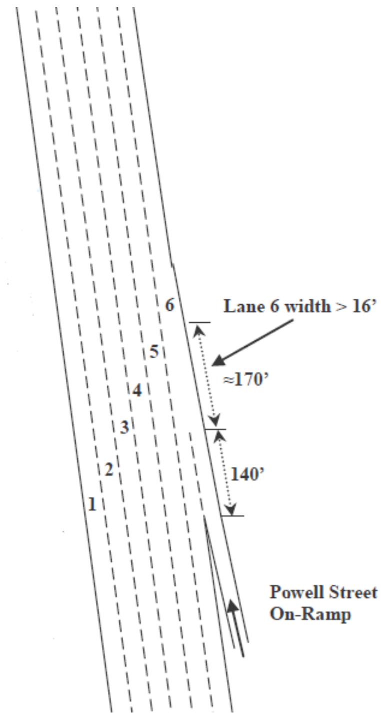

FIGURE 3 Schematic representation of the on-ramp section, with lanes 1 to 6 marked. Measurements are in feet. Source: US DOT FHWA

Descriptive statistics of the dataset are presented in Table 2. The variable values are collected at the time of interaction.

1 **TABLE 2** Descriptive statistics of the total sample; n=397 interactions, 794 players

Variable	Description	Mean	Standard deviation
d_{onramp}	The remaining distance between the front of the on-ramp player to the end of the acceleration lane (m)	45.35	22.96
$d_{mainline}$	The remaining longitudinal distance between the front of the mainline player to the of the acceleration lane (m)	50.08	23.02
d_{lm}	Distance between the leading vehicle on the mainline and the mainline player (m)	10.30	7.03
v_{onramp}	Velocity of the on-ramp player (km/hr)	31.38	12.07
$v_{mainline}$	Velocity of the mainline player (km/hr)	16.65	6.57
v_{leader}	Velocity of the leading vehicle on the mainline (km/hr)	17.18	7.05
ΔV_{lm}	Velocity difference between the leading vehicle on the mainline and the mainline player	0.52	4.07
ΔV_{om}	Velocity difference between the on-ramp player and the mainline player (km/hr)	14.73	9.56
ΔV_{lo}	Velocity difference between the leading vehicle on the mainline and the on-ramp player (km/hr)	-14.20	10.87
a_{onramp}	Acceleration of the on-ramp player (m/s/s)	-0.61	1.29
$a_{mainline}$	Acceleration of the mainline player (m/s/s)	-0.13	0.88
mainline_density	An estimate of vehicle density on the mainline lane (vehicles/km). It is calculated based on distance headways between four vehicles on the mainline; two putative leaders and two putative followers with respect to the on-ramp player	83.68	17.59
Vehicle_length	The length of a vehicle (m)	4.73	1.67
motorcycle	A binary variable taking the value 1 if the vehicle is a motorcycle; 0 otherwise	0.001	0.04
car	A binary variable taking the value 1 if the vehicle is a car; 0 otherwise	0.97	0.16
truck	A binary variable taking the value 1 if the vehicle is a truck; 0 otherwise	0.02	0.15
merge	A binary variable taking the value 1 if the on-ramp player merged in the interaction; 0 otherwise	0.390	0.488
giveway	A binary variable taking the value 1 if the mainline player gave way in the interaction; 0 otherwise	0.539	0.499

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5. RESULTS AND DISCUSSION

TABLE 3 Parameter estimates of the full dataset

	Coefficient	Robust Std. Err	z	P>|z|
$\boldsymbol{a_0}$				
constant	-1.530955	0.40353	-3.79	0.000
$\boldsymbol{a_1}$				
ΔV_{lo}	0.00010	0.00003	4.13	0.000
constant	0.00157	0.00063	2.47	0.014
$\boldsymbol{a_2}$				
constant	0.04316	0.00649	6.65	0.000
$\boldsymbol{b_0}$				
constant	-4.93997	0.70767	-6.98	0.000
$\boldsymbol{b_1}$				
d_{lm}	0.00027	0.00008	3.46	0.001
constant	-0.00998	0.00236	-4.23	0.000
$\boldsymbol{b_2}$				
constant	0.03893	0.00894	4.35	0.000
$\boldsymbol{b_3}$				
constant	-0.17256	0.03903	-4.42	0.000
$\boldsymbol{b_4}$				
constant	0.04088	0.01602	2.55	0.011

The seed values of p_{merge} and $p_{giveway}$ were chosen as 0.4 and 0.5 respectively. These values were closest to the observed mean decision probabilities in the full sample to the nearest 0.1.

Given these initial values the QRE fixed point problem was iterated through 10 times to have the probabilities p_{merge} and $p_{giveway}$ converge. The full sample, and each of the calibration and verification datasets were subject to the convergence.

Each of the 70% calibration samples had estimated values of model parameters, signs and significance levels consistent with those estimated using the full 100% sample. These sets of parameter results from each of the training samples are displayed in the Appendix. Therefore the estimates of model parameters using the full sample are used for discussion of parameter results.

Further, the impact of vehicle trajectories upon game theoretic driver interactions apart from individual manoeuvres is displayed in Figures 4 to 10. These charts illustrate the elasticity of QRE model probabilities to merge and give-way estimated using the full sample. Standardised values of trajectory variables (determined by $(X - b)/a$, where X is the trajectory variable value, b is its average and a is the range) are plotted against p_{merge} and $p_{giveway}$, bracketed at 10% intervals.

The kinetic energy parameter a_1 exhibits heterogeneity when subject to effects from ΔV_{lo}, and similarly so does kinetic energy parameter b_1 with d_{lm} (Table 3). The effect associated with ΔV_{lo} against a_1 suggests that for a given kinetic energy of on-ramp players, greater relative velocity of putative mainline leaders provides further utility to merge. This result is coherent with speed of leaders on the mainline reducing rear-end crash probability and magnitude of crash consequences for on-ramp vehicles looking to merge.

Amongst mainline players, the parameter for kinetic energy b_1 being heterogeneous with d_{lm} suggests for a given kinetic energy, greater gap distance to leading vehicles lowers the probability to give-way. This may be occurring as mainline players desire to maintain a suitable car-following distance with mainline leaders, ultimately providing greater utility to ‘do not give-way’.

The net effect of on-ramp player kinetic energy upon interactions is shown in Figure 4. Higher kinetic energy is correlated to harmonised merge give-way behaviour with $p_{merge} < p_{giveway}$, whereas at lower kinetic energies interactions are incoordinate with $p_{merge} >$

$p_{giveway}$. This may occur as mainline players are less intimidated by slower on-ramp vehicles, whilst on-ramp vehicles are more likely to merge at these lower speeds.

The parameters a_2 and b_2 describing remaining distance to the end of the acceleration lane are both positive. On-ramp vehicles prefer to merge later towards the end of the acceleration lane, whereas mainline players look to give-way earlier. This represents an incoordination, visualised in Figures 6 and 7 where at smaller remaining distances $p_{merge} > p_{giveway}$. On-ramp vehicles may prefer to merge later along the acceleration lane as they look to move up to the front of mainline queues.

A negative coefficient for b_3 indicates mainline players are less likely to give-way with greater relative velocity amongst their leading vehicle. Mainline players desire to maintain a suitable car-following velocity which affects their give-way behaviour. Concurrently, greater relative velocity of leading mainline vehicles may encourage on-ramp players to merge as they anticipate gaps ($p_{merge} > p_{giveway}$ in Figure 9). An indication is uniform mainline speeds encourages safer interaction.

Relative speeds between on-ramp and mainline players also impact the safety of interactions, with Figure 10 displaying conflicts at lower relative velocity.

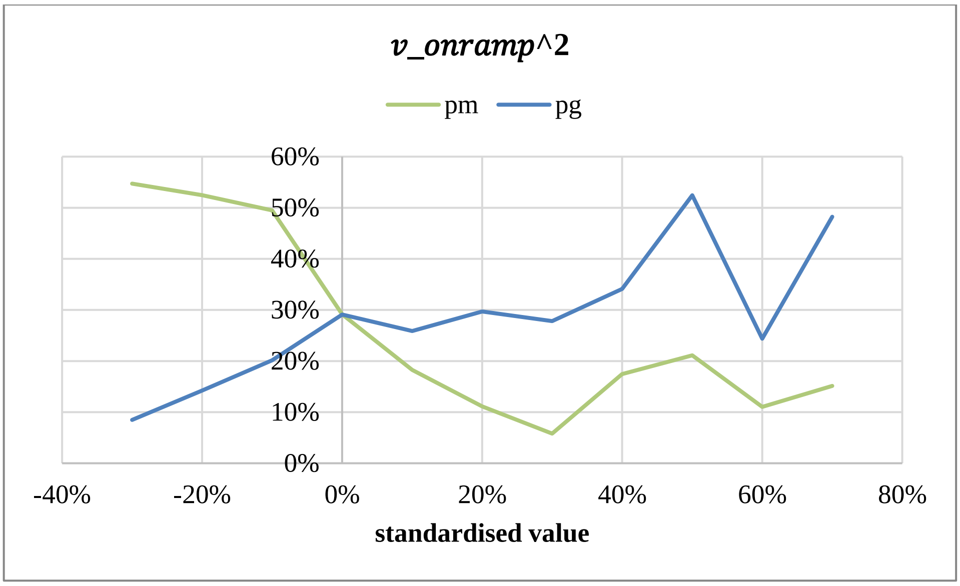

FIGURE 4 Impact of v_{onramp}^2 upon merge give-way interactions

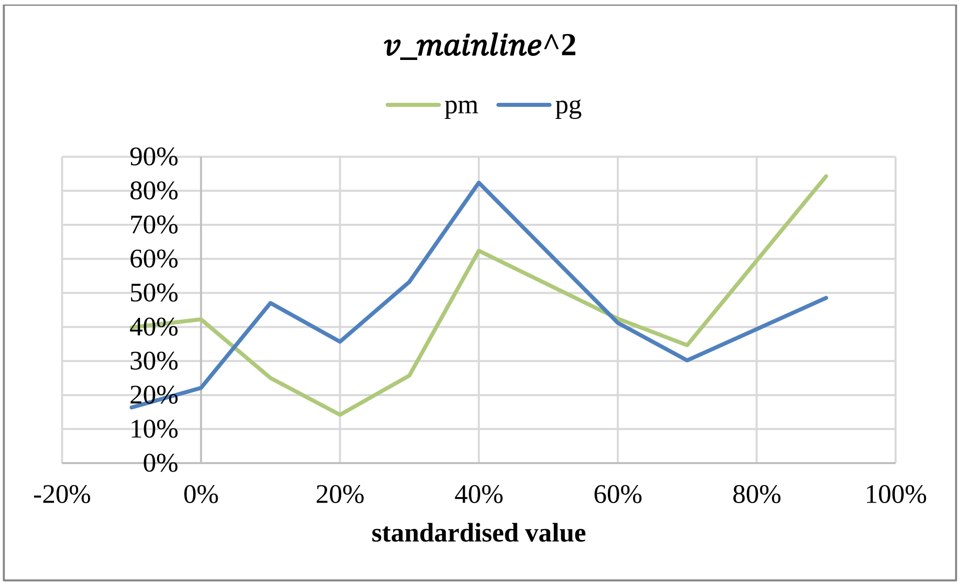

FIGURE 5 Impact of $v_{mainline}^2$ upon merge give-way interactions

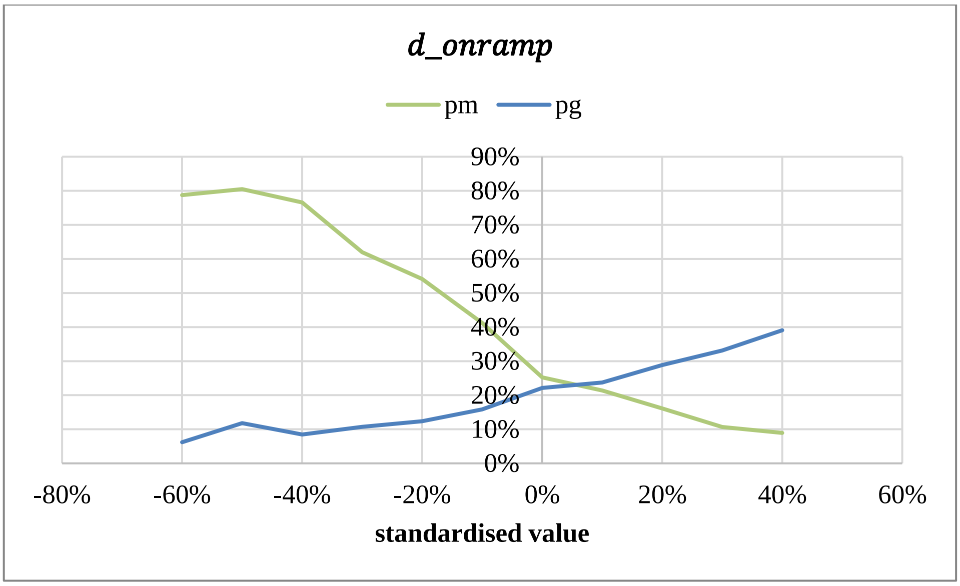

FIGURE 6 Impact of d_{onramp} upon merge give-way interactions

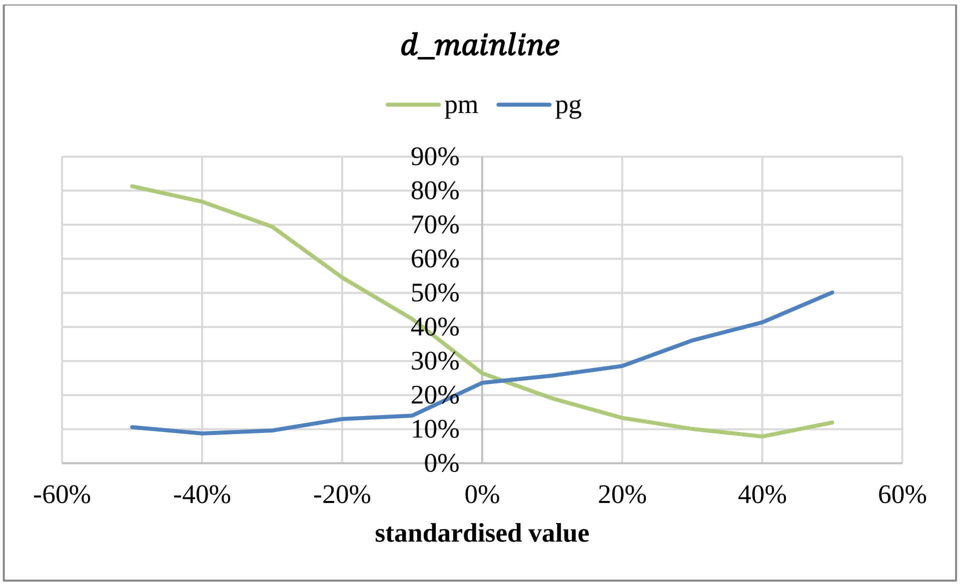

FIGURE 7 Impact of $d_{mainline}$ upon merge give-way interactions

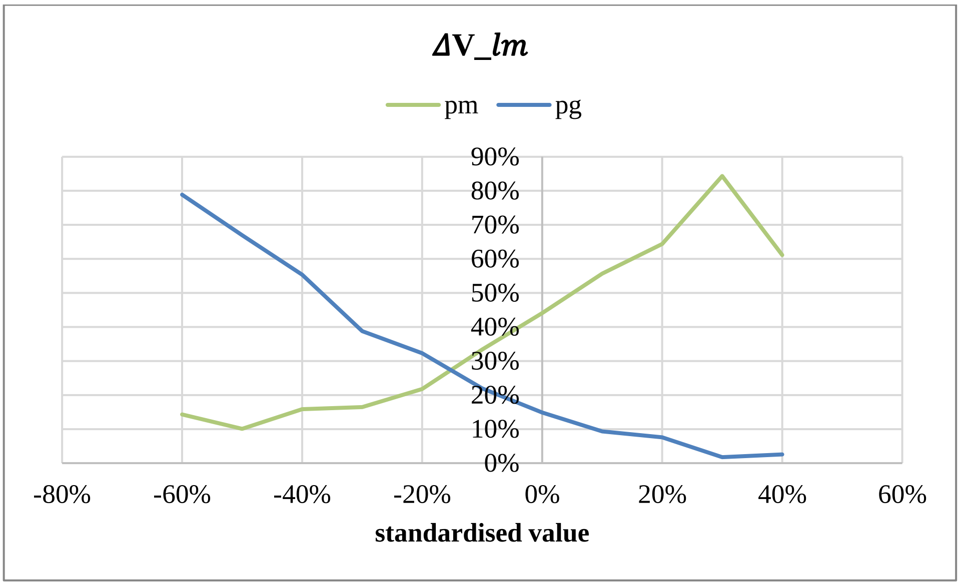

FIGURE 8 Impact of ΔV_{lm} upon merge give-way interactions

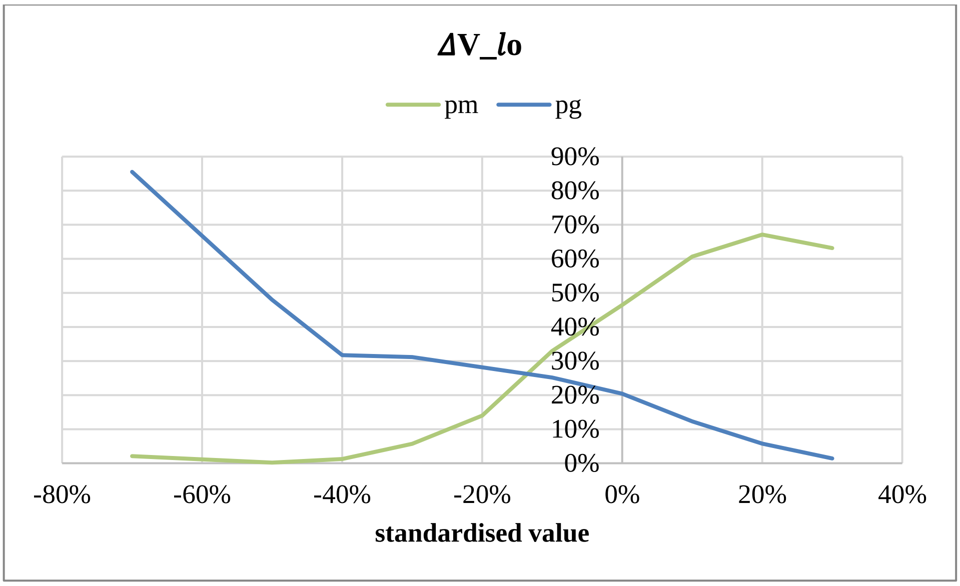

FIGURE 9 Impact of ΔV_{lo} upon merge give-way interactions

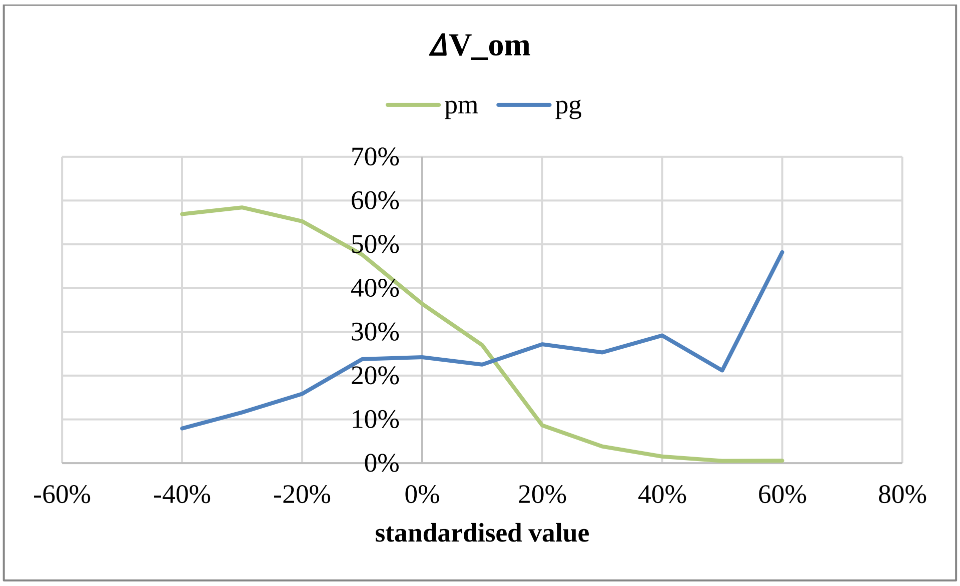

FIGURE 10 Impact of ΔV_{om} upon merge give-way interactions

The 30% verification datasets were used to compare QRE predictions of equilibrium behaviour to the observed equilibrium (Table 4). First, parameter values were arrived using a 70% training dataset. These parameter values were applied to the corresponding 30% testing dataset, whereby the logit functions of Equations 5 and 6 were used to estimate the probabilities p_{merge} and $p_{giveway}$ 'merge' and 'give-way'. The estimated probabilities in the verification testing data arrived by using parameter values from the training data were used to determine the expected number and variance of decisions in the verification testing datasets. The expected number was the number of defined interactions multiplied by the decision probability (Np). The standard deviation in number of decisions was calculated as $\sqrt{Np(1-p)}$. The expected output was compared to reality.

TABLE 4 Comparison of observed equilibrium with QRE. The averaged results derived from ten verification test datasets are presented below.

	On-ramp player merge decisions (# of interactions = 119)		**Mainline player give way decisions (# of interactions = 119)**	
Equilibrium	**Average expected**	**Average stdev**	**Average expected**	**Average stdev**
Observed	45.30	5.27	23.00	4.30
QRE	44.67	5.27	25.44	4.47

The QRE was able to accurately estimate the expected number and standard deviation in interactive decisions. It is demonstrated that a QRE framework can be used effectively to model operational decisions in aggregate. Dixit and Denant-Boemont (2014) were able to show that Strategic User Equilibrium (analogous to Quantal Response Equilibrium for strategic decisions) is able to likewise model mean and variability in strategic route choice decisions.

6. CONCLUSION

The aim of this paper was to assess Quantal Response Equilibrium as a game solution for interactions in lane changing manoeuvres. Prior studies of interaction in manoeuvring decisions have pre-eminently assumed Nash equilibrium solutions to behaviour, with one study testing Quantal Response Equilibrium (Barmpounakis et al., 2016). The Quantal Response Equilibrium approach adopted in the study in this paper inter-relates the utilities of player decisions, and calculates game payoff functions simultaneously as game solutions are arrived. In this way, values for payoff functions explicitly reflect interactions instead of only individual decisions.

QRE assumes drivers have stochastic instead of deterministic perceptions of competing players' decisions. This differentiates QRE from Nash Equilibrium. In particular, the equilibrium modelled in the study was that of merging and give way decisions at a freeway on-ramp. The calibration and verification data used against the proposed model was the NGSIM trajectory dataset collected in April 2005 (Federal Highway Administration, 2006). The lane changing interactions were identified in the trajectory dataset using an automated manner.

The decision models developed in the study incorporate incentives for time savings, and collision avoidance. Therefore as players are motivated beyond collision risks, the QRE equilibrium game solution achieved is non-trivial (Liu et al. 2007).

The parameter estimation approach allowed for payoff function parameter estimates to be heterogeneous across trajectory variable effects, allowing for interaction insights across these effects. In particular, the study found that interactions were affected by velocities and gap distances in the mainline. Mckelvey et al. (2000) and Rogers et al. (2009) adopt a similar approach where behavioural attributes across agents are allowed to be heterogeneous, finding improved QRE model estimation.

The study finds through cross-validation testing that QRE is able to accurately model not only means but also variance in choices. It demonstrates QRE as a suitable theoretical framework to model operational decision making. QRE takes into account errors in perception of other drivers' payoffs, whether they are caused by mistakes in judgement or lack of vision. With the advent of V2V and V2I communication technologies to improve driver awareness, future studies may test QRE game solutions to model their improvements to driver perceptions and safety.

This study builds upon the existing methods to mathematically calculate game solutions in driver manoeuvres. Future studies may investigate applying QRE as a game solution for modelling interaction in a range of driving scenarios.

Limitations in this study include those elements of game model formulation highlighted by Zhang et al. (2010). These are the identification of players and their strategy sets. For one, the number of interacting players analysed in this study was only two, however future research may consider more players. Mainline players could have their game strategy set expanded to include a manoeuvre 'change lane' as in Talebpour et al. (2015), ancillary to their acceleration behaviour. However, limitations in the number of explanatory variables in the dataset and the data sample size were barriers to its inclusion.

Furthermore, defining the time of an interaction instance and what qualifies as an action is an arbitrary practice within game theoretic research for driving manoeuvres. Establishing definitions that yield the best model fit to reality is integral. Future studies for instance may investigate alternative timings to define when an interaction takes place, such as when the front of the on-ramp vehicle matches the same longitudinal position as the mainline vehicle.

Moridpour et al. (2010) mentions it is important to improve lane changing decision models for trucks. In the full sample used for the investigation presented in this paper, only 19 trucks were observed out of 794 total vehicles which rendered insignificant any statistical analyses for the subgroup. If the sample size were large enough in the dataset for trucks, their parameter values for interactive merge or give way decisions could easily be made as heterogeneous effects with the model estimation method presented within this chapter. Subsequent investigations may test for statistical differences in truck interactive behaviour compared to other vehicle classes if vehicle trajectory datasets include large enough samples.

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APPENDIX Parameter estimates of the 10 training datasets

	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	ΔV_{lo} (vector of effect for $\boldsymbol{a_1}$)	$\boldsymbol{a_2}$	$\boldsymbol{b_0}$	$\boldsymbol{b_1}$	d_{lm} (vector of effect for $\boldsymbol{b_1}$)	$\boldsymbol{b_2}$	$\boldsymbol{b_3}$	$\boldsymbol{b_4}$
1	-1.075221**	0.0028097*	0.0001513*	0.0398763*	-4.272658*	-0.0087288*	0.0002435*	0.0319793*	-0.13841*	0.036291**
2	-0.9705581**	0.0024342*	0.0001215*	0.0407284*	-5.519255*	-0.0140168*	0.0006422*	0.0473344*	-0.1371338*	0.0449081**
3	-1.633824*	0.0015744**	0.0001108*	0.0496235*	-5.092362*	-0.010291*	0.0002719*	0.0424085*	-0.1933731*	0.0339948***
4	-1.293333*	0.0018427**	0.0001175*	0.0364012*	-5.061979*	-0.0099956*	0.0002349*	0.0385667*	-0.1679707*	0.0431448**
5	-1.305078*	0.0017756**	0.0001041*	0.0396278*	-4.204278*	-0.0086193*	0.0002676*	0.03176*	-0.1655011*	0.02888
6	-1.962715*	0.001185***	0.0000918*	0.0492981*	-5.160431*	-0.0099157*	0.0003275**	0.0365974*	-0.21866*	0.0609013*
7	-1.781591*	0.0014233**	0.0000855*	0.048948*	-4.452004*	-0.0104964*	0.0004486**	0.0323207*	-0.1682229*	0.0406523**
8	-1.524949*	0.0015176***	0.0000966*	0.0484909*	-4.658318*	-0.0073638**	0.0001868***	0.0386258*	-0.1567176*	0.0382055**
9	-1.323212*	0.0021935*	0.0001274*	0.04126*	-4.909822*	-0.0125107*	0.0004136*	0.0345682*	-0.2114646*	0.043576**
10	-1.908911*	0.00120	0.0000916*	0.0514509*	-4.743899*	-0.0099046*	0.0002643**	0.0392787*	-0.172561*	0.0319741***

***0.10 significance level

**0.05 significance level

*0.01 significance level